magnetic contribution at the resonance[7] one may compare directly the $e-W_0$ and $\mu-W_0$ coupling constants. This would provide, therefore, a clear test of the universality of the weak interactions.

We have also considered the possibility that a similar resonant effect could be produced by a boson coupled to the electron-positron field electromagnetically (similar to the ρ or ω coupling to photons). However, our estimate shows that because of its much larger width ($\Gamma \gtrsim 10$ MeV) its effect is negligible compared to the Bhabha cross section.

Note added in proof. Dr. R. Gatto has kindly informed us (private communication, 10 June 1964) that the processes $e_+ + e_- \to W_0 \to e_+ + e_-$ and $e_+ + e_- \to W_0 \to \mu_+ + \mu_-$ have already been suggested [N. Cabibbo and R. Gatto, Phys. Rev. **124**, 1577 (1961), Sec. 7]. The brief calculations done earlier are essentially in agreement with the results given above. Dr. Gatto has also conveyed to us the information that recent work at Frascati indicates their Adone storage ring should be capable of a resolution of 0.5 MeV at 2-BeV total energy. The consequence of such an improved resolution would be to multiply the result given in Eq. (3) above by a factor of 4.

[7] Unlike the W_0 mediated processes, the electromagnetic contributions are different for the muon and electron final states.

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Dynamics of the $d_{3/2}$ Unitary Multiplets

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A dynamical model of pseudoscalar meson-baryon scattering in the $J^P = 3/2^-$ state is proposed to support the conjecture of Glashow and Rosenfeld that the $N^{**}(1512)$ resonance is a member of a unitary-symmetry octet. The dynamical mechanism analyzed here is based on the Cook-Lee model of the higher pion-nucleon resonances. It is shown that the coupling to inelastic vector meson-baryon states in a generalized two-channel formalism yields an octet of $d_{3/2}$ resonances and a unitary singlet as well. The baryon, pseudoscalar-meson, and vector-meson octets are each assumed degenerate, so that the Cook-Lee model is immediately adaptable to an analysis of resonant unitary multiplets as a function of f, the Yukawa mixing parameter. It is found that, for $f = 0.326$, the attraction is slightly greater for the octet than for the singlet. The 2×2 octet amplitude is diagonalized by a rotation through an angle $\theta^* = 45°$; the Yukawa mixing parameter is $f^* = 0.428$. The $N^{**}N\pi$ coupling constant is computed to be $g^{*2}/(4\pi) = 0.150\ m_\pi^{-2}$, which may be compared with the observed N^{**} width.

I. INTRODUCTION

THE conjecture has been made by Glashow and Rosenfeld[1] that the $N^{**}(1512)$ resonance is a member of a unitary-symmetry octet with spin-parity $3/2^-$. According to them, its partners in the octet should be the $Y_0^*(1520)$, the $Y_1^*(1660)$, and a Ξ^* (undiscovered). These assignments were based chiefly on an analysis of partial widths for two-body decay modes. Martin[2] has also analyzed widths, and his approach, a different one, reveals discrepancies in the octet assignment. He argues in particular that the Y_0^* is more likely a unitary singlet than a member of an octet. It is the purpose of this paper to present a dynamical mechanism which yields both singlet and octet systems of $d_{3/2}$ resonances.

[1] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

[2] A. W. Martin, Nuovo Cimento (to be published).

In Sec. II the model is presented which leads to the results cited above. In Sec. III the analysis of resonant unitary multiplets is given.

II. DYNAMICAL MODEL

The prototype of the mechanism adopted here is the Cook-Lee model[3] of the higher πN resonances. For the $d_{3/2}$ state their model exploits the circumstance that an s-wave ρN system may be coupled by unitarity to the d-wave πN system. Virtual ρ production feeds the elastic channel and provides enough attraction to produce the N^{**} below the inelastic threshold. The dominant force driving the left-hand cuts in their two-channel model is assumed to arise from one-pion-exchange coupling the πN and ρN channels. That it is allowable to neglect a specific exchange force, so

[3] L. F. Cook and B. W. Lee, Phys. Rev. **127**, 283 (1962); **127**, 297 (1962).

essential in the lower energy $\frac{3}{2}^+$ system,[4] is borne out in the spirit of a sufficiency point of view by the success of their model. Furthermore it is observed, in a recent paper by Freedman,[5] that the fact that the nucleon Regge trajectory and the trajectory on which the N^{**} lies seem to be very close together implies the relative unimportance of the exchange force.

The Cook-Lee model suggests the mechanism to be adopted for baryon (B, mass M), pseudoscalar meson (P, mass μ) scattering in the $d_{3/2}$ state. The unitarity relations are assumed to include s-wave baryon (B), vector meson (V, mass m) states along with d-wave BP states. The elegant development in terms of helicity states is described exhaustively in the Cook-Lee papers, and need not be reiterated here. The B, P, and V octets are each assumed degenerate so that the generalized two-channel problem is described by the matrix amplitude

$$\mathfrak{F}=\begin{pmatrix} F & G^{\xi} \\ \bar{G}^{\zeta} & F'^{\zeta\xi} \end{pmatrix} \tag{1}$$

in which

$$\begin{aligned} F_{ij}&=hM_{ij}, \\ G_{ij'}{}^{\xi}&=(hh')^{1/2}M_{ij'}{}^{\xi}, \\ \bar{G}_{i'j}{}^{\zeta}&=(hh')^{1/2}M_{i'j}{}^{\zeta}, \\ F'_{i'j'}{}^{\zeta\xi}&=h'M_{i'j'}{}^{\zeta\xi}. \end{aligned} \tag{2}$$

Unprimed (primed) indices denote BP (BV) states; the dimensionality of the submatrices in (1) depends on the choice of isospin and strangeness. The h factors incorporate the proper threshold behavior:

$$h=\frac{2M}{p_0+M}\left(\frac{p_0+M}{p}\right)^4, \quad \text{for} \quad d \text{ waves},$$

and

$$h'=\frac{2M}{q_0+M}, \quad \text{for} \quad s \text{ waves}, \tag{3}$$

where p and q are the c.m. momenta in the BP and BV states, respectively, and p_0 and q_0 the respective baryon energies. The M's are the parity eigenamplitudes defined in terms of helicity amplitudes by Cook and Lee; in their notation the superscripts ζ and ξ may be 1, 3, or 5 and refer to the spin multiplicity of the BV states. The coupling of the BV and BP channels is assumed to arise from P exchange as shown in Fig. 1.

The model is defined in a pole approximation by prescribing the following left-hand cut discontinuites:

$$\begin{aligned} \mathrm{disc}_L F&=0=\mathrm{disc}_L F'^{\zeta\xi}, \\ \mathrm{disc}_L \bar{G}^{\zeta}&=2\pi i\alpha_{\zeta}\sigma(\rho^2)\beta\delta(w-w_0), \\ \mathrm{disc}_L G^{\xi}&=2\pi i\alpha_{\xi}\sigma^*(\rho^2)\beta^T\delta(w-w_0), \end{aligned} \tag{4}$$

in which α_{ζ} is a parameter depending on the polarization

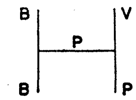


FIG. 1. P exchange diagram.

of V,[6] and $\sigma(\rho^2)$ describes $T=1$, $J=1\pi\pi$ scattering:

$$\sigma(\rho^2)=-96\pi^2 g(8\pi/3)^{1/2}\rho(\rho^2-4\mu^2)^{-1}e^{i\delta}\sin\delta. \tag{5}$$

Here, δ is the phase shift for $\pi\pi$ scattering in the $T=J=1$ state. This construction follows Cook and Lee in which the $\pi\pi\rho$ vertex is so expressed; the function σ incorporates g, the πNN coupling constant, so that the generalization employed here is represented by β, a matrix of isospin factors and ratios of octet model coupling constants to g. The dimensionality of β depends on the choice of isospin and strangeness.

The N over D method can be invoked by defining

$$\mathfrak{N}=\begin{pmatrix} N & O^{\xi} \\ \bar{O}^{\zeta} & N'^{\zeta\xi} \end{pmatrix} \tag{6}$$

and

$$\mathfrak{D}=\begin{pmatrix} D & E^{\xi} \\ \bar{E}^{\zeta} & D'^{\zeta\xi} \end{pmatrix} \tag{7}$$

such that $\mathfrak{N}=\mathfrak{F}\mathfrak{D}$ and $\mathrm{Im}\,\mathfrak{D}=-\pi\bar{\rho}\mathfrak{N}$ in which

$$\bar{\rho}=\begin{pmatrix} \rho & 0 \\ 0 & \rho' \end{pmatrix}; \tag{8}$$

the phase-space factors are (with θ functions suppressed):

$$\rho=\frac{1}{4(2\pi)^3}\frac{p^5}{w}\frac{1}{(p_0+M)^3}, \quad \text{for} \quad d \text{ waves},$$

and (9)

$$\rho'=\frac{1}{32(2\pi)^6}\frac{q}{w}\frac{(\rho^2-4\mu^2)^{1/2}}{\rho}(q_0+M), \quad \text{for} \quad s \text{ waves},$$

each multiplied by unit matrices of the appropriate dimensionality. The matrix multiplication $\mathfrak{N}=\mathfrak{F}\mathfrak{D}$ may be written out in detail for each submatrix as follows:

$$\begin{aligned} N(w)&=F(w)D(w)+\sum_{\eta}\int d\lambda G^{\eta}(w,\lambda)\bar{E}^{\eta}(w,\lambda), \\ O^{\xi}(w,\mu)&=F(w)E^{\xi}(w,\mu) \\ &\quad+\sum_{\eta}\int d\lambda G^{\eta}(w,\lambda)D'^{\eta\xi}(w,\lambda,\mu), \\ \bar{O}^{\zeta}(w,\nu)&=\bar{G}^{\zeta}(w,\nu)D(w) \\ &\quad+\sum_{\eta}\int d\lambda F'^{\zeta\eta}(w,\nu,\lambda)\bar{E}^{\eta}(w,\lambda), \\ N'^{\zeta\xi}(w,\nu,\mu)&=\bar{G}^{\zeta}(w,\nu)E^{\xi}(w,\mu) \\ &\quad+\sum_{\eta}\int d\lambda F'^{\zeta\eta}(w,\nu,\lambda)D'^{\eta\xi}(w,\lambda,\mu), \end{aligned} \tag{10}$$

[4] For example, G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956); S. C. Frautschi and J. D. Walecka, *ibid.* **120**, 1486 (1960).
[5] D. Z. Freedman, Phys. Rev. **134**, B652 (1964).
[6] As the following analysis will reveal, the α's turn out to be real.

TABLE I. Summary of states and factors for Eq. (17).

S, T	BP states	BV states	Representations	Determinants
1, 0	NK	NK^*	$\overline{10}$	$D(\overline{10})$
1, 1	NK	NK^*	27	$D(27)$
0, ½	$N\pi$, $N\eta$, ΛK, ΣK	$N\rho$, $N\varphi$, ΛK^*, ΣK^*	8, 8′, $\overline{10}$, 27	$D(8,8')D(\overline{10})D(27)$
0, $\frac{3}{2}$	$N\pi$, ΣK	$N\rho$, ΣK^*	10, 27	$D(10)D(27)$
−1, 0	$N\bar{K}$, $\Lambda\eta$, $\Sigma\pi$, ΞK	$N\bar{K}^*$, $\Lambda\varphi$, $\Sigma\rho$, ΞK^*	1, 8, 8′, 27	$D(1)D(8,8')D(27)$
−1, 1	$N\bar{K}$, $\Lambda\pi$, $\Sigma\pi$, $\Sigma\eta$, ΞK	$N\bar{K}^*$, $\Lambda\rho$, $\Sigma\rho$, $\Sigma\varphi$, ΞK^*	8, 8′, 10, $\overline{10}$, 27	$D(8.8')D(10)D(\overline{10})D(27)$
−1, 2	$\Sigma\pi$	$\Sigma\rho$	27	$D(27)$
−2, ½	$\Lambda\bar{K}$, $\Sigma\bar{K}$, $\Xi\pi$, $\Xi\eta$	$\Lambda\bar{K}^*$, $\Sigma\bar{K}^*$, $\Xi\rho$, $\Xi\varphi$	8, 8′, 10, 27	$D(8,8')D(10)D(27)$
−2, $\frac{3}{2}$	$\Sigma\bar{K}$, $\Xi\pi$	$\Sigma\bar{K}^*$, $\Xi\rho$	$\overline{10}$, 27	$D(\overline{10})D(27)$
−3, 0	$\Xi\bar{K}$	$\Xi\bar{K}^*$	10	$D(10)$
−3, 1	$\Xi\bar{K}$	$\Xi\bar{K}^*$	27	$D(27)$

in which λ, μ, ν stand for the $\pi\pi$ mass squared. Equations (4), (6), (7), and (10) lead to separable integral equations for each element of $\mathfrak{N}$; the solution of these and the resulting expressions for $\mathfrak{F}$ can readily be obtained as generalizations of Cook and Lee's results. The submatrix solutions for $\mathfrak{F}$ are

$$\begin{aligned} F(w)&=v(w)[1-(w-w_0)^2u(w)v(w)\beta^T\beta]^{-1}\beta^T\beta\,,\\ G^{\xi}(w,\mu)&=\frac{\alpha_\xi\sigma^*(\mu)}{w_0-w}[1-(w-w_0)^2u(w)v(w)\beta^T\beta]^{-1}\beta^T\,,\\ \bar{G}^{\zeta}(w,\nu)&=\frac{\alpha_\zeta\sigma(\nu)}{w_0-w}\beta[1-(w-w_0)^2u(w)v(w)\beta^T\beta]^{-1}\,,\\ F'^{\zeta\xi}(w,\nu,\mu)&=u(w)\alpha_\zeta\sigma(\nu)\alpha_\xi\sigma^*(\mu)\\ &\quad\times\beta[1-(w-w_0)^2u(w)v(w)\beta^T\beta]^{-1}\beta^T. \end{aligned} \tag{11}$$

In these expressions u and v are defined as

$$u(w)=\int_{M+\mu}^{\infty}\frac{\rho dx}{(x-w)(x-w_0)^2} \tag{12}$$

and

$$v(w)=\sum_\eta \alpha_\eta^2\int d\rho^2|\sigma(\rho^2)|^2\int_{M+m}^{\infty}\frac{\rho' dx}{(x-w)(x-w_0)^2}.$$

The $\pi\pi\rho$ coupling constant $f_{\rho\pi\pi}$ may be introduced by setting

$$|\sigma(\rho^2)|^2=16\rho(2\pi)^5(\rho^2-4\mu^2)^{-1/2}g^2f^2{}_{\rho\pi\pi}\delta(m^2-\rho^2). \tag{13}$$

As in Cook and Lee, the parameters α_η and w_0 are determined after explicit evaluation of the $d_{3/2}$ $\eta=1, 3, 5$ projections of the diagram of Fig. 1. The calculation yields a pole at the elastic threshold so the choice $w_0=M+\mu$ is made. Since resonances are anticipated between $M+\mu$ and $M+m$, α_η is determined by matching $\alpha_\eta(w-w_0)^{-1}$ to the value of the projection at the inelastic threshold; at this point in the range of interest the BP phase space factor has its greatest value. The results of the matching procedure are

$$\alpha_1=-\frac{1}{\sqrt{2}}\alpha_3=\frac{1}{\sqrt{3}}\alpha_5=-\frac{m-\mu}{3m}\frac{(2M+m)^2-\mu^2}{\mu^2+Mm}. \tag{14}$$

The numbers used in the calculation then are

$$\begin{gathered}\mu=2.9m_\pi\,,\quad m=2.06\mu\,,\quad M=2.85\mu\\ \textstyle\sum_\eta \alpha_\eta^2=13\\ g^2/(4\pi)=15\,,\quad f_{\rho\pi\pi}^2/(4\pi)=2.\end{gathered} \tag{15}$$

This completes the description of the model. It is structurally identical with that of Cook and Lee; the generalization of the elastic and inelastic channels is the only feature added for the analysis of resonant unitary multiplets. Under the assumption of degenerate B, P, and V octets this generalization is achieved by the inclusion of the factor β, a matrix the rows and columns of which are labeled by inelastic (BV) and elastic (BP) states, respectively. The approach is in the spirit of that adopted by Martin and Wali.[7] A dynamical model which has proven successful in the explanation of a πN scattering resonance is enlarged upon to treat BP scattering and to indicate which of the unitary multiplets may be resonant.

Cook and Lee allowed the parameter which represents $\sum\alpha_\eta^2$ to be variable in order to fit experiment; they subsequently compared its determination with the result of calculating the diagram. For the $d_{3/2}$ resonance agreement was not bad. In this problem more freedom is allowed in the calculation of the input diagram. An enormous number of channels are assumed to be coupled and, under the assumption of unitary symmetry,[8] all the input coupling constants are known in terms of a single adjustable parameter. In the next section the results as a function of this parameter are given.

III. RESONANT UNITARY MULTIPLETS

Equations (11) give the solutions for scattering in this model. For energies below $w=M+m$, the inelastic threshold, resonances occur when

$$\operatorname{Re}\det[1-(w-w_0)^2u(w)v(w)\beta^T\beta]=0. \tag{16}$$

[7] A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963).
[8] M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

Since the central issue in this work is the occurrence of resonant unitary multiplets, the equivalent equation is more relevant and easier to compute:

$$\operatorname{Re}\det[1-(w-w_0)^2u(w)v(w)\tilde{\beta}^T\tilde{\beta}]=0. \quad (17)$$

Here the tilde means that the matrix has been transformed from the particle basis to the unitary basis. Under the assumption of unitary symmetry, the representations 1, 10, $\overline{10}$, and 27 then may occur only on the diagonal of $\tilde{\beta}$; the octet representation may occur as a symmetric 2×2 submatrix [labeled $(8,8')$]. Table I lists, for each choice of strangeness and isospin (S,T), the BP and BV states which occur as well as the unitary multiplets; in the last column is given the form of the left-hand side of Eq. (17). The factors listed there are defined by

$$D(R)=1-(w-w_0)^2u(w)v(w)a_R^2 \quad (18)$$

for $R=1$, 10, $\overline{10}$, and 27, and

$$D(8,8')=\det[1-(w-w_0)^2u(w)v(w)\tilde{\beta}_8{}^T\tilde{\beta}_8], \quad (19)$$

where

$$\tilde{\beta}_8=\begin{pmatrix} a_8 & b_8 \\ b_8 & a_{8'} \end{pmatrix}. \quad (20)$$

The coefficients a_R and b_8 depend only on the Yukawa mixing parameter f for the BBP vertex[9]; they are

$$\begin{aligned} a_1 &= 6f, \\ a_{10} &= -a_{\overline{10}}=2(f-1), \\ a_{27} &= -2f, \\ a_8 &= a_{8'}=3f, \\ b_8 &= -5^{1/2}(1-f). \end{aligned} \quad (21)$$

An orthogonal transformation parameterized by the angle θ^* diagonalizes the $(8,8')$ submatrix of $\tilde{F}$[10]; if the new basis is denoted by $(8_1,8_2)$ the rotation is defined by:

$$\begin{pmatrix} 8_1 \\ 8_2 \end{pmatrix}=\begin{pmatrix} \cos\theta^* & -\sin\theta^* \\ \sin\theta^* & \cos\theta^* \end{pmatrix}\begin{pmatrix} 8 \\ 8' \end{pmatrix}. \quad (22)$$

The $(8,8')$ determinant becomes

$$D(8,8')=D(8_1)D(8_2), \quad (23)$$

where $D(8_1)$ and $D(8_2)$ are expressions of the form of Eq. (18) with

$$\begin{aligned} a_{8_1} &= a_8-b_8, \\ a_{8_2} &= a_8+b_8. \end{aligned} \quad (24)$$

The circumstance that $a_8=a_{8'}$ leads to the result that

$$\theta^*=45°. \quad (25)$$

[9] The notation of Martin and Wali (Ref. 7) is followed. See also A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1324 (1964); J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).
[10] R. E. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963).

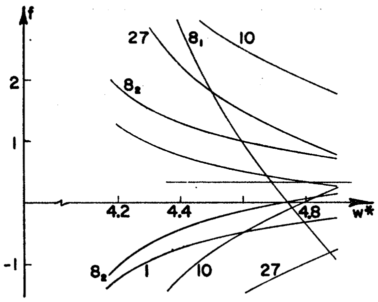


FIG. 2. Values of f giving a resonance at w^* for each representation. 10 and $\overline{10}$ fall on the same curve. The horizontal line at $f=0.326$ corresponds to $\theta=33°$.

This is the result one would obtain from a diagonalization of the octet portion of the box diagram obtained by folding the diagram of Fig. 1 back-to-back with itself.[11] As such, this finding does not require the full dynamical content of multichannel unitarity; the existence of a resonance, its position and width of course do. The result (25), and the determination of the $B^{**}BP$ mixing parameter, have been also obtained by Freedman.[5]

In Fig. 2 are plotted the values of f producing a resonance at $w=w^*$ in each of the representations 1, 10, $\overline{10}$, 27, 8_1, and 8_2. If values of f are disallowed which lie outside the range obtained by Martin and Wali[7] for the occurrence of the $p_{3/2}$ decuplet then the possibility of a 27-plot can be rejected. If the lack of experimental evidence for an $NK(T=0)$ resonance is used to reject $\overline{10}$, then 10 is also rejected and the Martin and Wali range can be restricted to $0.25<f<0.56$. In this range only 1 and 8_1 resonate; for a BBP mixing angle $\theta=33°$[10] ($f=0.326$) the positions of the resonances are

$$\begin{aligned} w_1{}^* &= 4.80\mu, \\ w_8{}^* &= 4.69\mu, \end{aligned} \quad (26)$$

so that for this case 8_1 is slightly more attractive than 1.

The $(8,8')$ submatrix for the diagram of Fig. 3 is

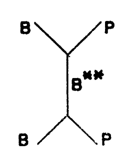
$$\begin{bmatrix} \frac{20}{3}(1-f^*)^2 & -4(5)^{1/2}f^*(1-f^*) \\ -4(5)^{1/2}f^*(1-f^*) & 12f^{*2} \end{bmatrix}, \quad (27)$$

where f^* is the $B^{**}BP$ mixing parameter. If the angle parameterizing the orthogonal transformation which diagonalizes (27) is identified with θ^* then $\theta^*=45°$ implies that

$$f^*=(5)^{1/2}/(3+(5)^{1/2})=0.428. \quad (28)$$

A width parameter may be assigned to the octet

FIG. 3. B^{**} pole diagram.



[11] A. W. Martin (private communication).

resonance by the definition

$$\Gamma_8=2\left[\mathrm{Im}D(8_1)\Big/\left(\frac{d}{dw}\mathrm{Re}D(8_1)\right)\right]_{w_8^*}, \qquad (29)$$

and similarly for the singlet. Numerically these are

$$\Gamma_1=68.6\ \mathrm{MeV},$$
$$\Gamma_8=72.8\ \mathrm{MeV}. \qquad (30)$$

The coupling constant g^* for the $N^{**}N\pi$ vertex can be defined in terms of the residue at the pole. In a neighborhood of $w=w_8^*$, F, evaluated in the $\pi N(T=\frac{1}{2})$ state, is

$$F_{\pi N,\pi N}{}^{(T=1/2)}=\frac{9\pi^2\Gamma_8[(w_8^*+M)^2-\mu^2]^3}{w_8^{*2}[p(w_8^*)]^5(w-w_8^*+i\Gamma_8/2)}. \qquad (31)$$

The coupling constant is given by

$$g^{*2}/(4\pi)=(7+3(5)^{1/2})\Gamma_8 w(p_0+M)/(20p^5)=0.150\ m_\pi^{-2}. \qquad (32)$$

Martin[2] has shown that the value 0.247 m_π^{-2} corresponds to the observed N^{**} width.

IV. CONCLUDING REMARKS

The occurrence of unitary singlet and octet systems of resonances has been shown to be the consequence of the appropriate generalization of the Cook-Lee model. The assignments for the multiplets deduced here have been discussed elsewhere.[1,2] The fact that both a singlet and an octet occur in this model may lend some theoretical support to the intriguing conjecture[2] that the Y_0^* (1520) is a manifestation of singlet-octet mixing. There are some aspects of the experimental situation which do not tend to support the Glashow and Rosenfeld assignments,[12] but these should not detract from the issue treated here, as long as there remain members of the multiplet with the established quantum numbers.

The dynamical model of Freedman[5] bears some resemblance to that described here. His method is based on single-channel unitarity, but with an input which contains an element of inelastic scattering. There is a difference of principle. As pointed out by Cook and Lee, the inclusion of inelastic states in the unitarity relations is essential to an accurate treatment of elastic scattering at these energies. In particular, the decrease in cross section above the inelastic threshold must be insured.

Finally it should be mentioned that the Martin and Wali model[9] also yields a $d_{3/2}$ unitary singlet, but by means of entirely different forces and with elastic unitarity alone. Perhaps one could conclude from their work and from that described here that an admixture of the two, including inelastic unitarity, would always yield the singlet and introduce the octet as well, and that a final accurate positioning of the resonances results. To establish this would entail quite a formidable program of calculation.

ACKNOWLEDGMENTS

It is a pleasure to thank R. H. Capps, A. W. Martin, and K. C. Wali for several conversations contributing considerably to this work.

[12] The allusion is to the as yet undiscovered Ξ^*, to the evidence that the Y_1^* has the wrong parity, and to reinvestigations of the N^{**} parity. Some references on these developments are P. E. Schlein, D. D. Carmony, G. M. Pjerrou, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **11**, 167 (1963); M. Taher-Zadeh, D. J. Prowse, P. E. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, *ibid.* **11**, 470 (1963); P. L. Connolly, E. L. Hart, D. C. Rahm, D. L. Stonehill, S. S. Yamamoto, and W. J. Willis, Bull. Am. Phys. Soc. **9**, 23 (1964); R. D. Eandi, T. J. Devlin, R. W. Kenney, P. G. McManigal, and B. J. Moyer, *ibid.* **9**, 410 (1964).