PHYSICAL REVIEW VOLUME 136, NUMBER 5B 7 DECEMBER 1964

Higher Baryon Resonances in the Static Model*

E. S. ABERS,† L. A. P. BALÁZS,‡ AND Y. HARA§
California Institute of Technology, Pasadena, California
(Received 17 July 1964)

The N/D solution to the static model, with linear approximation to the D function, is applied to a sequence of meson-baryon scattering problems. As in the old strong-coupling model, it is found that the N and Δ are the first two members of a sequence of particles with $I=J=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \cdots$, and that an analogous sequence is obtained when strange particles and SU(3) symmetry are included. In both cases, the $J=\frac{5}{2}$ particle is discussed in some detail. In the SU(2) case, general expressions are also derived for the widths of all the other members the sequence.

I. INTRODUCTION

THE simplest model for the P-wave scattering of baryons and pseudoscalar mesons is the static limit of the N/D method, in which only baryon-exchange forces are considered and the D function is approximated by a straight line. The essential element in this approximation is that the baryons are considered so heavy compared to the mesons that their recoil can be neglected. This was the model used by Chew[1] to illustrate the reciprocal bootstrap between the nucleon (N) and the $(\frac{3}{2},\frac{3}{2})$ isobar (Δ). Within the same calculational model, an analogous scheme has been shown recently to work for the SU(3) multiplets to which the N and the Δ belong.[2,3]

In this model, only ratios of coupling constants can be calculated. Unless additional dynamical assumptions are made,[4] we must give up the possibility of determining mass differences. However, we can always check whether the signs and relative magnitudes of the forces are consistent with a given scheme of particles. By considering the scattering of mesons from excited states of nucleons, we find that the method predicts a sequence of baryon states in both the SU(2) and SU(3) models, of which the familiar N and Δ (spin-$\frac{1}{2}$ octet and spin-$\frac{3}{2}$ decimet) are the first two.

The calculational scheme is elementary. A meson-baryon state is specified by its spin J, isotopic spin I, and total energy W. As is customary, let us use as variable $\omega=W-M_i$, where M_i is the mass of the baryon and $i=2\times$(spin of the baryon). Only P waves occur in our calculations, so we use the amplitude

$$f_{IJ}(\omega)=e^{i\delta}\sin\delta/q^3\,, \tag{1.1}$$

where $q^2=(\omega^2-1)$ and δ is the phase shift. The meson mass is 1. The dynamics are provided by the crossing relation

$$f_{IJ}(\omega)=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}f_{I'J'}(-\omega)\,. \tag{1.2}$$

Here, α and β are the crossing matrices for isotopic spin and spin, respectively. In our static model, only P states occur, and spin is conserved just like isotopic spin. Thus both α and β are finite-dimensional.

If a particle (which can be either a bound state or a resonance) occurs in a state with quantum numbers I and J, the corresponding amplitude has a pole $\gamma_{IJ}{}^i/(\omega_{IJ}-\omega)$. The force (or Born term), for which we take the exchange of all P-wave particles, then has the form

$$B_{IJ}(\omega)=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}\gamma_{I'J'}{}^i/(\omega_{I'J'}+\omega)\,. \tag{1.3}$$

If we define $\gamma_{IJ}{}^i=0$ whenever there is no particle in the (I,J) state, the sum in (1.3) may run over all possible I' and J'. We can now use $B_{IJ}(\omega)$ as the input to an N/D calculation of f_{IJ} to obtain

$$N_{IJ}(\omega)=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}\gamma_{I'J'}{}^iD(-\omega_{I'J'})/\omega_{I'J'}+\omega)\,, \tag{1.4}$$

$$D_{IJ}(\omega)=1-\frac{\omega-\omega_0}{\pi}\int_1^\Lambda d\omega'\frac{(\omega'^2-1)^{3/2}N_{IJ}(\omega')}{(\omega'-\omega_0)(\omega'-\omega)}\,, \tag{1.5}$$

where ω_0 is some subtraction point and Λ is a cutoff representing our ignorance of high-energy effects. The expressions (1.4) and (1.5) guarantee elastic unitarity and can thus be used whenever a one-channel approximation[5] is valid.

Suppose Eqs. (1.4) and (1.5) give a dynamical particle in the (I,J) state. Following Chew,[1] we can approximate Eq. (1.5) by a straight line

$$D_{IJ}(\omega)=(\omega_{IJ}-\omega)/(\omega_{IJ}-\omega_0)\,. \tag{1.6}$$

Then

$$\gamma_{IJ}{}^i=-N(\omega_{IJ})/D'(\omega_{IJ})=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}\gamma_{I'J'}{}^i\,. \tag{1.7}$$

* Work supported in part by the U. S. Atomic Energy Commission.
† Present address: CERN, Geneva, Switzerland.
‡ Present address: Tata Institute of Fundamental Research, Bombay, India.
§ On leave of absence from Physics Department, Tokyo University of Education, Tokyo, Japan. Present address: Physics Department, University of Chicago, Chicago, Illinois.
[1] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).
[2] Y. Hara, Phys. Rev. **135**, B1079 (1964).
[3] R. F. Dashen, Phys. Letters **11**, 89 (1964).
[4] One possible set of assumptions is made in the Appendix, where the masses of the $(\frac{3}{2},\frac{3}{2})$ and $(\frac{5}{2},\frac{5}{2})$ isobars are calculated in terms of the nucleon mass.

[5] The general arguments of this section are unchanged by the presence of well-behaved inelastic effects, which can be taken into account by replacing $N_{IJ}(\omega')$ in Eq. (1.5) by $R_{IJ}(\omega')N_{IJ}(\omega')$, where R_{IJ} is the ratio of total to elastic partial-wave cross sections. This does not change any of the subsequent equations. Similarly, the particular type of cutoff we have chosen is not crucial.

This result suggests that a reasonable measure of the force which is independent of the unknown masses is

$$F_{IJ}{}^{i}=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}\gamma_{I'J'}{}^{i}. \tag{1.8}$$

Thus, $F_{IJ}{}^{i}>0$ immediately provides us with a necessary condition for the existence of a particle, since all the residues $\gamma_{IJ}{}^{i}$ must be positive. Of course, this condition, which corresponds roughly to the force being attractive, is not sufficient, but we cannot do any better without actually calculating the D function in Eq. (1.6). From Eqs. (1.4) and (1.5), it is evident that, with reasonably smooth high-energy behavior, particles in those states in which F_{IJ} is largest may be expected to have the lowest mass, and be therefore important in the dynamics. On the other hand, if F_{IJ} is small, we expect the particle, if it exists at all, to have a very high mass and be therefore unimportant. Thus, from F_{IJ} we shall predict what states may have a particle and roughly the order of the masses, while from Eq. (1.7) we shall obtain relations among the coupling constants.

Chew's reciprocal bootstrap[1] is essentially Eq. (1.7) applied to the πN system assuming dynamical particles in the $(\frac{1}{2},\frac{1}{2})$ and $(\frac{3}{2},\frac{3}{2})$ states. The crossing matrices α and β are the same in this case:

$$\alpha=\beta=\begin{pmatrix}-\frac{1}{3} & \frac{4}{3}\\ \frac{2}{3} & \frac{1}{3}\end{pmatrix} \tag{1.9}$$

for $I,J=\frac{1}{2},\frac{3}{2}$. The two equations (1.7) in this case turn out to be identical.

$$\gamma^{1}{}_{1/2,1/2}=2\gamma^{1}{}_{3/2,3/2}, \tag{1.10}$$

which is consistent with experiment. If this result is now substituted into Eq. (1.8), we find the measure of the forces in the three states

$$\begin{aligned}F^{1}{}_{3/2,3/2}&=\gamma^{1}{}_{3/2,3/2},\\ F^{1}{}_{3/2,1/2}&=F^{1}{}_{1/2,3/2}=0,\\ F^{1}{}_{1/2,1/2}&=2\gamma^{1}{}_{3/2,3/2}.\end{aligned} \tag{1.11}$$

Thus the assumed system of particles is consistent with the force criterion of the preceding paragraph. Furthermore, we may expect $\omega_{1/2,1/2}<\omega_{3/2,3/2}$, in agreement with experiment.

But one must be careful not to exaggerate the predictive powers of this method. For, in fact, if we had assumed the existence of $(\frac{1}{2},\frac{3}{2})$ and $(\frac{3}{2},\frac{1}{2})$ particles as well, we would have obtained four equations (1.7), which reduce to the two conditions

$$\begin{aligned}\gamma^{1}{}_{1/2,3/2}&=\gamma^{1}{}_{3/2,1/2},\\ 2\gamma^{1}{}_{3/2,3/2}&=\gamma^{1}{}_{1/2,1/2}+\gamma^{1}{}_{1/2,3/2}.\end{aligned} \tag{1.12}$$

All four F_{IJ}'s are positive in this case and Chew's solution (1.10) is seen to be only one of a continuum of possible solutions.

The generalization of the πN reciprocal bootstrap to the corresponding SU(3) multiplets is straightforward, although complicated slightly by the fact that the octet state has to be treated as a two-channel problem, so that Eq. (1.7) cannot be used. This problem is discussed in detail in Refs. 2 and 3.

II. $\pi\Delta$ SCATTERING

Now we apply the same method to the scattering of pions by the $I=J=\frac{3}{2}$ baryon (Δ). Here, forces are provided at least by N and Δ exchange; both these particles are $\pi\Delta$ P waves. However, our method is valid only in these states in which $I=\frac{5}{2}$ or $J=\frac{5}{2}$, since the others communicate with the lower lying πN channel and therefore cannot be treated as single-channel problems. The crossing matrices α and β are once again the same:

$$\alpha=\beta=\begin{bmatrix}1/6 & -2/3 & 3/2\\ -1/3 & 11/15 & 3/5\\ 1/2 & 2/5 & 1/10\end{bmatrix} \tag{2.1}$$

for $I,J=\frac{1}{2},\frac{3}{2},\frac{5}{2}$.

What forces are there? We must certainly include N and Δ exchange, which we would expect to dominate. From the crossing matrix (2.1), $F^{3}{}_{5/2,5/2}$ is evidently positive, while the sign in the remaining single-channel states depends on the ratio $\gamma^{3}{}_{3/2,3/2}/\gamma^{3}{}_{1/2,1/2}$. Thus, we can predict an $I=J=\frac{5}{2}$ particle; if we also exchange it, Eq. (1.7) gives

$$\gamma^{3}{}_{5/2,5/2}=(1/4)\gamma^{3}{}_{1/2,1/2}+(4/25)\gamma^{3}{}_{3/2,3/2}+(1/100)\gamma^{3}{}_{5/2,5/2} \tag{2.2}$$

or

$$\frac{99}{100}\frac{\gamma^{3}{}_{5/2,5/2}}{\gamma^{3}{}_{1/2,1/2}}=\frac{1}{4}+\frac{4}{25}\frac{\gamma^{3}{}_{3/2,3/2}}{\gamma^{3}{}_{1/2,1/2}}. \tag{2.3}$$

To check on the possibility of particles in the other states, we need the ratio $(\gamma^{3}{}_{3/2,3/2}/\gamma^{3}{}_{1/2,1/2})$. Since $\gamma^{3}{}_{1/2,1/2}$ is known in terms of the $\pi N\Delta$ coupling constant, we then could also estimate from Eq. (2.3) the absolute magnitude of $\gamma^{3}{}_{5/2,5/2}$, thereby obtaining a prediction for the width of the decay of this resonance into $\pi+\Delta$.

Consider the amplitude A_{IJ} for the inelastic process $\pi N\rightarrow\pi\Delta$. When there is a particle in the (I,J) state, this amplitude has a pole $(\gamma_{IJ}{}^{3}\gamma_{IJ}{}^{1})^{1/2}/(\omega_{IJ}-\omega)$, where $\omega=W-M_1$. The Born term from the exchange of these particles is then

$$B_{IJ}(\omega)=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}(\gamma_{I'J'}{}^{3}\gamma_{I'J'}{}^{1})^{1/2}/(\omega_{I'J'}{}'+\omega), \tag{2.4}$$

where $\omega_{IJ}{}'=\omega_{IJ}-(M_3-M_1)$. The crossing matrices for this case are

$$\alpha=\beta=\begin{pmatrix}2/3 & -(\sqrt{10})/3\\ -(\sqrt{10})/6 & -2/3\end{pmatrix} \tag{2.5}$$

for $I,J=\frac{1}{2},\frac{3}{2}$.

As in the case of elastic scattering, let us assume that the lowest intermediate state, i.e., the πN state, is the most important. This means that the unitarity relation for A_{IJ} is

$$\mathrm{Im}A_{IJ}\propto f_{IJ}{}^*A_{IJ}=(N_{IJ}(\omega)/D_{IJ}{}^*(\omega))A_{IJ}. \quad (2.6)$$

Since $\mathrm{Im}A_{IJ}$ and N_{IJ} are real, A_{IJ} must have the same phase[6] as $D_{IJ}{}^{-1}$ on the right-hand cut. Thus it has the form

$$A_{IJ}(\omega)=n_{IJ}(\omega)/D_{IJ}(\omega), \quad (2.7)$$

where $n_{IJ}(\omega)$ has only left-hand singularities, $\mathrm{Im}n_{IJ}=\mathrm{Im}A_{IJ}D_{IJ}=\mathrm{Im}B_{IJ}D_{IJ}$. Therefore, $n_{IJ}(\omega)$ satisfies

$$n_{IJ}(\omega)=\sum_{I'J'}\alpha_{II'}\beta_{JJ'}[(\gamma_{I'J'}{}^3\gamma_{I'J'}{}^1)^{1/2}/(\omega_{I'J'}{}'+\omega)] \times D_{I'J'}(-\omega_{I'J'}). \quad (2.8)$$

If we use Eq. (1.6) for D_{IJ}, a direct-channel pole residue is

$$(\gamma_{IJ}{}^3\gamma_{IJ}{}^1)^{1/2}=-n_{IJ}(\omega_{IJ})/D_{IJ}{}'(\omega_{IJ}) =\sum_{I'J'}\alpha_{II'}\beta_{JJ'}(\gamma_{I'J'}{}^3\gamma_{I'J'}{}^1)^{1/2}, \quad (2.9)$$

a result analogous to Eq. (1.7).

Since $\gamma_{IJ}{}^1=0$ unless $I=J=\frac{1}{2},\frac{3}{2}$, the $\pi N\to\pi\Delta$ forces are provided by N and Δ exchange. The two equations (2.9) then turn out to be identical, and are

$$2(\gamma^3{}_{3/2,3/2}\gamma^1{}_{3/2,3/2})^{1/2}=(\gamma^3{}_{1/2,1/2}\gamma^1{}_{1/2,1/2})^{1/2}. \quad (2.10)$$

If this result is combined with Eq. (1.10), we obtain

$$\gamma^3{}_{3/2,3/2}/\gamma^3{}_{1/2,1/2}=\tfrac{1}{2}. \quad (2.11)$$

Finally, substituting (2.11) into (2.3), we get

$$\gamma^3{}_{5/2,5/2}/\gamma^3{}_{1/2,1/2}=\tfrac{1}{3}. \quad (2.12)$$

But $\gamma^3{}_{1/2,1/2}$ is related kinematically to the $\pi N\Delta$ coupling constant and hence to $\gamma^1{}_{3/2,3/2}$ through $\gamma^3{}_{1/2,1/2}=4\gamma^1{}_{3/2,3/2}$. Using Eq. (1.10), we therefore obtain finally

$$\gamma^3{}_{5/2,5/2}=\tfrac{2}{3}\gamma^1{}_{1/2,1/2}=2f^2 \quad (2.13)$$

as the prediction of our static model for the width of the $I=J=\frac{5}{2}$ resonance. Here, f^2 is the πNN pseudovector coupling constant and is numerically about 0.08.

Recently, two $p\pi^+\pi^+$ resonances have been observed, one at 1560 MeV[7] and one at 2400 MeV.[8] Let us assume that the former can be identified with our $(\frac{5}{2},\frac{5}{2})$ particle (see the Appendix). In that case, the latter may be its Regge recurrence with $J=\frac{9}{2}$. Then the slope of a straight-line Regge trajectory[9] in the energy variable would be roughly the same as the slopes of N, Δ, and Λ trajectories.[10] Plotting a Breit-Wigner formula for the $J=I=\frac{5}{2}$ $\pi\Delta$ elastic cross section,

$$\sigma_{5/2,5/2}=\frac{12\pi q^4(\gamma^3{}_{5/2,5/2})^2}{(\omega-\omega_{5/2,5/2})^2+q^6(\gamma^3{}_{5/2,5/2})^2}, \quad (2.14)$$

we find a full width at half-maximum of about 210 MeV, if $\omega_{5/2,5/2}$ is chosen so that the maximum is at 1560 MeV. The width of the resonance of Ref. 7 is of the order of 200 MeV.

Finally we can use Eq. (2.12) to check on the forces in the other single-channel states. The result, in remarkable analogy to (1.11), is

$$F^3{}_{5/2,1/2}=F^3{}_{1/2,5/2}=F^3{}_{5/2,3/2}=F^3{}_{3/2,5/2}=0. \quad (2.15)$$

Thus it is consistent to assume that there are no resonances in the other $\pi\Delta$ states with $I=\frac{5}{2}$ or $J=\frac{5}{2}$.[11]

III. OCTET-DECIMET SCATTERING

In this section we shall repeat the above calculation with all the particles involved promoted to SU(3) multiplets. Thus we consider the scattering of an octet of pseudoscalar P-wave mesons off a decimet of spin-$\frac{3}{2}$ baryons. The reduction of the direct product meson-baryon states is

$$\mathbf{8}\times\mathbf{10}=\mathbf{8}+\mathbf{10}+\mathbf{27}+\mathbf{35}. \quad (3.1)$$

Since no representation occurs twice on the right side of Eq. (3.1), we have to deal only with one-channel problems and therefore may apply the same scheme we used in the preceding sections.

The crossing matrix $\alpha_{II'}$ in Eq. (1.7) is now replaced by[12]

$$\alpha=\begin{pmatrix} 1/5 & -1/2 & -9/20 & 7/4 \\ -2/5 & 3/4 & -9/40 & 7/8 \\ -2/15 & -1/12 & 37/40 & 7/24 \\ 2/5 & 1/4 & 9/40 & 1/8 \end{pmatrix}, \quad (3.2)$$

where the rows and columns are labeled by the representation dimension $F=8, 10, 27, 35$. Since all the entries in the bottom line of Eq. (3.2) are positive, as are those in the bottom row of (2.1), we find from our force criterion that a quinquetrigesimet of spin-$\frac{5}{2}$ baryons may occur. If we also exchange these 35 isobars, then the SU(3) versions of (2.2) and (2.3) are

$$\gamma_{35}{}^3=(1/5)\gamma_8{}^3+(1/10)\gamma_{10}{}^3+(1/80)\gamma_{35}{}^3 \quad (3.3)$$

or

$$\frac{79}{80}\frac{\gamma_{35}{}^3}{\gamma_8{}^3}=\frac{1}{5}+\frac{1}{10}\frac{\gamma_{10}{}^3}{\gamma_8{}^3}, \quad (3.4)$$

where the isospin index in the residues has been re-

[6] R. Omnès, Nuovo Cimento 8, 316 (1958).

[7] G. Goldhaber (private communication).

[8] This resonance is listed by M. Roos, Nucl. Phys. 52, 1 (1964).

[9] S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. 126, 2204 (1962); G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961) and 8, 41 (1962); R. Blankenbecler and M. L. Goldberger, Phys. Rev. 126, 766 (1962).

[10] These Regge trajectories are shown in G. F. Chew, M. Gell-Mann, and A. H. Rosenfeld, Sci. Am. 210, 74 (February 1964).

[11] This should be contrasted with the conclusions of A. Messiah, Phys. Letters 1, 181 (1962), who also considered π-Δ scattering but predicted $(\frac{3}{2},\frac{5}{2})$ and $(\frac{5}{2},\frac{3}{2})$ resonances. This appears to be a consequence of his neglect of nucleon exchange. He also neglected the πN intermediate state in the $(\frac{1}{2},\frac{1}{2})$, $(\frac{1}{2},\frac{3}{2})$, $(\frac{3}{2},\frac{1}{2})$, and $(\frac{3}{2},\frac{3}{2})$ states.

[12] D. E. Neville, Phys. Rev. 132, 844 (1963).

placed by an F index and the spin index has been dropped for brevity.

To find the ratio $\gamma_{10}{}^3/\gamma_8{}^3$, we may generalize the $\pi N\to\pi\Delta$ process to SU(3) and then follow exactly the procedure in the previous section. The isotopic spin crossing matrix α in (2.5) is replaced by

$$\alpha=\begin{bmatrix} 0 & 1/\sqrt{5} & -(\sqrt{10})/4 & 9/4\sqrt{5} \\ 1/\sqrt{5} & 2/5 & -\sqrt{2}/4 & -27/20 \\ -\sqrt{(2/5)} & -\sqrt{2}/5 & -1/2 & -9/10\sqrt{2} \\ 2/3\sqrt{5} & -2/5 & -\sqrt{2}/6 & 1/10 \end{bmatrix}, \tag{3.5}$$

where now the rows are labeled by $F=8_A,\,8_S,\,10,\,27$. Here, the amplitudes labeled 8_A and 8_S are for transitions from the antisymmetric and symmetric octets appearing in $\mathbf{8}\times\mathbf{8}$ into the octet appearing in $\mathbf{8}\times\mathbf{10}$. Their residues are $(\gamma_8{}^3\gamma_{8A}{}^1)^{1/2}$ and $(\gamma_8{}^3\gamma_{8S}{}^1)^{1/2}$, where, in terms of the πNN coupling constant f^2,[2,3]

$$\gamma_{8A}{}^1=12(1-\alpha)^2f^2, \tag{3.6}$$

$$\gamma_{8S}{}^1=(20/3)\alpha^2f^2, \tag{3.7}$$

where $\alpha/(1-\alpha)$ is the usual D to F ratio. Applying the generalization of Eq. (2.9) to the spin-$\frac{3}{2}$ decimet state, we obtain

$$\gamma_{10}{}^3/\gamma_8{}^3=(1-\tfrac{2}{3}\alpha)^2 3f^2/\gamma_{10}{}^1. \tag{3.8}$$

The ratio $(f^2/\gamma_{10}{}^1)$ has already been calculated by considering the amplitude in the spin-$\frac{3}{2}$ decimet state of elastic octet-octet scattering.[2,3] The result is

$$\gamma_{10}{}^1=(16/11)[\tfrac{4}{3}\alpha^2+4\alpha(1-\alpha)]f^2. \tag{3.9}$$

Substituting this into (3.8), we obtain

$$\gamma_{10}{}^3/\gamma_8{}^3=(11/64)(3-2\alpha)/\alpha. \tag{3.10}$$

To get an absolute number, we need the D to F ratio. This has been calculated in Refs. 2 and 3 using slightly different methods. The results are all in agreement with the poorly known experimental value, so in Table I we list the values of $(\gamma_{10}{}^3/\gamma_8{}^3)$ and $(\gamma_{35}{}^3/\gamma_8{}^3)$ for all the calculated values of α; these ratios are related to the residue ratios for the nonstrange components by simple kinematical factors. We see that the ratio $(\gamma^3{}_{5/2,5/2}/\gamma^3{}_{1/2,1/2})$ is not very sensitive to the value of α and is approximately the same as the ratio $\frac{1}{3}$ which we obtained using the SU(2) model; therefore, we predict essentially the same width for the $I=J=\frac{5}{2}$ $\pi\Delta$ resonance as before.

TABLE I. Values of $(\gamma_{10}{}^3/\gamma_8{}^3)$ and $(\gamma_{35}{}^3/\gamma_8{}^3)$, using Eqs. (3.4) and (3.10) for various values of α. The values $\alpha=0.57$, 0.78 correspond to the two solutions obtained in Ref. 3, while $\alpha=0.69$ was the value calculated in Ref. 2. The values of $(\gamma^3{}_{5/2,5/2}/\gamma^3{}_{1/2,1/2})=(5/4)(\gamma_{35}{}^3/\gamma_8{}^3)$ and $(\gamma^3{}_{3/2,3/2}/\gamma^3{}_{1/2,1/2})=(25/32)(\gamma_{10}{}^3/\gamma_8{}^3)$ should be compared with the results of the SU(2) model of Sec. II.

α	$(\gamma_{10}{}^3/\gamma_8{}^3)$	$(\gamma_{35}{}^3/\gamma_8{}^3)$	$(\gamma^3{}_{3/2,3/2}/\gamma^3{}_{1/2,1/2})$	$(\gamma^3{}_{5/2,5/2}/\gamma^3{}_{1/2,1/2})$
0.57	0.56	0.26	0.44	0.32
0.69	0.40	0.24	0.32	0.31
0.78	0.32	0.24	0.25	0.29

TABLE II. The masses (in MeV) of the multiplets contained in the 35-dimensional representation calculated from Eq. (3.11) with $a=-191$ MeV and $b=32$ MeV and with m_0 fixed by the requirement that $m_{1,5/2}=1560$ MeV. The corresponding lowest strong thresholds are listed for comparison.

(Y,I)	m_{YI}	Strong threshold
(2, 2)	1260	πNK (1575)
(1, $\frac{3}{2}$)	1400	πN (1080)
(0, 1)	1540	$\pi\Lambda$ (1255)
(-1, $\frac{1}{2}$)	1690	$\pi\Xi$ (1460)
(-2, 0)	1830	$\pi\pi\Omega$ (1955)
(1, $\frac{5}{2}$)	1560	$\pi\pi N$ (1220)
(0, 2)	1670	$\pi\Sigma$ (1330)
(-1, $\frac{3}{2}$)	1780	$\pi\Xi$ (1460)
(-2, 1)	1890	$\pi\Omega$ (1815)
(-3, $\frac{1}{2}$)	2000	$\bar{K}\Omega$ (2170)

Finally, we can use our force criterion to check on the possibility of other resonances in the one-channel octet-decimet states. Again we find that the other forces $F_{FJ}{}^3$ are negative or small compared with $F_{35,5/2}{}^3$. For instance, with $\alpha=0.57$, we have

$$F^3{}_{8,5/2}=0.033\gamma_8{}^3,\quad F^3{}_{10,5/2}=-0.009\gamma_8{}^3,$$
$$F^3{}_{27,5/2}=-0.074\gamma_8{}^3,\quad F^3{}_{35,1/2}=0.023\gamma_8{}^3,$$
$$F^3{}_{35,3/2}=-0.011\gamma_8{}^3,\quad F^3{}_{27,1/2}=0.122\gamma_8{}^3,$$

and

$$F_{27,3/2}{}^3=0.055\gamma_8{}^3,$$

while $F^3{}_{35,5/2}=0.259\gamma_8{}^3$. Therefore, either there are no resonances in the other states or else they lie so high that their effects are probably unimportant.

Thus our model predicts 35 new spin-$\frac{5}{2}$ even-parity baryons. Since the properties of a quinquetrigesimet are relatively unfamiliar,[13] we conclude this section by describing some of them briefly. The states may be classified by their hypercharge Y and isospin I and are listed in Table II. In addition to the $I=\frac{5}{2}$ nonstrange multiplet, our supermultiplet contains another $Y=1$ group with $I=\frac{3}{2}$. This $I=\frac{3}{2}$, $J=\frac{5}{2}$ multiplet, which did not appear in our SU(2) model, will be seen in elastic πN scattering only to the extent that SU(3) is violated.

The masses of the 35 particles should satisfy the Gell-Mann–Okubo mass rule

$$m_{YI}=m_0+aY+b[I(I+1)-Y^2/4]. \tag{3.11}$$

In general, a and b depend on j and two Casimir operators

$$F_iF_i=\tfrac{1}{3}(m_1{}^2+m_1m_2+m_2{}^2)+(m_1+m_2)$$

and

$$d_{ijk}F_iF_jF_k=(1/18)(m_1-m_2)\times[9+9(m_1+m_2)+2m_1{}^2+5m_1m_2+2m_2{}^2],$$

where $[m_1,m_2]$ is the highest weight of the representation and is explained in the next section.

[13] Some of these properties are discussed by H. Harari and H. J. Lipkin, who noticed that some of the particles might be stable against strong decays, Phys. Rev. Letters **13**, 345 (1964).

Now it has been suggested by Glashow and Sakurai[14] that the constants a and b may be the same for all SU(3) multiplets with the same baryon number. This is consistent with our knowledge of the masses of the spin-$\frac{1}{2}$ octet and the spin-$\frac{3}{2}$ decimet, provided $a \approx -191$ MeV and $b \approx 32$ MeV. Using these values and the value 1560 MeV for $m_{1,5/2}$ we obtain the masses listed in Table II.[15]

From the table we see that three of the (Y,I) multiplets are stable against strong decays. The $(-2,0)$ would decay electromagnetically, while the $Y=2$ isotopic quintet and the $Y=-3$ isotopic doublet states would have to decay weakly. Therefore, provided the 1560-MeV $\pi^+\pi^+p$ peak is really a member of a quinquetrigesimet (which is the smallest representation containing $I=\frac{5}{2}$), the existence of these metastable particles is a test of the conjectured universality of the constants a and b.

IV. HIGHER SPIN STATES

Let us forget the strange particles for a moment and turn back to our SU(2) model of Secs. I and II. So far, we have found particles in the $(\frac{1}{2},\frac{1}{2})$, $(\frac{3}{2},\frac{3}{2})$, and $(\frac{5}{2},\frac{5}{2})$ states. This sequence suggests that if we calculated in a similar way the scattering of the pion and the $(\frac{5}{2},\frac{5}{2})$ state, we might obtain a $(\frac{7}{2},\frac{7}{2})$ particle, and repeating the same procedure indefinitely, continue the sequence to $(\frac{9}{2},\frac{9}{2})$, $(\frac{11}{2},\frac{11}{2})$, etc. Indeed, such an infinite sequence was obtained by Wentzel in the old strong-coupling model,[16] and by Tomonaga[17] who used an intermediate coupling solution of the static model. Of course, we cannot predict the masses without going beyond our approximation,[4] but the masses of the first three suggest an increase each time of the order of two pion masses.

To see that we can indeed get such a sequence, consider the $(n+1, n+1)$ state in the scattering of a pion off an (n,n) baryon (n is half-integral). Presumably, this is the lowest state with these quantum numbers, so we have a one-channel problem. The crossing matrices are

$$\alpha=\beta=\begin{pmatrix} \dfrac{1}{n(2n+1)} & -\dfrac{1}{n} & \dfrac{2n+3}{2n+1} \\ -\dfrac{2n-1}{n(2n+1)} & \dfrac{n^2+n-1}{n(n+1)} & \dfrac{2n+3}{(n+1)(2n+1)} \\ \dfrac{2n-1}{2n+1} & \dfrac{1}{n+1} & \dfrac{1}{(n+1)(2n+1)} \end{pmatrix} \quad (4.1)$$

for I (or J)$=n-1, n, n+1$. From this matrix it is evident that no matter what particles are exchanged, $F^{2n}{}_{n+1,n+1}$ is always positive since the elements of α and β appearing on the right side of Eq. (1.8) are always in the bottom row of Eq. (4.1) and therefore positive. Thus, one may always expect a resonance in the $(n+1, n+1)$ state.

To show that it is consistent to have only $I=J$ particles in our model, suppose we also exchange this $(n+1, n+1)$ state together with the (n,n) and $(n-1, n-1)$ particles, which have already been produced at a previous stage. Then Eq. (1.7) gives

$$\gamma^{2n}{}_{n+1,n+1}=\left(\frac{2n-1}{2n+1}\right)^2\gamma^{2n}{}_{n-1,n-1}+\left(\frac{1}{n+1}\right)^2\gamma_{n,n}{}^{2n}+\left(\frac{1}{(n+1)(2n+1)}\right)^2\gamma^{2n}{}_{n+1,n+1}. \quad (4.2)$$

Now we consider the process

$$\pi+(n,n)\rightarrow\pi+(n+1,\, n+1).$$

The generalization of Eq. (2.9) is

$$(\gamma^{2n+2}{}_{n+1,n+1}\gamma^{2n}{}_{n+1,n+1})^{1/2}=\frac{n(n+2)(2n+1)}{(n+1)^2(2n+3)}(\gamma_{n,n}{}^{2n+2}\gamma_{n,n}{}^{2n})^{1/2}+\frac{1}{(n+1)^2}\times(\gamma^{2n+2}{}_{n+1,n+1}\gamma^{2n}{}_{n+1,n+1})^{1/2}, \quad (4.3)$$

since only the (n,n) and $(n+1, n+1)$ isobars can be exchanged. It is not difficult to show that the only pair of ratios which can satisfy Eqs. (4.2) and (4.3) and at the same time be consistent with the results of Secs. I and II for $n=\frac{1}{2}, \frac{3}{2}$ is

$$\gamma^{2n+2}{}_{n+1,n+1}/\gamma_{n,n}{}^{2n+2}=\gamma^{2n}{}_{n+1,n+1}/\gamma_{n,n}{}^{2n}=(2n+1)/(2n+3). \quad (4.4)$$

If we now use this result in Eq. (1.8), we find that $F^{2n}{}_{n+1,n}=F^{2n}{}_{n+1,n-1}=F^{2n}{}_{n,n+1}=F^{2n}{}_{n-1,n+1}=0$. Thus we do not obtain any dynamical particles with $I\neq J$.

Equation (4.4) can also be used to get the width of an $(n+1, n+1)$ isobar when it decays into a pion and an (n,n) particle. We use the fact that both $\gamma_{n,n}{}^{2n+2}$ and $\gamma^{2n}{}_{n+1,n+1}$ are uniquely determined in terms of the coupling constant between a pion, an (n,n) particle and an $(n+1, n+1)$ particle. This leads to the relation

$$\gamma_{n,n}{}^{2n+2}=[(2n+3)/(2n+1)]^2\gamma^{2n}{}_{n+1,n+1}. \quad (4.5)$$

Combining this with Eq. (4.4), we immediately obtain $\gamma^{2n+2}{}_{n+1,n+1}=\gamma_{n,n}{}^{2n}$. In particular, this implies $\gamma_{n,n}{}^{2n}=\gamma^1{}_{1/2,1/2}$. Equation (4.4) therefore gives

$$\gamma^{2n}{}_{n+1,n+1}=(2n+1)/(2n+3)\gamma^1{}_{1/2,1/2}, \quad (4.6)$$

which is just the reduced width in terms of $\gamma^1{}_{1/2,1/2}=3f^2$.

Can we find an analogous sequence of SU(3) multi-

[14] S. L. Glashow and J. J. Sakurai, Nuovo Cimento **25**, 337 (1962).

[15] This calculation of the masses in the quinquetrigesimet was carried out by M. Gell-Mann, who also noticed that some of the particles might be stable against strong decays.

[16] G. Wentzel, Helv. Phys. Acta **13**, 269 (1940); see also, W. Pauli and S. Dancoff, Phys. Rev. **62**, 85 (1942).

[17] S. Tomonaga, Progr. Theoret. Phys. (Kyoto) **1**, 83, 109 (1946).

plets? In Sec. III we studied the scattering of the *PS* octet off a spin-$\frac{3}{2}$ decimet and discovered the strongest attraction in the product state of highest J and highest SU(3) representation. This suggests that if we scatter the mesons off the new spin-$\frac{5}{2}$ multiplet, we may obtain a spin-$\frac{7}{2}$ multiplet transforming according to the largest representation which appears in $\mathbf{8}\times\mathbf{35}$, and so forth, leading to a sequence of states just as in the SU(2) case. To identify these representations, let us name them by their highest weights $[m_1,m_2]$, which are defined so that the highest hypercharge which occurs is $Y_{\max}=(m_1+2m_2)/3$, while the isotopic multiplet with $Y=Y_{\max}$ has $I=m_1/2$. Thus the octet is a $[1,1]$, the decimet a $[3,0]$, and the quinquetrigesimet a $[4,1]$. The dimension of a general representation is $(m_1+1)\times(m_2+1)(m_1+m_2+2)/2$. Therefore, the representation for that multiplet in our sequence which has spin n is $[n+\frac{3}{2}, n-\frac{3}{2}]$, since this is the highest representation occurring in the product of $[1,1]$ with $[n+\frac{1}{2}, n-\frac{5}{2}]$. In fact, the representation $[n+\frac{3}{2}, n-\frac{3}{2}]$ contains $I=n$ with $Y=1$ only once. Starting with $n=\frac{3}{2}$, the dimensions of the first few of these multiplets are 10, 35, 81, 154, $\cdots$.

The same dynamical arguments as in the SU(2) case can be used to show that our model will indeed produce this sequence. Using the SU(3) generalization of Eq. (1.8), the force F_{FJ} in the highest state is given entirely in terms of crossed-channel residues with coefficients which are products of an element from the bottom row of the angular-momentum crossing matrix (4.1) with one from the bottom row of the appropriate SU(3) crossing matrix. These elements are always positive, since in both cases they are squares of Clebsch-Gordan coefficients. Therefore, F_{FJ} is always positive in the state of highest J and highest SU(3) representation, which means that we can always expect a particle in such a state. We cannot of course argue as we could for the SU(2) case, that these are in general the only particles which our model would give rise to.

We conclude with some speculative observations. A sequence of multiplets for which I (or F) as well as J increases with the mass suggests that it might be fruitful to study continuation in these internal quantum numbers in analogy to the Regge continuation in angular momentum. Our results may shed a little light on the nature of this continuation.

The most striking feature of our sequences of states is that it is not I as a function of W for fixed J [in the SU(2) case] which most resembles a Regge trajectory, but rather a curve obtained by increasing both J and I simultaneously. Furthermore, if the $I=J$ versus W curve is interpolated between the points 940, 1240, and 1560 MeV, which we assume are the first three physical points lying at $I=J=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$, then the slope of this new type of trajectory is roughly the same as the slopes of the usual baryon Regge trajectories of which these particles are the lowest members. In other words, if the first Regge recurrences of the N, Δ, and $\frac{5}{2}$, $\frac{5}{2}$ particles are indeed at 1690, 1920, and 2400 MeV, respectively, all the Regge trajectories lie on top of each other to within 100 MeV or so. If this notion is extended to SU(3) multiplets, we arrive at the rule that all Regge trajectories with $B=1$ are degenerate. This rule is broken by only 10–20%, the principal manifestation of the breaking being that higher I (nonstrange) trajectories lie somewhat higher than those with lower I.

ACKNOWLEDGMENTS

We wish to thank Professor Murray Gell-Mann for suggesting the use of the static model to study sequences of particles, and for several fruitful conversations; in particular, for pointing out the relation of our work to the possibility of continuing in internal quantum numbers. We also thank Dr. Yuval Ne'eman and Mr. Douglas Beder for some interesting discussions.

APPENDIX

Calculation of the Lowest Particle Masses in the SU(2) Model

To calculate the masses of our isobars, we have to make one additional assumption. For example, let us assume that the cutoff Λ in Eq. (1.5) is the same in all cases. It can then be calculated by requiring that the D function vanish at $\omega=0$ in the $(\frac{1}{2},\frac{1}{2})$ state in πN scattering. For simplicity, we shall also assume that the D function is linear with the value and slope normalized at the physical threshold $\omega=1$. In other words, we approximate Eq. (1.5) by

$$D_{IJ}(\omega)=1-\frac{\omega-1}{\pi}\int_1^\Lambda d\omega'(\omega'^2-1)^{3/2}\frac{N_{IJ}(\omega')}{(\omega'-1)^2}, \quad \text{(A1)}$$

taking $\omega_0=1$. This approximation is consistent with Eq. (1.6) and so all our results on coupling constant ratios can be taken over.

The equations $D_{IJ}(\omega_{IJ})=0$ in the πN scattering for $I=J=\frac{1}{2}$ and $I=J=\frac{3}{2}$ can now be solved simultaneously for Λ and $\omega_{3/2,3/2}$, since $\omega_{1/2,1/2}=0$ and since we know $\gamma^1{}_{1/2,1/2}$ and $\gamma^1{}_{3/2,3/2}$ in terms of the πNN coupling constant $f^2=0.08$. The result is $\Lambda=7.1$ and $\omega_{3/2,3/2}=1.88$. If we take this Λ as well as the values of $\gamma_{IJ}{}^3$ obtained in Sec. II and solve $D_{5/2,5/2}(\omega_{5/2,5/2})=0$ in $\pi\Delta$ scattering, we obtain $\omega_{5/2,5/2}=3.14$.

The above process can be continued indefinitely. Thus we can calculate the mass of the $(\frac{7}{2},\frac{7}{2})$ particle in $\pi-(\frac{5}{2},\frac{5}{2})$ scattering, the $(\frac{9}{2},\frac{9}{2})$ mass in $\pi-(\frac{7}{2},\frac{7}{2})$ scattering, etc. At every stage we can use the general formulas of Sec. IV to obtain the needed residues. We obtain $\omega_{7/2,7/2}=4.8$, $\omega_{9/2,9/2}=11.5$, $\cdots$. However, we see that $\omega_{9/2,9/2}>\Lambda$, which means that our simple cutoff model cannot be applied in this case. Since the masses of all the higher isobars depend on this mass, they cannot be calculated correctly either.

Therefore, we can predict only the $(\frac{3}{2},\frac{3}{2})$, $(\frac{5}{2},\frac{5}{2})$, and

$(\frac{7}{2},\frac{7}{2})$ masses within our scheme. Assuming the pion and nucleon masses and the πNN coupling constant to be known, these turn out to be 1200, 1640, and 2310 MeV. It is a curious fact that, together with the nucleon, which has a mass of 940 MeV, these masses m_J obey to a few percent the rigid rotator formula $m_J = AJ(J+1)+B$, where A and B are constants. This is exactly the prediction of the strong-coupling model[16] which, however, had an additional arbitrary parameter.

The above masses of the $(\frac{3}{2},\frac{3}{2})$ and $(\frac{5}{2},\frac{5}{2})$ particles should be compared with the experimental values of 1240 and 1560 MeV, respectively. In the latter case we are assuming, of course, that we can identify our particle with the resonance of Ref. 7. [Actually, the value $\omega_{5/2,5/2}$ in Eq. (2.14) does not coincide with the maximum of the cross section; it corresponds to 1650 MeV, which may be the more appropriate quantity to compare with our calculated value.]

PHYSICAL REVIEW VOLUME 136, NUMBER 5B 7 DECEMBER 1964

Impact-Parameter K-Matrix Approach to High-Energy Peripheral Interactions*

Richard C. Arnold
Department of Physics, University of California, Los Angeles, California
(Received 24 July 1964)

An approximate dispersion-theoretic treatment of peripheral inelastic processes is introduced with the aid of a K-matrix formalism based on the impact-parameter representation of Blankenbecler and Goldberger. The method allows the use of one-meson exchange poles as a framework for constructing a multichannel scattering amplitude which satisfies unitarity in the high-energy region, allowing for an indefinitely large number of open channels. The reaction matrix is time-reversal symmetric and exhibits any other symmetries of the pole terms. Applications are numerically worked out for models of high-energy $\bar{K}p$ and np charge exchange, and in the former case satisfactory agreement with experiments is achieved. A qualitative discussion is given of peripheral isobar production models. The high-energy $\bar{p}p$ and $\bar{K}p$ diffraction scattering is examined, as well as the agreement of the small-momentum-transfer behavior with a simple model not involving Regge poles. The method sheds no light on the difference between $\bar{p}p$ and pp scattering at high energies.

I. INTRODUCTION

ELASTIC and inelastic reaction amplitudes of elementary particles and isobars at high energies characteristically exhibit a peak in the forward direction. In some reactions, such as proton-antiproton elastic scattering,[1] the form of the amplitude can be readily interpreted by analogy with optical diffraction patterns, suggesting a semiclassical picture of the nucleon with an absorptive core and a diffuse boundary, phenomenologically of Gaussian shape. In some other cases, for example[2] $K^+ + p \to K^0 + N_{3/2}^{*++}$, the center-of-mass angular distribution of the production reaction is clearly consistent with a one-meson exchange formula. The most common high-energy reaction behavior seems to be intermediate between these extremes.

Phenomenological corrections to one-particle exchange formulas based on the introduction of form factors have been widely used in the analysis of peripheral inelastic processes,[3] but these form factors have at least two objectionable properties. The first is lack of generalizability; evidence has accumulated that such a form factor appropriate to the vertex $\rho\pi\pi$ has a behavior much different from that for the ρKK vertex,[2] while a close relation between these form factors would be expected in various symmetry schemes such as unitary symmetry.

The second is a lack of theoretical foundation within the framework of dispersion, or on-the-mass-shell, techniques. A form factor may be expected to have an important influence in a perturbation-theoretic approach, but even then it is difficult to see the source of such large variations as are required to fit the data. This point has been discussed by Durand and Chiu,[4] Ross and Shaw,[5] and earlier by Baker and Blankenbecler.[6]

The authors (particularly Refs. 4 and 5) also point out that the inclusion of initial and final-state interactions, usually taken to be strong elastic scattering with a diffraction character, is very important in the analysis of peripheral inelastic processes; and, in fact, these corrections may be quite sufficient to explain the deviations from one-meson exchange previously ascribed to form factors. Essentially the same conclusion has been reached by Dar and Tobocman in a slightly different language; a detailed discussion of the mechanism has been given by Gottfried and Jackson.[7]

* Partially supported by the National Science Foundation.
[1] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **11**, 503 (1963).
[2] V. Barger and E. McCliment, Phys. Letters **9**, 191 (1964).
[3] E. Ferrari, Nuovo Cimento **30**, 240 (1963); E. Ferrari and F. Sellari, Nuovo Cimento **27**, 1450 (1963).
[4] L. Durand and Y. T. Chiu, Phys. Rev. Letters **12**, 399 (1964).
[5] M. H. Ross and G. L. Shaw, Phys. Rev. Letters **12**, 672 (1964).
[6] M. Baker and R. Blankenbecler, Phys. Rev. **128**, 415 (1962).
[7] A. Dar and W. Tobocman, Phys. Rev. Letters **12**, 511 (1964); A. Dar, *ibid.* **13**, 91 (1964). K. Gottfried and J. D. Jackson, CERN paper, 1964 (unpublished).