$\mu^- + He^3 \rightarrow H^3 + \nu$ Capture Rate*

R. J. Oakes†
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California

(Received 14 August 1964)

The muon capture rate in He^3 for the process $\mu^- + He^3 \rightarrow H^3 + \nu$ has been computed using nuclear wave functions suggested by recent analyses of e-He^3 and e-H^3 scattering experiments.

I. INTRODUCTION

Recent analyses of the He^3 and H^3 electromagnetic form factors,[1] the inelastic scattering cross section for the process[2] $e + He^3 \rightarrow d + p + e'$, and the photodisintegration of[3] He^3 have yielded considerable information concerning the three-body nuclear wave function. It has been shown that the dominant component of the three-body wave function is the completely spatially symmetric S state and some specific forms of this wave function have been examined. The Irving-Gunn wave function has been found to give the best agreement with the data. The Irving wave function is also reasonably successful. In the following the rate for the muon capture process $\mu^- + He^3 \rightarrow H^3 + \nu$ is calculated as a further test of these three-body wave functions.

II. CAPTURE RATE

The capture rate for the process $\mu^- + He^3 \rightarrow H^3 + \nu$ will be calculated using the conserved vector-current theory of weak interactions including both induced pseudoscalar and class II[4] axial vector currents. We assume the μ^- is captured from the ground state of the μ-He^3 atom and will consider only the spatially symmetric S state of the nuclear wave function.[5] Under these assumptions the nonrelativistic reduction of the matrix element leads to

$$\begin{aligned}\mathfrak{M} = \int d^3r_1 \int d^3r_2 \int d^3r_3 \sum_{k=1}^{3} \exp\{-i[\mathbf{p}\cdot\mathbf{r}_k + \tfrac{1}{3}\mathbf{P}\cdot(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)]\}\,\varphi(\mathbf{r}_k)\\
\times u^*(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\{g_V F_{1V}(q^2)(1+p/2M)\chi_\nu^\dagger(1-\boldsymbol{\sigma}\cdot\hat{p})\chi_\mu \Phi_f^\dagger \tau_k^- \Phi_i\\
+[g_A F_A(q^2) - (p/2M) g_V[F_{1V}(q^2)+\kappa_V F_{2V}(q^2)]]\chi_\nu^\dagger(1-\boldsymbol{\sigma}\cdot\hat{p})\boldsymbol{\sigma}\chi_\mu\cdot\Phi_f^\dagger\boldsymbol{\sigma}_k\tau_k^-\Phi_i\\
-(p/2M)\{g_A F_A(q^2)+g_V[F_{1V}(q^2)+\kappa_V F_{2V}(q^2)]-g_P F_P(q^2)+g_{\rm II}F_{\rm II}(q^2)\}\chi_\nu^\dagger(1-\boldsymbol{\sigma}\cdot\hat{p})\chi_\mu\Phi_f^\dagger\boldsymbol{\sigma}_k\cdot\hat{p}\tau_k^-\Phi_i\\
+g_V F_{1V}(q^2)/M\chi_\nu^\dagger(1-\boldsymbol{\sigma}\cdot\hat{p})\boldsymbol{\sigma}\cdot\mathbf{p}_k\chi_\mu\Phi^\dagger\tau_k^-\Phi_i + g_A F_A(q^2)/M\chi_\nu^\dagger(1-\boldsymbol{\sigma}\cdot\hat{p})\chi_\mu\Phi_f^\dagger\boldsymbol{\sigma}_k\cdot\mathbf{p}_k\tau_k^-\Phi_i\}u(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3), \quad (1)\end{aligned}$$

where $\mathbf{p}$ and $\mathbf{P}$ are the momenta of the neutrino and H^3, respectively. The momentum and spin operators $\mathbf{p}_k$ and $\boldsymbol{\sigma}_k$ act on the kth nucleon whose position is $\mathbf{r}_k$. The neutrino and muon two-component spinors are χ_ν and χ_μ, the muon atomic wave function is $\varphi(\mathbf{r})$, and the three-body nuclear wave function is $u(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\Phi$ with $u(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ the completely symmetric spatial part and Φ the completely antisymmetric $J=\frac{1}{2}^+$, $I=\frac{1}{2}$ spin-isospin part. The vector, axial vector, induced pseudoscalar, and class II axial vector, coupling constants, and form factors are g_V, g_A, g_P, $g_{\rm II}$ and F_{1V}, F_{2V}, F_A, E_P, and $F_{\rm II}$, respectively. The form factors are evaluated at the four-momentum transfer to the nucleon $q^2 \simeq m_\mu(2p - m_\mu)$. In the conserved vector-current theory F_{1V} and F_{2V} are the isovector electric and magnetic form factors of the nucleon, respectively. Adopting the convention usually followed for the electromagnetic form factors, each form factor has been normalized to $\frac{1}{2}$ at $q^2=0$. τ_k^- is the usual isospin operator that changes the kth nucleon from proton to neutron.

After squaring this matrix element, averaging over initial states assuming a statistical distribution, and summing over final states, the capture rate is found to be[6]

$$\begin{aligned}\Gamma = \frac{2}{\pi}\frac{p^2}{(1+p/M_{H^3})}\varphi^2(0)I^2(p)\Bigg\{g_V^2F_{1V}^2(q^2)\left(1+\frac{p}{6M}\right)^2 + 3\left[g_A F_A(q^2) - \frac{p}{2M}g_V[F_{1V}(q^2)+\kappa_V F_{2V}(q^2)]\right]^2\\
+\frac{p}{M}\left[g_A F_A(q^2) - \frac{p}{2M}g_V[F_{1V}(q^2)+\kappa_V F_{2V}(q^2)]\right]\{\tfrac{1}{3}g_A F_A(q^2)+g_V[F_{1V}(q^2)+\kappa_V F_{2V}(q^2)]-g_P F_P(q^2)+g_{\rm II}F_{\rm II}(q^2)\}\\
+\left(\frac{p}{2M}\right)^2\{\tfrac{1}{3}g_A F_A(q^2)+g_V[F_{1V}(q^2)+\kappa_V F_{2V}(q^2)]-g_P F_P(q^2)+g_{\rm II}F_{\rm II}(q^2)\}^2\Bigg\}. \quad (2)\end{aligned}$$

* Supported in part by the U. S. Air Force through Air Force Office of Scientific Research Contract AF 49(638)-1389.
† National Science Foundation Postdoctoral Fellow.

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[3] B. L. Berman, L. J. Koester, and J. H. Smith, Phys. Rev. **133**, B117 (1964).
[4] S. Weinberg, Phys. Rev. **112**, 1375 (1958).
[5] Admixtures of the other nine possible states are thought to be present with only a few percent probability.
[6] A. Fujii and H. Primakoff, Nuovo Cimento **12**, 327 (1959); H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959). The present result differs from that derived in these references in that a class II axial-vector current is included here and the relativistic terms proportional to the nucleon momentum have been retained. The effect of these relativistic terms is to reduce the capture rate a few percent.

In obtaining Eq. (2) we have approximated the muon atomic wave function by its value at the origin

$$\varphi(0)=[2e^2m_\mu(1+m_\mu/M_{He^3})^{-1}]^{3/2}/\sqrt{\pi}, \quad (3)$$

where $e^2 \approx 1/137$ is the fine-structure constant. The muon, nucleon, He^3, and H^3 masses are denoted by m_μ, M, M_{He^3}, and M_{H^3}, respectively. The dependence on the nuclear wave function is contained in the integral $I(p)$ which is defined by

$$I(p)=\int d^3\rho \int d^3r \exp[-i\tfrac{2}{3}\mathbf{p}\cdot\mathbf{r}]|u(\mathbf{r},\boldsymbol{\rho})|^2, \quad (4)$$

where $\mathbf{r}=\mathbf{r}_1-\frac{1}{2}(\mathbf{r}_2+\mathbf{r}_3)$ and $\boldsymbol{\rho}=\mathbf{r}_3-\mathbf{r}_2$. The nuclear wave functions we are concerned with are the Irving and the Irving-Gunn. The Irving wave function is[7]

$$u(\mathbf{r},\boldsymbol{\rho})=A \exp[-\tfrac{1}{2}\alpha(2r^2+\tfrac{3}{2}\rho^2)^{1/2}], \quad (5)$$

where the normalization constant is given by

$$A=3^{3/4}\alpha^3/(120)^{1/2}\pi^{3/2}. \quad (6)$$

The Irving-Gunn wave function is[8]

$$u(\mathbf{r},\boldsymbol{\rho})=\frac{A \exp[-\frac{1}{2}\alpha(2r^2+\frac{3}{2}\rho^2)^{1/2}]}{(2r^2+\frac{3}{2}\rho^2)^{1/2}} \quad (7)$$

and the corresponding normalization constant is

$$A=3^{1/4}\alpha^2/\sqrt{2}\pi^{3/2}. \quad (8)$$

For these wave functions the integrals $I(p)$ can be carried out using the techniques given in Appendix B of Ref. 2. For the Irving wave function one finds

$$I(p)=(1+Q^2)^{-7/2} \quad (9)$$

and for the Irving-Gunn wave function one finds

$$I(p)=\frac{\frac{4}{3}[1+2(1+Q^2)^{1/2}]}{(1+Q^2)^{3/2}[1+(1+Q^2)^{1/2}]^2}, \quad (10)$$

where Q^2 is given by

$$Q^2=2p^2/9\alpha^2. \quad (11)$$

III. NUMERICAL RESULTS

In evaluating the capture rate the vector and axial-vector coupling constants quoted by Wu[9] were used. These are $g_V=1.15\times10^{-11}$ MeV^{-2} and $g_A/g_V=-1.19$. The isovector electromagnetic form factors are[10] $F_{1V}(q^2)=0.492$ and $F_{2V}(q^2)=0.485$. The isovector anomalous magnetic moment is $\kappa_V=\kappa_p-\kappa_n=3.706$ nucleon magnetons. Little is known about the axial-vector form factor but it seems reasonable to put $F_A(q^2)=F_A(0)=0.5$, for such small momentum transfer ($q^2\approx0.27$ F^{-2}). Dispersion calculations[11] in which the induced pseudoscalar term arises from the one-pion-pole contribution leads to $g_PF_P(q^2)\approx7g_AF_A(q^2)$. For the moment we will neglect the class II axial-vector current and put $g_{II}=0$.

Finally, for the parameter α appearing in the nuclear wave function we take the values arrived at from the analysis of the e-He^3 and e-H^3 scattering data and the He^3 photodisintegration data. Schiff[1] finds $\alpha=250$ MeV for the Irving wave function, and Berman, Koester, and Smith[3] report $\alpha=152$ MeV for the Irving-Gunn wave function.

The resulting capture rates are $\Gamma=1.50\times10^3$ sec^{-1} for the Irving wave function and $\Gamma=1.41\times10^3$ sec^{-1} for the Irving-Gunn wave function.

IV. DISCUSSION

Three measurements of the $\mu^-+He^3\rightarrow H^3+\nu$ capture rate have been reported. The observed values are $(1.41\pm0.14)\times10^3$ sec^{-1},[12] $(1.52\pm0.05)\times10^3$ sec^{-1},[13] and $(1.44\pm0.09)\times10^3$ sec^{-1}.[14] Comparing these with the values obtained above for the Irving wave function (1.50×10^3 sec^{-1}) and the Irving-Gunn wave function (1.41×10^3 sec^{-1}) we conclude that the agreement in the case of the Irving wave function is quite good, while the Irving-Gunn wave function leads to a capture rate which seems a little too small.

These results can also be understood in terms of the mean-square radius of the nucleus. The dependence of the capture rate on the three-body wave function is primarily through the mean-square radius as can be seen by writing $I(p)$ in the approximate form

$$I(p)\approx1-\tfrac{1}{6}p^2\langle r^2\rangle, \quad (12)$$

where

$$\langle r^2\rangle\equiv(4/9)\int d^3\rho\int d^3r\, r^2u^2(\boldsymbol{\rho},\mathbf{r}). \quad (13)$$

For the Irving wave function one finds

$$\langle r^2\rangle=14/3\alpha^2, \quad (14)$$

while for the Irving-Gunn wave function we have

$$\langle r^2\rangle=20/9\alpha^2. \quad (15)$$

Numerically, the radii are 1.70 F for the Irving ($\alpha=250$ MeV) and 1.93 F for the Irving-Gunn ($\alpha=152$ MeV). Consequently, the Irving-Gunn wave function with

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[14] R. M. Edelstein, D. Clay, J. W. Keuffel, and R. L. Wagner, Jr., *International Conference on Fundamental Aspects of Weak Interactions at Brookhaven National Laboratory* (U. S. Department of Commerce, Washington D. C., 1963), p. 303.

$\alpha = 152$ MeV gives too small a $\mu^- + He^3 \rightarrow H^3 + \nu$ capture rate since it corresponds to too large a mean-square radius. (Raising α by about 20 MeV to obtain the correct radius would spoil the agreement with the photodisintegration data.[3])

We note that the previous calculations[6,15] of the $\mu^- + He^3 \rightarrow H^3 + \nu$, which have ranged from 1.40×10^3 to 1.66×10^3 sec^{-1}, differ primarily in the assumed nuclear wave function. The capture rate essentially depends only on the nuclear wave function through the mean-square radius, and the measurements of the capture rate lead to a radius of 1.6 to 1.7 F which is in agreement with values found by Hofstadter and collaborators in elastic e-He^3 and e-H^3 scattering.[16]

Finally we observe that *the class II axial-vector current enters in the muon-capture matrix element Eq. (1) in the same manner as the induced pseudoscalar term.* Consequently, unless the induced pseudoscalar contribution is accurately known, the presence of a small amount of the class II current cannot be detected.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor J. S. Bell and Dr. T. A. Griffy for interesting discussions. We gratefully acknowledge the hospitality of the Physics Division of the Aspen Institute for Humanistic Studies where part of this work was performed.

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PHYSICAL REVIEW VOLUME 136, NUMBER 6B 21 DECEMBER 1964

Theory of Hidden Variables

DAVID KERSHAW
Harvard College, Cambridge, Massachusetts

(Received 1 July 1964)

It is shown that the stationary states of the nonrelativistic Schrödinger's equation are just the stationary states of a classical-mechanical system which is subject to random submicroscopic fluctuations of position. The proof covers the case (1) of a single particle moving in a potential, and (2) of two particles interacting through a potential $V(x_1 - x_2)$. The results can be easily generalized to the case of n interacting particles.

INTRODUCTION

In his theory of hidden variables in quantum mechanics, Bohm[1] has suggested that the uncertainty expressed by

$$(\Delta p)(\Delta q) \geq h \tag{1}$$

might be due to the presence of some random submicroscopic fluctuations which would introduce uncertainty into the otherwise classical equations of motion.

PART I

Let us then consider a function $\rho(x,t)$ such that

$$\int d^3x\, \rho(x,t) = 1. \tag{2}$$

$\rho(x,t)$ is to be viewed either as the probability of finding the particle at the point x at time t, or as a function such that $m\rho(x,t)$ is the mass density of a continuous distribution of matter of total mass m. The two points of view will be interchangeable throughout the paper.

The particles are subject to random fluctuations, so in general there exists no velocity (the paths of the particles may be discontinuous). However, let us assume that if the particle was at x at time t, then at time $t+dt$ it will have a probability $w(t,x,dt,dx)$ of being found at the point $x+dx$. Since w is a probability distribution we have

$$\int w(t,x,dt,dx) d^3(dx) = 1.$$

Then we define the velocity at x at time t by

$$v(x,t) = \lim_{dt \to 0} \left(\frac{1}{dt}\right) \int (dx) w(t,x,dt,dx) d^3(dx).$$

[1] D. Bohm, in *Quantum Theory, Radiation and High Energy Physics*, edited by D. Bates (Academic Press Inc., New York, 1962), Vol. III, p. 345.