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Departures from the Eightfold Way. II. Baryon Electromagnetic Masses

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The leading nontadpole contributions to the electromagnetic mass splittings of the baryons are calculated. Best fits to experiment are used for the nucleon form factors; the strange baryon form factors are obtained from these by unitary symmetry. Account is taken of the breakdown of unitary symmetry by using experimental masses in intermediate states and by including the effects of $\theta-\omega$ mixing. These calculations and the assumption of tadpole dominance are used to fit the six mass splittings of baryons and pseudoscalar mesons with one free parameter. All of the splittings are in good agreement with experiment except for the kaon splitting, which has the right sign but only $\frac{2}{3}$ of the proper magnitude. The results of a preliminary version of this calculation were reported in the first paper in this series.

I. INTRODUCTION

SEVERAL years ago, Cini, Ferrari, and Gatto[1] obtained a formula for the electromagnetic self-mass of the nucleon in terms of its electromagnetic form factors.[2] The formula is represented diagrammatically by Fig. 1, where the blobs represent form factors, and analytically by the expression

$$\bar{u}\delta m u = \frac{-ie^2}{(2\pi)^4}\bar{u}\int \frac{d^4k}{k^2+i\epsilon}\left[F_1(k^2)+\frac{i}{2m}F_2(k^2)\sigma_{\mu\nu}k_\nu\right] \times\frac{p_\lambda\gamma_\lambda+k_\lambda\gamma_\lambda+m}{k^2+2p\cdot k+i\epsilon}\left[F_1(k^2)-\frac{i}{2m}F_2(k^2)\sigma_{\mu\nu}k_\nu\right]u, \quad (1)$$

where m is the mass of the nucleon, e is its charge, and F_1 and F_2 are the form factors, normalized such that $F_1(0)$ is one and $F_2(0)$ is the anomalous moment in nuclear magnetons.

Equation (1) is obtained by writing the self-mass in terms of nucleon-photon scattering (with unphysical photons), and then approximating the scattering amplitude by the contribution from one-nucleon intermediate states (pole term). It has been often applied to calculations of the neutron-proton mass difference, with a notorious lack of success.[1,3] Several reasons have been advanced for this failure:

(1) Equation (1) neglects the contributions from intermediate states heavier than one nucleon. Perhaps these contributions are large.[4]

(2) Equation (1) is critically dependent on the behavior of the form factors at high momentum transfers, of which we know nothing. Indeed, if we use form factors with hard cores, the formula diverges quadratically. In order to obtain convergent results, we must use a cutoff; widely different mass differences may be obtained, depending on the choice of cutoff.[5]

(3) Equation (1) neglects the contributions from possible scalar meson tadpole diagrams.[6] This is not, as might appear, a special case of the intermediate-state criticism stated above; the scalar tadpoles simply add a constant to the unphysical photon-nucleon scattering amplitude, and make no contribution to the absorptive part.

This third viewpoint was proposed in the first paper in this series, and it is the one we adhere to here. The main body of this paper (Sec. II) is a calculation of the leading nontadpole contributions to the electromagnetic mass splittings, not only of the nucleon, but of all the baryons, using Eq. (1). Experimental best-fit form factors are used for the nucleon,[7] and unitary symmetry is used to obtain the strange baryon form factors from these.[8] Account is taken of departures from unitary symmetry by using physical masses for intermediate states and by correcting the form factors for ϕ-ω mixing.[9]

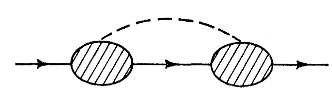

FIG. 1. The leading nontadpole contribution to the electromagnetic mass splittings of the baryons. The blobs represent electromagnetic form factors.

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† Partially supported by the National Science Foundation.

[1] M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters **2**, 7 (1959).

[2] A closely related formula had been obtained earlier from a somewhat different view point by R. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954).

[3] S. Sunakawa and K. Tanaka, Phys. Rev. **115**, 754 (1959).

[4] W. N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963). Of course, the contribution from the next state, the 3–3 resonance, vanishes because of isospin symmetry; but still higher resonances may make nonvanishing contributions. The contributions of other members of the $3/2^+$ decuplet to other baryon splittings do not vanish because of symmetry; preliminary calculations indicate that they may be significant [R. Socolow (private communication)].

[5] L. Pande, Nuovo Cimento **26**, 1063 (1962).

[6] S. Coleman and S. Glashow, Phys. Rev. **134**, B671 (1964).

[7] L. Hand, D. Miller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963).

[8] S. Coleman and S. Glashow, Phys. Rev. Letters **6**, 423 (1961).

[9] S. Coleman and H. Schnitzer, Phys. Rev. **134**, B863 (1964).

TABLE I. Calculated values of the leading nontadpole contributions to electromagnetic mass splittings, in MeV. "Electric-electric" indicated that term in the mass splitting which is quadratic in the electric form factor, etc. The nucleon form factors used in the calculation are best fits to experimental data. The strange baryon form factors are derived from these by unitary symmetry.

	Electric-electric	Electric-magnetic	Magnetic-magnetic	Total
p-n	1.0	0.0	0.1	1.1
Σ^+-Σ^0	1.1	−0.4	0.0	0.7
Ξ^--Ξ^0	1.15	0.05	0.05	1.25
Σ^--Σ^0	1.1	0.1	0.2	1.4
π^+-π^0	...	...	...	5.0[a]
K^+-K^0	...	...	...	2.3[a]

[a] According to R. Socolow.

Since the current experimental form factors do not have a hard core, the integrals are unambiguous and contain no free parameters. A cutoff is neither needed nor used.

We offer no *a priori* reasons why the first and second possible sources of error should be small. The only cause we have to believe them small is that the numbers we obtain by neglecting them are in reasonable agreement with experiment.

II. CALCULATIONS

In this section we calculate the leading nontadpole contributions to the baryon electromagnetic mass splittings. The most recent fit to experimental form factors is that of Hand, Miller, and Wilson.[7] These authors have improved the analysis of the data by studying the charge and magnetic moment distributions, G_E and G_M, rather than the form factors,

$$G_E(k^2)=F_1(k^2)+(k^2/4m^2)F_2(k^2)\,, \qquad (2a)$$

and

$$G_M(k^2)=F_1(k^2)+F_2(k^2)\,. \qquad (2b)$$

Hand *et al.* obtain expressions for the isoscalar (isovector) G's in terms of two poles: the usual $\omega(\rho)$ pole and a "soft core" pole with a mass of 30 $(\text{fermi})^{-2}$. If these expressions were inserted directly into Eqs. (2), we would find expressions for the foım factors in terms of three poles: the two above, and a new pole at $4m^2$. The new pole is an artifact introduced by the method of analysis. If we were to take it seriously, the unitarity relation for the form factor would imply, quite independently of the experimental data with which we began the analysis, the existence of a vector resonance with mass equal to twice the nucleon mass. Instead of following this naive procedure, we use two-pole expressions for the form factors, with residues selected such that our expressions agree with the values and first derivatives at the origin of the form factors of Hand *et al.* In this way we obtain the following expressions:

$$F_{1S}=\frac{0.55}{q^2-1.3}-\frac{0.63}{q^2-0.69}\,, \qquad (3a)$$

$$F_{1V}=\frac{0.35}{q^2-1.3}-\frac{0.49}{q^2-0.63}\,, \qquad (3b)$$

$$F_{2S}=\frac{0.42}{q^2-1.3}-\frac{0.18}{q^2-0.69}\,, \qquad (3c)$$

$$F_{2V}=\frac{2.8}{q^2-1.3}-\frac{2.5}{q^2-0.63}\,, \qquad (3d)$$

where momentum is measured in units of nucleon masses. We may then calculate the form factors of the strange baryons using the predictions of unitary symmetry.[8]

The calculation of the mass splitting from these form factors is best done by assuming at first that the mass of the intermediate Λ that occurs in the Σ^0 self-energy is the same as that of the Σ. Then we may straightforwardly use the results of Cini *et al.*[2] to compute the electromagnetic self-masses. The results are displayed in Table I. To complete the calculation we must correct for the $\Lambda-\Sigma$ mass difference. The correction term is the difference of two diagrams of the form of Fig. 1, with the external mass the Σ mass, the blob the Σ^0 form factor, and the internal mass the Σ mass in one case and the Λ mass in the other. Unitary symmetry tells us that the Σ^0 form factor is equal to the neutron form factor; since the neutron electric form factor is small, we need only consider the part quadratic in F_2.

If

$$z=1-m(\Lambda)^2/m(\Sigma)^2\,, \qquad (4)$$

then, to first order in z, the correction term is given by the expression

$$\frac{m(\Sigma)ze^2}{128\pi^4}\int dxdy\,\rho(x)\rho(y)(x-y)^{-1}[f(x)-f(y)]\,, \qquad (5)$$

where ρ is the spectral function for the magnetic

TABLE II. Calculated values of the leading nontadpole contributions to electromagnetic mass splittings. Everything is as in Table I except that account has been taken of ω-ϕ mixing in calculating the strange baryon form factors.[a]

	Electric-electric	Electric-magnetic	Magnetic-magnetic	Total
p-n	1.1	0.2	0.1	1.4
Σ^+-Σ^0	1.0	−0.9	0.0	0.1
Ξ^--Ξ^0	1.75	−0.4	0.25	1.6
Σ^--Σ^0	1.5	0.3	0.1	1.9
π^+-π^0	...	...	...	4.5[b]
K^+-K^0	...	...	...	2.4[b]

[a] In order to do this, the isoscalar nucleon form factors had to be fitted to a ϕ pole and a ω pole, instead of to the empirical curves used in the preceding calculation. This is the origin of the slight shift in the calculated nucleon splitting.
[b] According to R. Socolow.

TABLE III. Calculated and observed values of electromagnetic mass splittings, in MeV.

	Theoretical value (without mixing)			Theoretical value (with mixing)			
	Nontadpole contribution	Tadpole contribution	Sum	Nontadpole contribution	Tadpole contribution	Sum	Experiment
p-n	1.1	−2.8	−1.7	1.4	−2.6	−1.2	−1.3
Σ^+-Σ^0	0.7	−4.2	−3.5	0.1	−3.9	−3.8	−3.6±0.5
Ξ^--Ξ^0	1.25	5.6	6.8	1.6	5.2	6.8	6.1±1.6
Σ^--Σ^0	1.4	4.2	5.6	1.9	3.9	5.8	4.5±0.4
π^+-π^0	5.0	0	5.0	4.5	0	4.5	4.6
K^+-K^0	2.3	−5.1	−2.8	2.4	−4.7	−2.3	−3.9±0.6

moment,[1]

$$f(y)=y+y^2+\tfrac{3}{2}y\ln y-y^2w(y)\,, \qquad (6)$$

and

$$w(y)=[y(4-y)]^{-1/2}\arctan[(4-y)/y]^{1/2} \qquad (7)$$

for $y^2<4$, the only range of interest to us. The calculated value of this quantity is 0.03 MeV—too small to have any effect on our result.

We also calculate the mass splittings taking account of $\omega-\phi$ mixing. We replace the soft core at 30 (fermi)$^{-2}$ in the nucleon isoscalar form factors by a pole at the ϕ mass, but retain the soft core in the isovector form factors. The new isoscalar form factors are

$$F_{1S}=\frac{-0.68}{q^2-0.69}+\frac{0.58}{q^2-1.15}\,, \qquad (3a')$$

and

$$F_{2S}=\frac{-0.23}{q^2-0.69}+\frac{0.43}{q^2-1.15}\,. \qquad (3c')$$

These fit the data as well as do Eqs. (3). The conventional particle mixing approximation is unsatisfactory for calculating the strange baryon form factors; it leads to the appearance of hard cores, which causes the self-energy integrals to diverge. Instead, we use the vector mixing approximation,[9] which does not have this deficiency. Then all we need to find the strange baryon form factors from those shown above is a single mixing parameter, which we have found elsewhere[9] (from Gell-Mann–Okubo-like arguments and the masses of the vector mesons) to be 29°. The electromagnetic mass splittings calculated from form factors obtained in this way are shown in Table II.

We do not feel that this calculation is appreciably more trustworthy than the preceding one. However, we feel that comparison of the two offers some insight into the sensitivity of this sort of computation to the methods used to estimate the strange baryon form factors.

Socolow[10] has performed the corresponding calculations for pseudoscalar mesons, assuming the pion form factor is given completely by a pole at the mass of the ρ. His results are shown in Tables I and II. His calculation, unlike ours, includes the contribution from the next heaviest intermediate state (one photon and one vector meson).

III. CONCLUSIONS

In the first paper in this series,[6] it was shown that the assumption of tadpole dominance led to expressions for the tadpole contributions to the electromagnetic masses of the strongly interacting particles in terms of their medium-strong mass splittings and one free parameter (the ratio of the π' tadpole to the η' tadpole). Table III shows theory and experiment[11] for some electromagnetic splittings, using the estimates of the nontadpole contributions obtained in Sec. II. The unknown parameter is adjusted to give the best fit to the data. *This table replaces the similar tables in Ref. 6.* The earlier tables were based on a preliminary version of our calculation which contained several numerical errors. As the reader may see from the table, we are able to obtain good agreement for all four of the baryon splittings as well as the pion splitting. For the kaon splitting, however, we only obtain $\frac{2}{3}$ of the proper value.

There is another way of expressing the same results, which perhaps reveals more clearly the possible sources of error. The expressions for the tadpole parts of the mass splittings may be used to write five formulas[6] for the splittings. Three of these, the intramultiplet formulas, involve electromagnetic splittings only. The remaining two, the hybrid formulas, involve both electromagnetic and medium-strong splittings. Since we have taken account of the nontadpole contributions to the electromagnetic splittings, but not to the medium-strong ones, we would expect our calculation to improve the agreement of the intramultiplet formulas with experiments much more than that of the hybrid formulas. This is indeed the case: If we insert the results of Tables I and II in these formulas, we find that they are all in agreement with experiment except for one of the hybrid formulas [(Eq. (10) of Ref. 6], for which

[10] R. Socolow, thesis, Harvard University, 1964 (unpublished).

[11] Our experimental masses are from H. Barkas and A. Rosenfeld, University of California Radiation Laboratory Report UCRL-8030 (rev.) (unpublished), except for the Ξ splitting, which is a private communication from H. Ticho.

one side is twice the other; this is what is responsible for our poor prediction of the kaon splitting.

Our estimate of the nontadpole contributions to the baryon self-masses depends on the input experimental form factors and on the method chosen to calculate the strange baryon form factors. Judging by the differences between Tables I and II, and also by several model calculations we have done, we feel that the sensitivity is such that our final results are not trustworthy to within more than one MeV. Any accuracy greater than this displayed by Table III is probably only coincidence.

We believe that the agreement we have obtained between theory and experiment offers considerable support both to the notion of tadpole dominance and to our policy of neglecting the first two possible sources of error cited in Sec. I, and that methods similar to those used here should give results of similar accuracy for other electromagnetic mass splittings (e.g., those within the Y_1^* multiplet) and mass-like electromagnetic transition matrix elements (e.g., that for the two pion decay of the ω).

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Threshold Regge Poles and the Effective-Range Expansion*

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It is shown, without any reference to potentials, that an infinite number of Regge poles approach $l=-\frac{1}{2}$ in the limit of zero energy. Unitarity and the assumption of a Sommerfeld-Watson transform play the crucial role. As a by-product an improved effective-range expansion is obtained.

It is well known in potential theory that an infinite number of Regge poles arrive at $l=-\frac{1}{2}$ as $\nu\rightarrow 0$, ν being the square of the c.m. momentum.[1] We shall show below that even without any reference to potentials this result will hold provided: (i) Partial wave amplitudes for a fixed energy are analytic except for a finite number of poles and branch cuts in the region to the right of $\mathrm{Re}\,l=-\frac{1}{2}-\epsilon$, where ϵ is an arbitrarily small positive number. (ii) There exists a Sommerfield-Watson transform in the same region. As a by-product we obtain an improved effective-range expansion.

For simplicity we shall consider the Mandelstam representation to hold even though it is possible that our results may be true also for a more complicated singularity structure.[2] The reduced partial wave amplitude $A(\lambda,\nu)$ satisfies the generalized unitarity relation[3]

$$A^{-1}(\lambda,\nu)-A^{*-1}(\lambda,\nu)=-2i\nu^{\lambda};\quad \lambda=l+\tfrac{1}{2},\quad \nu\geq 0. \quad (1)$$

For all real λ, A is real in the region between threshold and the branch point of the left-hand cut. One can, therefore, write[3,4]

$$A^{-1}(\lambda,\nu)=R(\lambda,\nu)+(\nu^{\lambda}e^{-i\pi\lambda}/\sin\pi\lambda)\,, \quad (2)$$

where $R(\lambda,\nu)$ has the left-hand cut and also the right-hand cuts beginning at the thresholds of inelastic channels.

Now, in the absence of cancellation from the first term, the second term in the right-hand side of (2) would make $A(\lambda,\nu)$ vanish at integral values of λ *independently* of the value of ν. This would be a rather extraordinary situation. In fact it follows from Carlson's theorem[5] and the assumption (ii) above that $A(\lambda,\nu)$ must then vanish identically. The only way to avoid this is for $R(\lambda,\nu)$ to supply the necessary terms that cancel the poles coming from $\sin\pi\lambda$. Separating out the pole parts, we have for small ν,[6]

$$R(\lambda,\nu)=\bar{R}(\lambda,\nu)-\sum_{n=0}\frac{\nu^n}{\pi(\lambda-n)}\,, \quad (2)$$

$$A^{-1}(\lambda,\nu)=\bar{R}(\lambda,\nu)-\sum_{n=0}\frac{\nu^n}{\pi(\lambda-n)}+\frac{\nu^{\lambda}e^{-i\pi\lambda}}{\sin\pi\lambda}\,. \quad (4)$$

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[1] B. R. Desai and R. G. Newton, Phys. Rev. **129**, 1445 (1963); and **130**, 2109 (1963); V. N. Gribov and I. Ya. Pomeranchuk, Phys. Rev. Letters **9**, 238 (1962), had independently indicated the existence of these poles.

[2] The information we really need is that $R(\lambda,\nu)$ in (2) can be expanded in a power series in ν, for $\mathrm{Re}\lambda>-\epsilon$.

[3] For the relativistic case, where one considers the invariant amplitude, the right-hand side should be multiplied by an appropriate energy-dependent factor. If the masses are equal $(=m)$ this factor would be $(\nu+m^2)^{-\frac{1}{2}}$.

[4] See, for instance, A. O. Barut and D. E. Zwanziger, Phys. Rev. **127**, 974 (1962).

[5] See E. C. Titchmarsh, *The Theory of Functions* (Oxford University Press, London, 1939), 2nd ed., p. 186.

[6] One actually requires all but a finite number of poles to be canceled so that some of poles coming from $\sin\pi\lambda$ may remain. Such cases, however, should be considered accidental.