scribed in Appendix I. Since l can be any non-negative integer, these crossing matrices correspond precisely to the "rational" solutions [Eq. (12)] for the S-matrix elements. Note, finally, that this derivation holds for $l=0$, even though one channel is a "nonsense" channel, because the recurrence relations between the Legendre polynomials may be formally extended to the $l=0$ case.

Note added in proof. After this article was submitted for publication, Professor K. Wilson directed our attention to a paper by J. Rothleitner [J. Rothleitner, Z. Physik **177**, 287 (1964)] in which general solution for the two-channel static model is obtained by a technique substantially different from our own. We wish to thank Professor Wilson for informing us of Rothleitner's interesting work.

PYSICAL REVIEW VOLUME 136, NUMBER 5B 7 DECEMBER 1964

Leptonic Decay of Hyperons in an Intermediate-Boson Theory

SEITARO NAKAMURA
Department of Physics, University of Tokyo and Nihon University, Tokyo, Japan

AND

KUNIO ITAMI
Department of Physics, University of Tokyo, Tokyo, Japan

(Received 11 May 1964)

The energy spectrum of electrons emitted in the beta decay of hyperons is calculated on the basis of the intermediate-boson theory of Tanikawa and Watanabe. In the bare-nucleon approximation, an appreciable deviation of the spectrum shape from that calculated on the local limit of the universal Fermi interaction is expected.

RECENTLY, Marshak *et al.*[1] proposed a scheme of the weak interactions in which the weak boson responsible for the leptonic decay of hyperons is different from that responsible for the β decay of nucleon. On the other hand, Sato and one of the authors (SN)[2] considered a scheme in which the weak boson of the Tanikawa type[3,4] (with the baryon number) is responsible for the leptonic decay of hyperons, while both the weak boson of the Yukawa type (without baryon number) and that of the Tanikawa type take part in the β decay of nucleon. These two schemes are alike in distinguishing the leptonic decay of hyperons from the β-decay of nucleon by the difference in the weak-boson channel. We shall show that the expected energy spectrum of the electron emitted in the leptonic decay of hyperons should differ appreciably in these two schemes, which could therefore be tested by precision measurements.

For the sake of simplicity let us calculate the energy spectrum of the electrons emitted in the β decay of the Λ hyperon when it is mediated by the Tanikawa boson by using the lowest-order perturbation. We shall limit ourselves to the case in which the Hamiltonian for the interaction among leptons, baryons, and boson leads to the $V\pm A$ coupling types

$$\bar{p}\gamma_\alpha(1\pm\gamma_5)\Lambda\cdot\bar{e}\gamma_\alpha(1\pm\gamma_5)\nu \quad \text{and} \quad \bar{p}\gamma_\alpha(1\mp\gamma_5)\Lambda\cdot\bar{e}\gamma_\alpha(1\pm\gamma_5)\nu$$

in the local limit.

(i) Spin-0 boson

(a) $V+A$

$$H_0=[f_{\Lambda 0}\bar{\Lambda}(1\pm\gamma_5)e+g_{\Lambda 0}\bar{p}(1\pm\gamma_5)\nu]\phi_\Lambda+\text{H.c.} \quad (1a)$$

(b) $V-A$

$$H_0'=[f_{\Lambda 0}'\bar{p}(1\pm\gamma_5)e^c+g_{\Lambda 0}'\bar{\Lambda}(1\pm\gamma_5)\nu^c]\phi_\Lambda'+\text{H.c.} \quad (1b)$$

(ii) Spin-1 boson

(a) $V-A$

$$H_1=i[f_{\Lambda 1}\bar{\Lambda}\gamma_\alpha(1\pm\gamma_5)e+g_{\Lambda 1}\bar{p}\gamma_\alpha(1\pm\gamma_5)\nu]\phi_{\Lambda\alpha}+\text{H.c.} \quad (2a)$$

(b) $V+A$

$$H_1'=i[f_{\Lambda 1}'\bar{p}\gamma_\alpha(1\pm\gamma_5)e^c+g_{\Lambda 1}'\bar{\Lambda}\gamma_\alpha(1\pm\gamma_5)\nu^c]\phi_{\Lambda\alpha}'+\text{H.c.} \quad (2b)$$

Here e and ν denote the annihilation operators of electron and neutrino, respectively. We assume that ν and its charge conjugation ν^c are described by the four-component Dirac spinors.

[1] R. E. Marhsak, C. Ryan, T. K. Radha, and K. Raman, Phys. Rev. Letters **11**, 396 (1963); Nuovo Cimento **16**, 408 (1964).
[2] S. Nakamura and S. Sato, Progr. Theoret. Phys. (Kyoto) **29**, 325 (1963).
[3] Y. Tanikawa, Progr. Theoret. Phys. (Kyoto) **3**, 338 (1948); Proc. Intern. Conf. Theoret. Phys. Kyoto Tokyo, Japan, 1953, 369 (1954); Progr. Theoret. Phys. Kyoto, **10**, 361 (1953); Y. Tanikawa and K. Saeki, Progr. Theoret. Phys. (Kyoto) **10**, 232 (1953); Y. Tanikawa, Phys. Rev. **108**, 1615 (1957); Y. Tanikawa and S. Watanabe, Phys. Rev. **113**, 1344 (1959).
[4] S. Nakamura and K. Itami, Progr. Theoret. Phys. (Kyoto) **26**, 274 (1961).

Now let us evaluate the matrix element S for the decay process $\Lambda \to p+e^-+\bar{\nu}$ in each case.
(i) Spin-0 case

$$S=-i(2\pi)^4\delta(p_\Lambda-p_p-p_e-p_\nu)\times\left(\frac{M_\Lambda M M_e}{2p_{\Lambda0}p_{p0}p_{e0}p_{\nu0}}\right)^{1/2}\mathbf{M}, \quad (3)$$

(a) $V+A$

$$\mathbf{M}=(f_{\Lambda0}g_{\Lambda0}/2)(M_x^2-M_\Lambda^2-2p_\Lambda p_e)^{-1}\bar{u}_p(p_p)\gamma_\alpha(1\mp\gamma_5)\times u_\Lambda(p_\Lambda)\cdot\bar{u}_e(p_e)\gamma_\alpha(1\pm\gamma_5)v_\nu(p_\nu), \quad (3a)$$

(b) $V-A$

$$\mathbf{M}=(-f_{\Lambda0}'g_{\Lambda0}'/2)\cdot(M_x^2-M_\Lambda^2-2p_\Lambda p_\nu)^{-1}\bar{u}_p(p_p)\times\gamma_\alpha(1\pm\gamma_5)u_\Lambda(p_\Lambda)\cdot\bar{u}_e(p_e)\gamma_\alpha(1\pm\gamma_5)v_\nu(p_\nu). \quad (3b)$$

Here p_Λ, p_p, p_e, and p_ν are the four-momenta of the Λ, proton, electron, and neutrino, respectively. M_Λ and M_e are the masses of the Λ and electron, respectively. The term $2p_\Lambda p_e$ in the propagator (3) will contribute to the deviation of the electron spectrum from the local limit. From the matrix element (3) we can easily calculate the decay rate Γ:

(a) $V+A$

$$\Gamma=\frac{f_{\Lambda0}^2g_{\Lambda0}^2}{8\pi^3}\beta^2XE_{\max}\int_0^1 d\epsilon\frac{1-2X\epsilon}{2X}F^2\frac{1}{(1+\beta\epsilon)^2}, \quad (4a)$$

(b) $V-A$

$$\Gamma=\frac{f_{\Lambda0}'^2g_{\Lambda0}'^2}{8\pi^3}\beta^2XE_{\max}\int_0^1 d\epsilon\frac{1}{\beta^2}\left[(2+\beta)\ln\frac{1+\beta(1-\epsilon+F)}{1+\beta(1-\epsilon)}-\beta F\left(1+\frac{1+\beta}{[1+\beta(1-\epsilon)][1+\beta(1-\epsilon+F)]}\right)\right], \quad (4b)$$

where

$$E_{\max}=(M_\Lambda^2-M^2)/2M_\Lambda,$$
$$\epsilon=E_e/E_{\max},$$
$$X=E_{\max}/M_\Lambda,$$
$$\beta=2M_\Lambda E_{\max}/(M_x^2-M_\Lambda^2),$$
$$F=2X\epsilon(1-\epsilon)/(1-2X\epsilon).$$

E_e ($E_{\max}$) is the energy (maximum energy) of the electron. In the above expression, M_e is neglected in comparison with E_e.

In the local limit, the decay rates (4a) and (4b) reduce to the well-known result[5]

$$\Gamma=(G^2E_{\max}^5/\pi^3)C(X), \quad (5)$$

where

$$C(X)=\frac{1}{X}\int_0^1 d\epsilon\frac{1-2X\epsilon}{2X}F^2$$
$$=-\frac{1}{16}\frac{1}{X^5}(1-2X)^2\ln(1-2X)-\frac{1}{24}\frac{1}{X^4}(1-X)(3-6X-2X^2), \quad (6)$$

$$\frac{G}{\sqrt{2}}=\frac{1}{2}\frac{f_{\Lambda0}g_{\Lambda0}}{M_x^2-M_\Lambda^2}=\frac{1}{2}\frac{f_{\Lambda0}'g_{\Lambda0}'}{M_x^2-M_\Lambda^2}. \quad (7)$$

(ii) Spin-1 case

(a) $V-A$

$$\mathbf{M}=f_{\Lambda1}g_{\Lambda1}(M_x^2-M_\Lambda^2-2p_\Lambda p_e)^{-1}\times\bar{u}_p(p_p)\left[\left(1+\frac{M_\Lambda M}{2M_x^2}\right)\gamma_\alpha\pm\left(1-\frac{M_\Lambda M}{2M_x^2}\right)\gamma_\alpha\gamma_5\right]\times u_\Lambda(p_\Lambda)\bar{u}_e(p_e)\gamma_\alpha(1\pm\gamma_5)v_\nu(p_\nu), \quad (8a)$$

(b) $V+A$

$$\mathbf{M}=f_{\Lambda1}'g_{\Lambda1}'\frac{1}{M_x^2-M_\Lambda^2-2p_\Lambda p_\nu}\times\bar{u}_p(p_p)\left[\left(1+\frac{M_\Lambda M}{2M_x^2}\right)\gamma_\alpha\mp\left(1-\frac{M_\Lambda M}{2M_x^2}\right)\gamma_\alpha\gamma_5\right]\times u_\Lambda(p_\Lambda)\bar{u}_e(p_e)\gamma_\alpha(1\pm\gamma_5)v_\nu(p_\nu). \quad (8b)$$

Here we neglect the terms of the order of M_e/M_x. It must be noted that the weight α of the V and A couplings is given by

$$\alpha=\left(1+\frac{M_\Lambda M}{2M_x^2}\right):\left(1-\frac{M_\Lambda M}{2M_x^2}\right)=\begin{cases}1.6 & \text{for } M_x\approx1.5\text{ BeV},\\ 1.3 & \text{for } M_x\approx1.8\text{ BeV}.\end{cases} \quad (9)$$

The decay rate for each case is given as follows.

(a) $V-A$

$$\Gamma=\frac{f_{\Lambda1}^2g_{\Lambda1}^2}{2\pi^3}\beta^2XE_{\max}\int_0^1 d\epsilon\left[\frac{1-X}{2X}F^2-\tfrac{1}{3}F^3+\left(\frac{M_\Lambda M}{2M_x^2}\right)^2\frac{1-2X\epsilon}{2X}F^2-\frac{M^2}{2M_x^2}\frac{1}{2X}F^2\right]\frac{1}{(1+\beta\epsilon)^2}, \quad (10)$$

[5] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

(b) $V+A$

$$\Gamma=\frac{f_{\Lambda 1}'^2 g_{\Lambda 1}'^2}{2\pi^3}\beta^2 X E_{\max}\int_0^1 d\epsilon\left\{\frac{1-2X\epsilon}{2X}F^2\frac{1}{[1+\beta(1-\epsilon)][1+\beta(1-\epsilon+F)]}+\left(\frac{M_\Lambda M}{2M_x^2}\right)^2\frac{1}{\beta^2}\right.$$
$$\times\left[(2+\beta)\ln\frac{1+\beta(1-\epsilon+F)}{1+\beta(1-\epsilon)}-\beta F\left(1+\frac{1+\beta}{[1+\beta(1-\epsilon)][1+\beta(1-\epsilon+F)]}\right)\right]$$
$$\left.-\frac{M^2}{2M_x^2}\frac{1}{X\beta^2}\left(\ln\frac{1+\beta(1-\epsilon+F)}{1+\beta(1-\epsilon)}-\frac{\beta F}{1+\beta(1-\epsilon+F)}\right)\right\}. \quad (11)$$

In the local limit, the decay rate reduces to (5), with allowance being made for the expression of G:

$$\frac{G}{\sqrt{2}}=\frac{f_{\Lambda 1}g_{\Lambda 1}}{M_x^2-M_\Lambda^2}=\frac{f_{\Lambda 1}'g_{\Lambda 1}'}{M_x^2-M_\Lambda^2}. \quad (12)$$

In Figs. 1(a) and 1(b) we illustrate the energy spectrum of electrons emitted in $\Lambda\to p+e^-+\bar{\nu}$ by assuming $M_x\approx 1.5$ BeV.

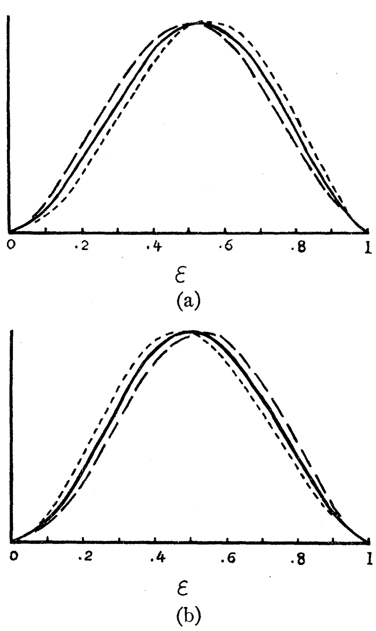

FIG. 1. Energy spectrum of the electron. (a) $V-A$ case. $V+A$ case. The solid line is the local limit ($M_x=\infty$), the dotted line is the spin-0 case, and the dashed line is the spin-1 case.

The spectrum shapes expected from the Tanikawa boson theory show appreciable deviation compared to that expected from the theory in the local limit ($M_x=\infty$); the point of maximum in the spectrum should shift towards the higher (lower) energy side for the cases spin-0 $V+A$, spin-1 $V-A$ (spin-0 $V-A$, spin-1 $V+A$). These tendencies are the outcome of the larger momentum transfer which is a characteristic feature of the Tanikawa boson channel. On the other hand, it is easily inferred that, as long as $M_{W_1}>1.5$ BeV, the W_1-boson channel in the theory of Marshak *et al.* will lead to essentially the same spectrum shape with the expected spectrum in the local limit.

In the above calculation, we did not take into account the possible effects of the strong interactions on the matrix elements. We have to deal with as many as six kinds (four kinds when we neglect the mass of electron before the mass of baryon) of form factors in the matrix elements even in the local limit. For the matrix elements in the nonlocal theory of Tanikawa boson, in which a lepton pair couples nonlocally, the effects of the strong interactions depend essentially on a function of two variables besides the momentum transfer. At present we have no clue to estimate the magnitudes of these form factors. The present calculation should, therefore, be regarded as an approximation which is valid when the form factors are negligible before the effects of the difference in momentum transfer between the local theory and the nonlocal theory become appreciable.