Distorted-Wave Born Approximation Analysis of $C^{12}(d,p)C^{13}$

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The $C^{12}(d,p)C^{13}$(g.s.) reaction at 25.9-MeV deuteron energy was investigated in terms of the distorted-wave Born approximation model. The dependence of the DWBA solutions on changes of the radial form factors of the imaginary optical-model potentials, consistent with the elastic-scattering data, was studied.

1. INTRODUCTION

SINCE its introduction in nuclear physics several years ago,[1] the distorted-wave Born approximation (DWBA) model has enjoyed a frequent application in the theoretical treatment of direct nuclear reactions.

In comparisons with simpler theories, it is sometimes asserted that although generally the agreement with experiment is improved, the DWBA model has so many parameters that it is difficult to recognize and to understand the implications of the fits. Systematic investigations are therefore of interest for a useful application of the theory.

Even in cases where the validity of the basic assumptions of the model, i.e., validity of first-order perturbation theory and dominance of elastic scattering for the in- and outgoing waves, appears to be justified, ambiguities are inherent to actual calculations, inasmuch as the optical potential which is used is not uniquely determined by the elastic scattering data. Fortunately, the DWBA results generally are not too sensitive to changes in the elastic-scattering wave functions as long as these are consistent with the experimental data and correspond to "reasonable" optical potential parameters. However, at least in the noncutoff DWBA model, the strength and the radial shape of the absorptive optical model potential appear to be particularly important for the resulting angular distribution, since these to a considerable extent determine the spatial localization of the reaction process.

In this paper we report on an investigation of this question in the case of the $C^{12}(d,p)C^{13}$ (g.s.) reaction at 25.9-MeV deuteron energy, from which experimental data became available recently.[2]

The elastic-scattering data used for the deduction of appropriate optical-model potentials were for the $C^{12}(d,d)$ case, the data from Ref. 2, while for the approximate description of the $C^{13}(p,p)$ scattering the $C^{12}(p,p)$ data from Wright[3] at 30.6 MeV were used. Our attention has been mainly concentrated on the variation of the DWBA solutions as a function of the ambiguities found in the radial form factors of the absorptive optical-model potentials.

The calculations were performed on the CDC-1604 computer of the Weizmann Institute.

2. ELASTIC SCATTERING (OPTICAL MODEL)

Standard techniques[4] were used in obtaining the optical-model fits to the elastic scattering data. The FORTRAN code ELSA,[5] which treats the scattering in the absence of a spin-dependent potential was used.

The program searches for solutions of the Hamiltonian

$$H=-\frac{\hbar^2}{2\mu}\Delta\mathbf{r}-\frac{V_v+iW_v}{1+\exp[(r-R_v)/a_v]}-4\frac{iW_s\exp[(r-R_s)/a_s]}{\{1+\exp[(r-R_s)/a_s]\}^2}+V_{\mathrm{Coulomb}}$$

that minimize the error function $\chi^2=\sum_i g_i(\sigma_{\mathrm{exp}}{}^i-\sigma_{\mathrm{th}}{}^i)^2$ for given weight factors g_i, where i runs over the angles where $\sigma_{\mathrm{exp}}{}^i$ is known. In addition to automatic searches it is possible to scan the χ^2 surface by calculating χ^2 values over a multidimensional grid in parameter space. Both techniques have been used in the present calculations.

In attempting to fit the elastic proton and deuteron data we found not just a single satisfactory fit but rather a series of solutions with different imaginary potentials. With these solutions the dependence of the DWBA result on the shape of the imaginary potential could be studied, the requirement being satisfied, that the asymptotic behavior of the corresponding optical-model wave functions is consistent with the experimental data.

For both the elastic deuteron and the elastic proton scattering a clear ambiguity in the shape of the imaginary part of the optical potential was found. While keeping the real potential fixed, the imaginary potential could be changed continuously, without the fit being disturbed, from a pure surface potential into essentially a pure volume potential. The different ambiguous shapes of the imaginary potential could be parametrized by only two parameters, namely W_v and W_s. Conse-

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[1] W. Tobocman, Phys. Rev. **115**, 98 (1959).
[2] R. van Dantzig and L. A. Ch. Koerts, Nucl. Phys. 48, 177 (1963).
[3] B. J. Wright, University of California Radiation Laboratory Report UCRL-2422, 1953 (unpublished).
[4] P. E. Hodgson, *The Optical Model of Elastic Scattering* (Clarendon Press, Oxford, 1963).
[5] C. C. Giamati, W. Tobocman, and D. V. Renkel, National Aeronautics and Space Agency Technical Note D-2120 (unpublished).

quently, the ambiguity is in both cases a pure shape ambiguity in the imaginary potential alone. Figure 1 shows the ambiguities for the proton and deuteron elastic scattering. The general character of the ambiguities is very similar in both cases. The χ^2 valley is sharp in the direction of W_s and rather flat in the direction of W_v. When the surface absorption is decreased, its role is gradually taken over by volume absorption. That the fits for the different imaginary potentials are very similar in form can be seen in Fig. 2, where the elastic angular distributions relative to Coulomb scattering for very different combinations of W_v and W_s and for fixed values of all other parameters are shown. Even better fits could be obtained by allowing minor variations of the other parameters, i.e., less than about 2% for V_v, 0.2% for R_v, 2.5% for a_v, 1% for R_s, and 2% for a_s (see Table I). But in order to obtain as clear as possible a picture of the effect of the variations in the shape of the imaginary potential alone, the values from Table I (except W_v and W_s) were averaged to the final values:

$$V_v=85.1 \text{ MeV}, R_v=2.61 \text{ F}, a_v=0.73 \text{ F},$$
$$R_s=3.0 \text{ F}, a_s=0.69 \text{ F for deuterons.}$$
$$V_v=59.4 \text{ MeV}, R_v=2.48 \text{ F}, a_v=0.54 \text{ F},$$
$$R_s=2.68 \text{ F}, a_s=0.44 \text{ F for protons.}$$

By varying W_s, with these final parameter values fixed, the error function χ^2 was again minimized for different values of W_v. The solutions in this way obtained not only correspond to minima in the subspace of W_v and W_s, but they are very nearly also minima in

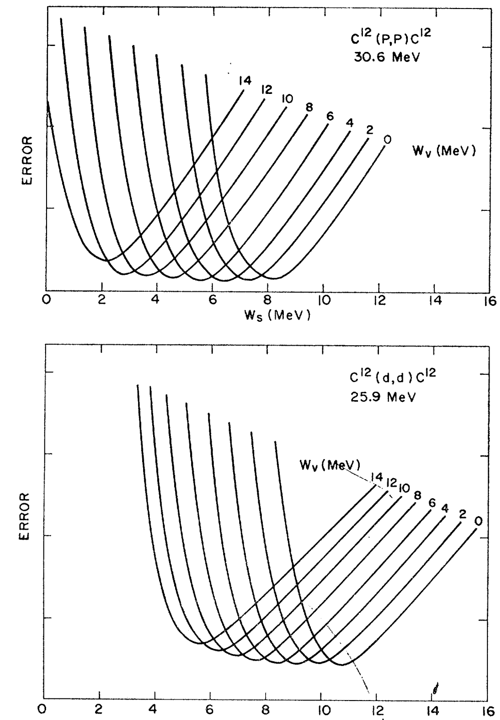

FIG. 1. Ambiguities in the shape of the imaginary part of the optical potential for elastic scattering of 25.9-MeV deuterons and 30.6-MeV protons on C^{12}. The shape is parametrized by the strengths of the volume potential W_v and of the surface potential W_s. Along the vertical scale the error function $\Sigma_i\, g_i(\sigma_{\exp}{}^i-\sigma_{\text{th}}{}^i)^2$ has been plotted in arbitrary units. The other optical model parameters have fixed values very close to those given in Table I.

TABLE I. Values of the parameters.

	V_v (MeV)	W_v (MeV)	R_v (F)	a_v (F)	W_s (F)	R_s (F)	a_s (F)
			$C^{12}(d,d)C^{12}$, $E_d=25.9$ MeV				
	83.4	0.0	2.62	0.71	10.3	3.00	0.70
	84.2	3.0	2.61	0.71	9.3	3.01	0.69
	85.2	6.0	2.62	0.73	8.4	2.95	0.69
	87.4	9.0	2.61	0.76	7.5	2.93	0.68
	87.1	12.0	2.61	0.76	6.3	3.09	0.64
	85.1		2.61	0.73		3.00	0.69
a		0			10.5		
b		6			8.2		
c		12			6.1		
			$C^{12}(p,p)C^{12}$, $E_p=30.6$ MeV				
	59.2	0.0	2.52	0.52	8.5	2.64	0.45
	59.3	3.5	2.47	0.55	6.5	2.66	0.44
	59.8	7.0	2.46	0.56	5.0	2.65	0.42
	59.4		2.48	0.54		2.65	0.44
p		0			8.2		
q		3.5			6.6		
r		6			5.5		
s		12			2.9		

the six-dimensional space of V_v, R_v, a_v, W_s, R_s, and a_s. The resulting combinations of W_v and W_s turn out to satisfy a linear relationship. This is shown in Fig. 3. That furthermore the ambiguities seem to be essential optical-model ambiguities, in the sense that they cannot simply be removed within the framework of the optical model itself, might also be indicated by the result that the total reaction cross section undergoes only a minor variation along the ambiguity, i.e., about 10% for the protons and about 2% for the deuterons (see Fig. 3). Although it might be possible that the inclusion of, for example, polarization data in the fitting procedure would tend to select special shapes of the absorptive potentials, it is improbable that this alone would lead to an unambiguous result. In that case at least one new adjustable parameter has to be introduced while the shape of the spin-dependent potential can be chosen with considerable freedom.

3. THE $C^{12}(d,p)C^{13}$ (g.s.) REACTION (DWBA)

The DWBA calculations were carried out using essentially the same numerical techniques as given in Ref. 1. A description of the FORTRAN code DRC, which

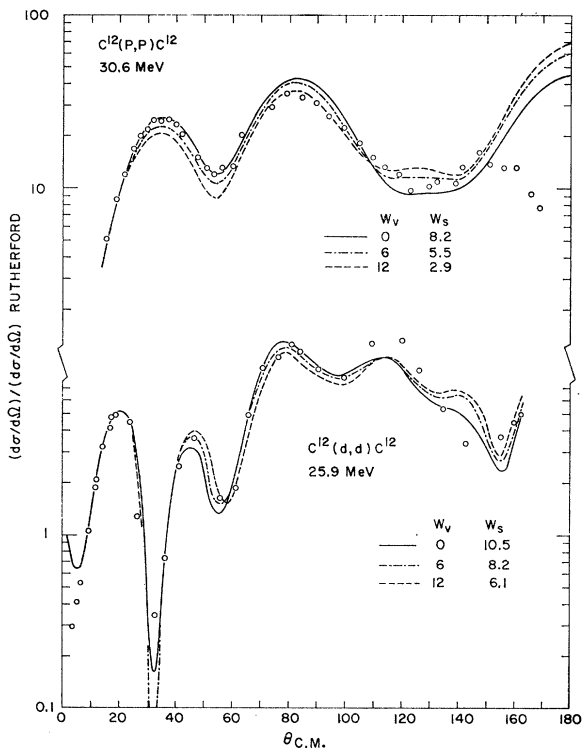

FIG. 2. Angular distributions of the ratio of the differential elastic-scattering cross section and the Rutherford cross section for 25.9-MeV deuterons and 30.6-MeV protons on C^{12}. The theoretical curves correspond to very different shapes of the imaginary optical potential. The experimental proton data are from Wright (Ref. 3) and the deuteron data from van Dantzig and Koerts (Ref. 2).

was used, is given elsewhere.[6] The nuclear-potential well for the calculation of the bound-state wave function of the captured neutron, taken to be a $1p$ wave function, had the Saxon–Woods form with radius parameter $R_N=3.0$ F and diffuseness parameter 0.6 F. The depth of the well was adjusted to 37 MeV so as to make the wave function correspond to the correct binding energy. The neutron-proton interaction was treated in the zero-range approximation. The optical-model radii R_v and R_s for the elastic scattering of protons on C^{12} were corrected for the $A^{1/3}$ difference between C^{12} and C^{13}. The optical-model ambiguities in the shape of the imaginary potential, discussed in Sec. 1, were used as a degree of freedom in the DWBA calculations. In Fig. 4 several of the results are presented.

The potentials are successively changed from a pure surface potential into a volume potential. The figure shows the following characteristic features:

(a) The main stripping peak is completely unaffected by the variation along the ambiguity.

(b) Marked differences appear as a result of the variation especially in the region of the second maximum from 30°–70° and at angles larger than 110°.

(c) The proton and the deuteron potentials are contravariant in their influence on the stripping curve. More volume absorption in the proton channel brings the second maximum down; more volume absorption in the deuteron channel raises the second maximum compared to the main stripping peak.

(d) The region near 95° is insensitive to the variations.

(e) The region of the second maximum is more sensitive to the deuteron potential than to the proton potential.

The contravariant behavior of the absorptive potentials can also be seen in Fig. 5, where the imaginary proton and deuteron potentials are varied simultaneously, their shapes being kept similar to each other. As a result of the opposite tendencies the differences between the curves become much less pronounced. In Fig. 6 for different cases the sensitive forward part of the angular distribution is reproduced on a linear scale. The theoretical curves are all normalized to the experimental points at the maximum of the main stripping peak. The errors given at the points in Fig. 6 represent almost entirely the error in this normalization which is due to the uncertainties in the forward

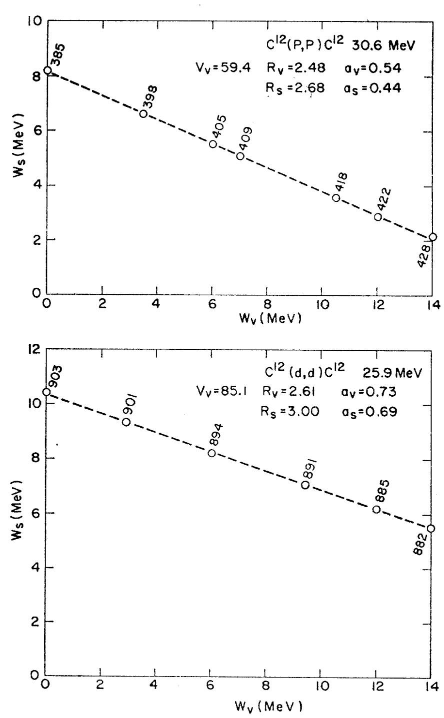

FIG. 3. Representation of the shape ambiguities of the imaginary optical potential in the two-dimensional parameter subspace of W_v and W_s, for fixed values of the other optical-model parameters. The points correspond to parameter sets W_v-W_s that minimize for fixed values of W_v the error function χ^2. The figures near the points are the values of the calculated total reaction cross section.

[6] W. R. Gibbs, V. A. Madson, J. A. Miller, W. Tobocman, and E. C. Cox, National Aeronautics and Space Agency Technical Note D-2170 (unpublished).

experimental points. The most important difference between theory and experiment is the presence of a clear minimum near 33° in all theoretical curves, which is absent in the experimental angular distribution.

Figure 7 gives a final comparison of one of the DWBA curves with the experimental data. The DWBA cutoff curve corresponding to the same optical-model wave functions is also given. To some extent this curve might be considered to be more realistic for the forward angles which semiclassically correspond to the higher order partial waves in the incident channel, because the zero-range neutron-proton force used tends to overemphasize the contribution of the partial waves localized in the nuclear interior.

4. DISCUSSION

In the preceding two sections we presented some results in which the application of the optical model in the DWBA model for stripping reactions plays an essential role. It is found that for the elastic scattering of 25.9-MeV deuterons and 30.6-MeV protons on C^{12}, a considerable freedom is present in the choice of the radial form factor of the imaginary optical-model potential. The ambiguities are almost purely ambiguities of the imaginary potential alone, so that when these are used as a degree of freedom in calculations with the DWBA model, the effect on the DWBA stripping results of taking different radial distributions of the imaginary potential could be studied. It turned out that a change in the radial form factor is not only important in backward directions, which might be expected from a semiclassical correspondence to small impact parameters, but that while the main stripping peak is completely unaffected, it also gives a marked variation of the stripping curve in the angular region between 30° and 70°.

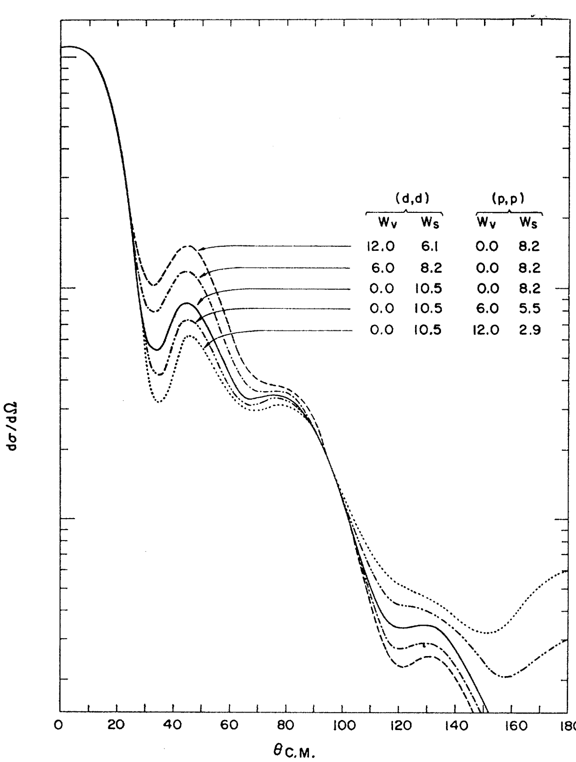

FIG. 4. DWBA angular distributions for the $C^{12}(d,p)C^{13}$ (g.s.) reaction ($l=1$, $Q=2.722$ MeV) at 25.9 MeV for different shapes of the imaginary optical potential, all of which are consistent with the elastic scattering angular distributions. All curves are normalized to each other at 5°.

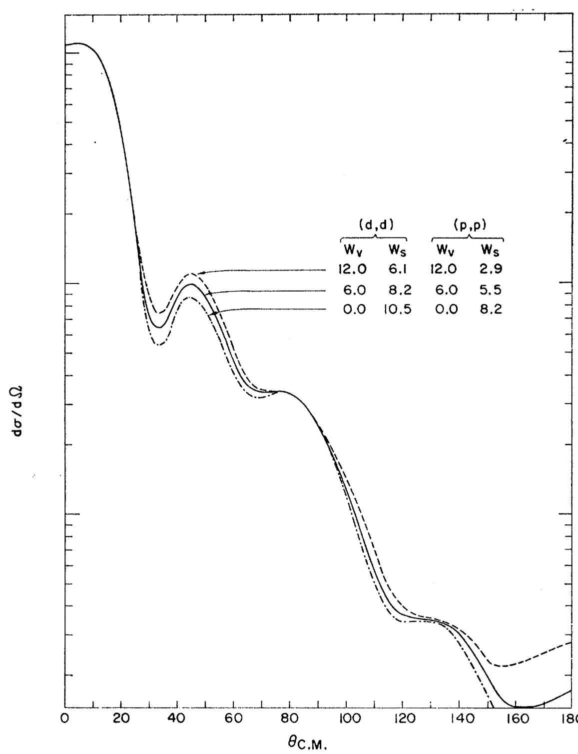

FIG. 5. DWBA angular distributions corresponding to simultaneous changes in the shape of the imaginary optical potential along the ambiguities.

Tobocman and Gibbs[7] have shown in a completely different case, i.e., the $Ca^{40}(d,p)Ca^{41}$ reaction at 4.13-MeV deuteron energy, that especially the angular region following that of the main stripping peak is sensitive to variation of the imaginary optical potentials in the elastic channels. Their explanation was based on the obtained evidence that this region, in which a secondary peak of appreciable height is found, may get an essential contribution from stripping in the interior of the nucleus, rather than at the nuclear surface. As such a contribution is likely to be much less in the present case because of the much smaller nuclear volume and because of the loss in nuclear transparency for higher energies, it can be understood that no important secondary peak in the experiment under consideration is found. On the other hand, since the

[7] W. Tobocman and W. R. Gibbs, Phys. Rev. **126**, 1076 (1962).

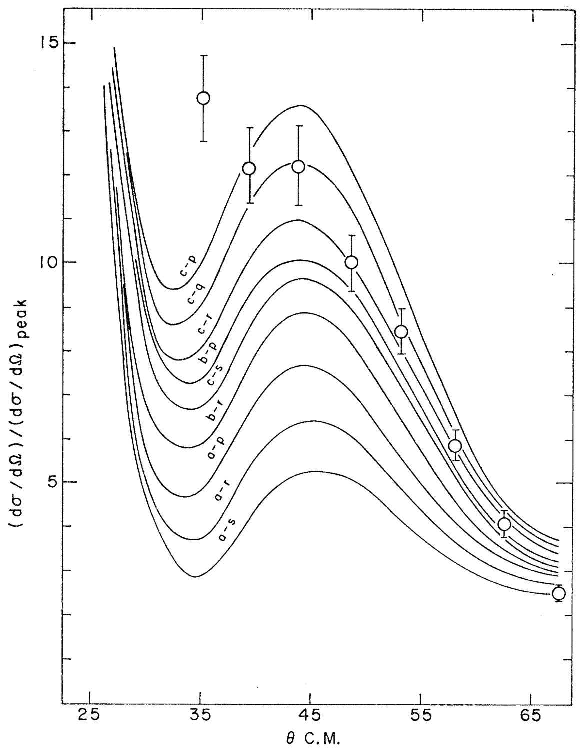

FIG. 6. Comparison of the theoretical and experimental ratio of the average differential cross section below 10° (at the stripping peak) and the differential cross section in the forward angular region sensitive to variation of the imaginary optical potentials. The errors almost entirely represent the experimental uncertainties in the points below 10° that enter into the normalization. (For parameter values see Table I.)

cross section in the considered angular region is much larger than predicted by a plane-wave theory (Fig. 7), it seems natural to assume that the distortion close to and inside the nucleus of the in- and outgoing waves manifests itself importantly in this part of the angular distribution. Because these distortions, taken into account in the framework of the optical model, bear a close and essential relationship to the form of the nuclear potential, a dependence of the stripping curve on the imaginary optical potential form factors used in the calculations seems to be quite natural and is actually found in our present work. The obtained variations in the curves appear to be significant because realistic changes in other parameters that are not *a priori* completely fixed, such as the radius and diffuseness parameter of the nuclear well for the bound particle, do not influence the result in the region with angles smaller than about 50° and cause relatively minor variations for larger angles.

One point should however be mentioned in this context. The fact that the calculations are carried out in the zero-range approximation for the neutron-proton force may lead to an overestimate of the dependence on the shape of the potentials. This results from the fact that a zero-range force, as compared to a finite range force, tends to overestimate the interaction inside the nuclear volume. The result may thus from this point of view be due to the zero-range approximation in the DWBA model. Calculations that correct for finite-range effects[8] would therefore improve systematic investigations within the framework of the combined optical and DWBA model. The conclusion that is indicated by the different shapes of the imaginary potentials may be that the DWBA results favor a volume absorptive potential in the deuteron channel rather than a surface potential. It is worthwhile noting that this result at first sight might be surprising, as one usually assumes that deuterons broken up at the nuclear surface constitute for this energy an important part of the flux being removed from the elastic channel. However, at least two circumstances in the present case may explain the obtained result. The fact that the target nucleus has only 12 constituent particles appears to make a physical distinction between "interior" and "surface" rather arbitrary since any surface process automatically involves the interior of the nucleus. Also

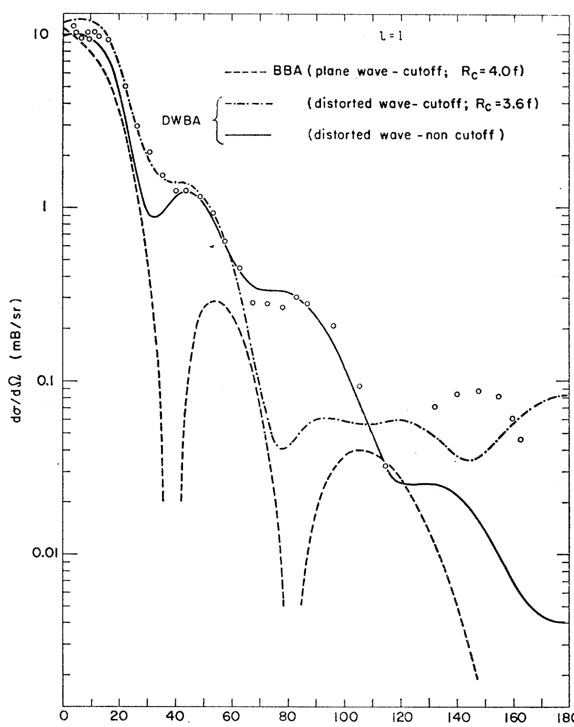

FIG. 7. Comparison of theoretical angular distributions of the $C^{12}(d,p)C^{13}$ (g.s.) reaction at 25.9-MeV deuteron energy with the experimental data from Ref. 2. The imaginary potential ambiguities were used as a degree of freedom in the fitting procedure. The optical model parameters used in the calculation of the DWBA curves (cutoff and noncutoff) are those labeled *c* and *q* in Table I. The absolute magnitudes of the calculated DWBA curves correspond to a spectroscopic factor $S=0.7$. A Butler curve normalized to the experimental points near 0° is given in order to show the importance of distortion effects.

[8] R. M. Drisko and G. R. Satchler, Phys. Letters **9**, 342 (1964).

a localization of absorption at the "surface" seems to be difficult to be understood physically, realizing that the C^{12} nucleus and the deuteron have a comparable size.

The final fit (Fig. 7) that has been obtained represents the general characteristics of the experimental data satisfactorily, especially when it is realized that both the optical model and the DWBA model appear not to be very good approximations for light nuclei. The main difference between the noncutoff DWBA and the experimental curve is the minimum near 33° that persists in all the theoretical curves. The fact that the cutoff DWBA curve fits the forward part of the angular distribution may very well indicate however that the inclusion of finite-range effects that tend to enhance the surface stripping contribution might improve the noncutoff fit in this region.

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$Li^7(Li^7,He^6)Be^8$ Reaction*

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Energy spectra of He^6 particles emitted in the nuclear reaction $Li^7(Li^7,He^6)Be^8$ were measured using a two-parameter E-dE/dx data-taking system which incorporated a general-purpose computer. Incident Li^7 ions had an energy of 3.0 MeV. The energy spectra showed peaks corresponding to the ground state and first (2.9 MeV) excited state of Be^8 and an anomalous yield between these states. This yield is simply explained as resulting from the two-step reaction $Li^7(Li^7,\alpha)Be^{10*}(He^6)\alpha$. No evidence for direct three-body breakup is found. He^6 angular distributions do not exhibit strong structure; the ground-state group shows slight anisotropy.

INTRODUCTION

The $Li^7(Li^7,He^6)Be^8$ reaction is an exothermic ($Q=7.229$ MeV) reaction involving identical particles. The reaction mechanism is of interest because of the information this reaction might give about the nuclear wave functions. If the reaction proceeds by nucleon transfer via tunneling, reduced widths could be extracted from differential cross-section data. The plausibility of this reaction mechanism is enhanced in the present work because the incident kinetic energy is only 0.6 of the Coulomb barrier ($r=2.7$ F). One of the objectives of the present work was to obtain angular distribution data to make possible a test of the applicability of this reaction mechanism.

Transfer reactions have been studied before. The $Be^9(Le^7,Li^8)Be^8$ is a prime example.[1] An interpretation of the results of this study by barrier tunneling has had some success.[2] The $N^{14}(N^{14},N^{13})N^{15}$ reaction[3] is another example and theoretical interpretation of this reaction has had success.[4,5] In these cases, there is little energy release resulting from the rearrangement of nucleons compared to the incident kinetic energy. The $Li^7(Li^7,He^6)Be^8$ reaction, if it is a direct transfer reaction, is an example of tunneling where semiclassical approximations are precluded by the large Q value.

Several experiments providing energy spectra of the Be^8 nucleus have indicated an anomalous yield at about 1 MeV excitation in Be^8. This effect has been observed in the following reactions: $B^{11}(p,\alpha)Be^8$,[6] $Be^9(p,d)Be^8$,[7] and $Be^9(He^3,\alpha)Be^8$.[8] It has been attributed to the

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† Work performed while United States Steel Foundation Postgraduate Fellow 1963–1964.

[1] E. Norbeck, J. M. Blair, L. Pinsonneault, and R. J. Gerbracht, Phys. Rev. **116**, 1560 (1959).

[2] S. K. Allison, *Reactions Between Complex Nuclei* (John Wiley & Sons Inc., New York, 1960), p. 213.

[3] L. C. Becker, F. C. Jobes, and J. A. McIntyre, in *Proceedings of the Third International Conference on Reactions Between Complex Nuclei, Asilomar, 1963* (University of California Press, Berkeley, 1963). p. 106.

[4] K. R. Greider, Phys. Rev. **133**, B1483 (1964).

[5] Breit, Chin, and Wahrswerber, Phys. Rev. **133**, B404 (1964).

[6] E. H. Beckner, C. M. Jones, and G. C. Phillips, Phys. Rev. **123**, 255 (1961).

[7] See Ref. 6.

[8] J. A. Weinman and R. K. Smithers, Nucl. Phys. **45**, 260 (1963).