Dichroism of the Vacuum

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Using Schwinger's expression for the Lagrangian density for the electromagnetic field in vacuum, including vacuum polarization effects, the imaginary part of the Lagrangian density is calculated to order e^2 from the pole contributions. It is found that in the presence of a strong electric field $E\sim\pi m^2/e\sim 10^{16}$ V/cm, there is strong absorption of light polarized in a direction parallel to the electric field, the vacuum thus exhibiting a phenomenon analogous to dichroism in polaroids.

I. INTRODUCTION

In a recent article it has been pointed out by Klein and Nigam[1] that since in quantum electrodynamics two photons can interact through the creation of virtual electron-positron pairs giving rise to terms in the Lagrangian density which are biquadratic in field intensities, the vacuum behaves like a birefringent medium. It is shown here that the Lagrangian density for the electromagnetic field in vacuum, including the effect of virtual pair creation, has imaginary contributions. Because of the fact that electron-positron pairs are created by a uniform electric field, it turns out that in the presence of a strong electric field $E\sim\pi m^2/e\sim 10^{16}$ V/cm, where m is the mass of the electron and e the charge, the imaginary part of the Lagrangian density is responsible for imparting the property of dichroism to the vacuum.

II. THEORY

Schwinger[2] has obtained an expression for the Lagrangian density of the electromagnetic field in vacuum in a closed form which takes into account the effect of vacuum polarization and satisfies the basic requirement of gauge invariance. The following is his result for the Lagrangian density when interaction only with a spin-$\frac{1}{2}$ charged field (electron-positron field) is taken into account:

$$\mathfrak{L}=-\mathfrak{F}-\frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3}$$

$$\times\exp(-m^2s)\left[(es)^2\mathcal{G}\frac{\mathrm{Re}\cosh esX}{\mathrm{Im}\cosh esX}-1-\tfrac{2}{3}(es)^2\mathfrak{F}\right] \quad (1)$$

$$=\tfrac{1}{2}(E^2-H^2)+\frac{2\alpha^2}{45}\frac{(\hbar/mc)^3}{mc^2}[(E^2-H^2)^2+7(E\cdot H)^2],\quad (2)$$

where

$$\mathfrak{F}=\tfrac{1}{4}F_{\mu\nu}{}^2=\tfrac{1}{2}(H^2-E^2),$$
$$\mathcal{G}=\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}{}^*=\tfrac{1}{8}iF_{\mu\nu}\epsilon_{\mu\nu\lambda\kappa}F_{\lambda\kappa}=\mathbf{E}\cdot\mathbf{H},\quad (3)$$
$$\mathbf{X}=\mathbf{H}+i\mathbf{E},$$

and the fine structure constant $\alpha=e^2/4\pi\hbar c$. In Eq. (2), the second term on the right is the two-photon contribution arising from the interaction of one photon with another and is in agreement with the result derived by Euler and Kockel, by Heisenberg and Euler, by Weisskopf, and by Karplus and Neuman.[3] Higher terms in the expansion will represent higher photon contributions.

In order to obtain the imaginary part of the Lagrangian density, we must determine the contributions due to the poles in Eq. (1). We carry out an expansion of the integrand in powers of E and H and retain terms up to order H^2. To this order we find that

$$\cosh es(\mathbf{H}+i\mathbf{E})=\cos esE+\frac{es}{2E}H^2\sin esE$$
$$+\frac{(\mathbf{E}\cdot\mathbf{H})^2}{2}\frac{es}{E}\left\{\frac{-\sin esE}{E^2}+\frac{es}{E}\cos esE\right\}$$
$$+i(\mathbf{E}\cdot\mathbf{H})\frac{es}{E}\sin esE+\cdots\quad (4)$$

so that

$$\mathfrak{L}=\tfrac{1}{2}(E^2-H^2)-\frac{1}{8\pi^2}\int ds\, s^{-3}\exp(-m^2s)\bigg[(es)^2$$
$$\times\left\{\frac{E}{es}\cot esE+\tfrac{1}{2}(\mathbf{E}\cdot\mathbf{H})^2\left(\frac{-1}{E^2}+\frac{es}{E}\cot esE\right)\right.$$
$$\left.+\tfrac{1}{2}H^2\right\}-1+\tfrac{1}{3}(es)^2(E^2-H^2)\bigg].\quad (5)$$

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[1] J. J. Klein and B. P. Nigam, Phys. Rev. **135**, B1279 (1964).

[2] J. Schwinger, Phys. Rev. **82**, 664 (1951).

[3] H. Euler and B. Kockel, Naturwiss. **23**, 246 (1935); W. Heisenberg and H. Euler, Z. Physik **98**, 714 (1936); V. Weisskopf, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **14**, 6 (1936); R. Karplus and M. Neuman, Phys. Rev. **80**, 380 (1950); **83**, 776 (1951).

In Eq. (5) only the two terms which contain $\cot esE$ have poles at $esE=n\pi$, where $n=1, 2, 3\cdots$ (there being no pole at $s=0$). Calculating the residues at $s=n\pi/eE$, we obtain the following expression for the Lagrangian density correct to order e^2.

$$\mathcal{L}=\tfrac{1}{2}(E^2-H^2)-\frac{ie^2}{8\pi}\left[E^2\Phi_1\left(\frac{eE}{m^2\pi}\right)+\frac{1}{2}\left(\frac{\mathbf{E}\cdot\mathbf{H}}{E}\right)^2\Phi_2\left(\frac{eE}{m^2\pi}\right)\right], \quad (6)$$

where

$$\Phi_1(x)=\sum_{n=1}^{\infty}\frac{\exp(-n/x)}{n^2\pi^2}$$
$$\Phi_2(x)=\sum_{n=1}^{\infty}\exp(-n/x)=[\exp(1/x)-1]^{-1}. \quad (7)$$

The permittivity and permeability constants ϵ_{ij} and μ_{ij} can easily be deduced from the relations

$$D_i=\partial\mathcal{L}/\partial E_i=\epsilon_{ij}E_j$$
$$B_i=-\partial\mathcal{L}/\partial H_i=\mu_{ij}H_j. \quad (8)$$

From Eqs. (6) and (8) we obtain that

$$\epsilon_{ij}=\delta_{ij}+\text{terms of order }(H/E)^2$$
$$\mu_{ij}=\delta_{ij}+\tfrac{1}{2}i\alpha\frac{E_iE_j}{E^2}\Phi_2\left(\frac{eE}{m^2\pi}\right). \quad (9)$$

Let us now consider that a strong external uniform electric field E is present along the x direction ($E_x=E$, $E_y=E_z=0$). Then from Eq. (9) we have

$$\mu_{\parallel}=\mu_{xx}=1+\tfrac{1}{2}i\alpha\Phi_2=1+\tfrac{1}{2}i\alpha(e^{m^2\pi/eE}-1)^{-1}$$
$$\mu_{\perp}=\mu_{yy}=\mu_{zz}=1, \quad (10)$$

where $\mu_{\parallel}$ and $\mu_{\perp}$ stand for the permeability components parallel and perpendicular to the electric field. It is to be noted that the absorptive part of $\mu_{\parallel}$ is very small unless $E\sim m^2\pi/e$. Also, the absorptive part arises for the case of a pure electric field since the electric field is capable of producing pairs. For the case of a pure magnetic field there is no absorptive part. Taking $\epsilon_{\parallel}=\epsilon_{\perp}\simeq 1$ and recalling that the refractive index $n=(\mu\epsilon)^{1/2}$, we have

$$n_{\parallel}=(1+\tfrac{1}{2}i\alpha\Phi_2)^{1/2}\simeq 1+\tfrac{1}{4}i\alpha\Phi_2$$
$$n_{\perp}\simeq 1. \quad (11)$$

Consider an unpolarized beam of light traveling in the z direction. The components of the electric vectors $E_{\parallel}$ and $E_{\perp}$ in the parallel and perpendicular planes are given by

$$E_{\parallel}=E_0\exp(i\omega t-ik_{\parallel}z)$$
$$E_{\perp}=E_0\exp(i\omega t-ik_{\perp}z), \quad (12)$$

where

$$k_{\parallel}=k_0n_{\parallel}$$
$$k_{\perp}=k_0n_{\perp}, \quad (13)$$

$k_0=2\pi/\lambda$ being the magnitude of the wave vector in field-free vacuum, and $n_{\parallel}$ and $n_{\perp}$ are the refractive indices given by Eq. (11). If $I_{\perp}$ and $I_{\parallel}$ are the intensities along perpendicular and parallel directions, the fractional polarization P occurring after traversal of thickness z of the region of space in which the strong electric field acts can be defined as follows:

$$P=(I_{\perp}-I_{\parallel})/(I_{\perp}+I_{\parallel})$$
$$=\tanh(\tfrac{1}{4}k_0\alpha\Phi_2z). \quad (14)$$

For small values of the argument, the quantity $2\pi\times(\alpha/4)\Phi_2$ is the fractional polarization per wavelength of the path. There is partial polarization due to unequal absorption of the two components of the electric field. The polarization is strongly dependent on the applied electric field strength E, being essentially zero for $E\ll m^2\pi/e$, and proportional to E for $E\gg m^2\pi/e$.

As a result of Eq. (11), the component of the light beam polarized parallel to the external electric field is absorbed, leaving in its path positron-electron pairs. The component polarized perpendicular to the pair-forming external electric field passes through unaltered, except for a change of speed of propagation due to birefringence[1] previously discussed. This behavior of the vacuum in a strong electric field is analogous to that of a dichroic crystal such as polaroid. An unpolarized light beam becomes partially polarized in passing through the polaroid because the component of the electric vibration perpendicular to the polaroid axis is absorbed whereas the other component is not. The vacuum, in the presence of a very strong electric field $E\sim\pi m^2/e\sim 10^{16}$ V/cm, behaves in a like manner.

III. DISCUSSION

The electric field strengths required for demonstrating the dichroism of the vacuum, being about 10^{16} V/cm (10^{10} MV/cm), poses a real problem in detecting this effect. Because of the phenomenon of field emission of electrons, neutral atoms (in the gaseous phase) become unstable when subjected to fields of the order of a few hundred MV/cm. A solid object, however, can be subjected to somewhat higher fields, provided that the emission of electrons is inhibited by raising the surface to a very high positive potential. Eventually, "dissolution" of the lattice of the material occurs, since the cohesive strength of the material (which is basically of electrical origin) may be exceeded. This is the so-called "field evaporation."[4] At a field strength $\sim$1000 MV/cm in tungsten, and $\sim$1400 MV/cm in carbon, for example, the field evaporation becomes significant. These fields

[4] Erwin W. Miller, in *Advances in Electronics and Electron Physics* (Academic Press Inc., New York, 1960), Vol. XIII, p. 102.

are 10^7 times lower than those required for the vacuum to become dichroic. Since the period of oscillation of a light wave is about 2×10^{-15} sec, only a very short-duration electrical field could be used to demonstrate this dichroism, and dissolution might be avoided.

Pulsed laser beams have been generated whose power is about 500 MW. If this beam could be focused to an area of the order λ^2, λ being the wavelength of light, the resulting intensity would be $\sim2\times10^{17}$ W/cm^2 corresponding to an alternating electric field of amplitude $\sim10^{10}$ V/cm. This is just comparable with the highest field encountered in field evaporation work.

The field strengths in question ($\sim10^{16}$ V/cm) are comparable with those existing at the surface of the nucleus of an atom. $E=e/r^2$, for a proton is $\sim2\times10^{18}$ V/cm, assuming a proton radius $\sim2.5\times10^{-13}$ cm. However, the nuclear electric field is confined to a very small volume. Furthermore, these fields are usually shielded by the surrounding clouds of negative charge due to bound electrons. Therefore a light wave having wavelength much greater than the size of an atom would not sense the strong nuclear field. It seems unlikely that at present the dichroism of the vacuum can be observed.

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Parity, Charge Conjugation, and Time Reversal in the Gravitational Interaction*

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Some consequences of possible violation of parity (P), charge conjugation (C), time reversal (T), and TCP invariance in the gravitation interaction are discussed and compared with existing experimental evidence. The evidence is suggestive of TCP (or CP or C) invariance, but one cannot rule out separate violation of P, C, and T.

I. INTRODUCTION

STRONG evidence[1] exists indicating that parity (P), time reversal (T), and charge conjugation (C) are absolute invariance properties of the strong and electromagnetic interactions. Further, P and C are known to be violated[2] in weak interactions, but T (or PC, related to T by the TCP theorem[3]) is apparently conserved.[4] It has long been conjectured that the relative lack of symmetry in the weak interactions is somehow connected with (or even responsible for) the relative weakness of the force. Additional evidence for such a symmetry-strength correlation[5] may be found if one considers other symmetries referring to internal properties such as isospin (I) and strangeness (S). For instance, only the strong interaction conserves both I and S; the electromagnetic (EM) interaction conserves S but not I, while the weak interaction violates both S and I.

A presumed symmetry-strength correlation provides the phenomenological justification for the "perturbation" approach toward understanding the known hierarchy of interactions. This approach attributes the breakdown of isotopic spin symmetry, i.e., multiplet mass splittings, to the isospin-violating, relatively weak EM interaction. Similarly, one usually attributes the supermultiplet mass splitting, i.e., the breakdown of the new SU_3 symmetries,[6] to an unspecified but presumably relatively weak part of the strong interactions. The assumption of weakness, together with transformation properties analogous to those of the EM interaction, led to the Gell-Mann–Okubo mass formula.[7] The recent

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[1] For references see, for example, K. Gotow and S. Okubo, Phys. Rev. **128**, 1921 (1962).

[2] For references see, for example, J. Jackson, *The Physics of Elementary Particles* (Princeton University Press, Princeton, New Jersey, 1958), Chap. 8–10.

[3] G. Lüders, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **28**, 5 (1954). Also see W. Pauli, in *Niels Bohr and the Development of Physics* (Pergamon Press, Ltd., London, 1955). For direct evidence of the validity of CPT, see R. G. Sachs, Phys. Rev. **129**, 2280 (1963).

[4] For evidence in β decay, see M. Burgy, V. Krohn, T. Novey, G. Ringo, and V. Telegdi, Phys. Rev. Letters **1**, 324 (1958). For evidence in Λ decay, Σ decay, and Ξ decay, see J. Cronin and O. Overseth, Phys. Rev. **129**, 1795 (1963). E. F. Beall, B. Cork, D. Keefe, P. Murphy, and W. Wentzel, Phys. Rev. Letters **8**, 75 (1962), and H. K. Ticko, Proceedings International Conference on Fundamental Aspects of Weak Interactions, Brookhaven National Laboratory p. 410, 1963 (unpublished).

[5] See, for example, the discussion of Pais at the Fifth Rochester Conference and of Gell-Mann and Schwinger at the Sixth Rochester Conference (unpublished).

[6] M. Gell-Mann, California Institute of Technology, Internal Report CTSL-20 (unpublished); Phys. Rev. **125**, 1067 (1962). Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[7] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).