Ground State of Ce^{143}

In $_{58}Ce_{85}{}^{143}$ the three neutrons outside the major closed shell of 82 neutrons, are expected to be in the $f_{7/2}$ orbital. These can couple to a resultant spin of either $\frac{7}{2}$ or $\frac{5}{2}$, the parity being negative. This is supported by the direct measurement of the spins of the neighboring nuclei with $N=83$ to 87.

The $\log ft$ values of β transitions from Ce^{143} to most of the levels of Pr^{143} indicate that they are of the first forbidden type ($\Delta J=0, \pm 1$, yes). This shows that these levels have even parity. The absence of the first-forbidden β transition to the ground state of Pr^{143} cannot be explained. It can only be mentioned that similar cases are present in the β decay of Nd^{147} and Nd^{149} where the ground-state β transition from $\frac{5}{2}^-$ to $\frac{7}{2}^+$ is absent or highly retarded.[21,22]

ACKNOWLEDGMENTS

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Nuclear Matrix Elements for First-Forbidden β Decay*

L. S. Kisslinger and Chi-Shiang Wu
Western Reserve University, Cleveland, Ohio
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The first-forbidden β decay from the ground states of odd-odd nuclei to the quadrupole vibrational states of even-even nuclei is studied. It is found that the collective motions introduce particle-hole correlations and thereby lead to cancellations, which is the dominant feature of these processes. The region of proton number between 50 and 60 is treated in detail.

I. INTRODUCTION

Although β decay has long played an important part in the study of nuclear structure, and the classification into allowed and forbidden transitions was correlated with nuclear shell model soon after it was proposed,[1] it has been only recently that experimental techniques have been adequate to determine the nuclear matrix elements of the operators leading to forbidden transitions. There is still uncertainty in some of the experimental results, but systematics are gradually being collected which are hard to understand in either the shell model or the collective model.

The first-forbidden β decays involve six matrix elements, all with parity change. There are two matrix elements (in the standard notation[2] $\int\gamma_5$ and $\int\boldsymbol{\sigma}\cdot\mathbf{r}/i$) for which the spins of the initial and final states must be the same ($\Delta J=0$), three $\Delta J=1$ matrix elements ($\int i\mathbf{r}$, $\int\boldsymbol{\sigma}\times\mathbf{r}$, and $\int\boldsymbol{\alpha}$), and $\int iB_{ij}$ for which the spin change can be as large as two. The single-particle estimates for the relativistic matrix elements $\int\gamma_5$ and $\int\boldsymbol{\alpha}$ are roughly 0.1, while for the other matrix elements they are roughly the nuclear radius R.

The experimental values are always considerably less than these. Typical experimental results are 1–10% of the single-particle estimate for the $\int iB_{ij}$ and $\frac{1}{10}$% for the other matrix elements. Moreover, there seem to be rapid variations in the matrix elements from isotope to isotope.

In this work we study the transitions from odd-odd to even-even spherical nuclei, the only ones for which there is experimental data, in terms of the shell model with particle correlations in order to try to learn what aspects of nuclear structure can lead to these unusual results. These decays are especially interesting from a nuclear structure point of view since the final states are collective states. The detailed calculations are restricted to the isotopes with proton number 50–60, a region in which many properties of the low-lying states have been successfully interpreted in terms of the modes of motion derived from a system of shell-model particles interacting with pairing and quadrupole forces.[3] We use these methods to calculate the first forbidden beta transitions to low-lying collective states, giving numerical results for the β^+ and β^- transitions to the first 2^+ states from the 2^- and 3^- states in the odd-odd nuclei.

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II. NUCLEAR STATES AND OPERATORS

A. Uncorrelated Shell Model

All of the first-forbidden β decays which have been studied in the region of proton number between 50 and 60 are for transitions from either a 2^- or a 3^- ground state of an odd-odd nucleus. As an example, the first matrix elements measured were for the Sb^{124} 3^- ground state to the 2^+ first excited state of Te^{124}. This transition has a $\log ft$ of[4] 10.2 compared to the more usual value of $\log ft \sim 8$ for similar transitions, and the spectrum shows a large deviation from the ξ approximation,[5–7] which has made easier the measurement of the four contributing matrix elements. The $\int iB_{ij}/R$ is about 10^{-2} while the $\Delta J=1$ matrix elements are about 10^{-3} of the single-particle value.[8]

Let us consider first the decays from the 3^- states. From the Sb odd-mass isotopes we know that the only important single-particle proton levels are the $d_{5/2}$ and $g_{7/2}$ even-parity levels, and from studies of the other odd-mass isotopes one expects that this will be true for this entire region.[3] The neutron must be in the $h_{11/2}$ level. If the 2^+ final state is also a pure configuration, it must be either $|g_{7/2}^2(2+)\rangle$ or $|d_{5/2}g_{7/2}(2+)\rangle$ (here only the particles not coupled to zero spin are being considered) if the transition is to be allowed.

Thus in the shell model the basic transition must be an $h_{11/2}$ neutron to a $g_{7/2}$ proton, which is forbidden for the $\int i\mathbf{r}$, $\int \boldsymbol{\alpha}$, and $\int \boldsymbol{\alpha}\times\mathbf{r}$ matrix elements but for which the $\int iB_{ij}$ matrix element is about the nuclear radius. Since the type of configuration admixture involved in the quasiparticles and in the phonons (see Sec. B) does not change the selection rule forbidding the first three matrix elements, this just reduces to the well-known result that those matrix elements are forbidden in the shell model. A crude estimate of additional perturbations which can give rise to these matrix elements is given below, but this is not central to the nuclear structure effects being considered.

On the other hand, it is not at all obvious why the $\int iB_{ij}$ should be so small. Although the microscopic structure of the states must be given as a mixture of a number of j levels, as is discussed below, the $h_{11/2}$ neutron and $g_{7/2}$ proton levels of importance are certainly well occupied in the initial and final states, respectively, and there is no basic selection rule to hinder the transition. This suggests an accidental cancellation, so that the Sb^{124} β decay displays both "selection rules" and "calcellation effects" in spite of the apparent unlikelihood of a cancellation, which has led to the conclusion by the previous authors who have studied this problem that there must be another selection rule in operation.[7] That a dynamic cancellation is occurring is even more strongly suggested by the recent experiments[9] in the β decay of the 2^- ground state to the 2^+ first excited state of Te^{122}. In this case the $\int iB_{ij}/R \approx 0.1$, a change of an order of magnitude from the quite analogous Sb^{124} decay.

The decays from the 2^- states should be very similar, except that only the $g_{7/2}$ protons could be coupled to the $h_{11/2}$ neutrons to give the 2^- state in the shell model, and that all six nuclear matrix elements can contribute. However, the $\int iB_{ij}$ is still the only nonzero matrix element in the absence of mixing of other major shells, and it is from the systematics of the $\int iB_{ij}$ that one can expect to extract most information about particle correlations.

B. Correlated States

We take as the nuclear Hamiltonian a pairing force between neutrons and protons separately and a quadrupole force of equal magnitude between all the particles. It has been shown that a pairing plus a quadrupole force can account for the essential systematic nuclear properties leading to the quadrupole vibrations,[10–13] and that on the basis of this Hamiltonian many systematic nuclear properties in this region can be semiquantitatively predicted, such as the positions and transition rates of the one-phonon states.[3,13,14]

The nuclear states are obtained as in Ref. 3; we simply sketch the procedure and give the pertinent definitions here. All nuclear properties are described in terms of quasiparticles and phonons, the microscopic composition of the latter being given as correlated quasiparticles. In terms of the quasiparticles the Hamiltonian is written

$$H=E_0+\sum_j E_j(2j+1)^{1/2}\eta_{jj}{}^0-\tfrac{1}{2}\chi Q\cdot Q\,, \qquad (1)$$

where E_0 is the ground-state energy of an even-even isotope; Q, the quadrupole operator, is

$$Q_M=\sum_{jj'}\frac{\langle j\|r^2Y^2\|j'\rangle}{5}\left[\frac{U_jV_{j'}+U_{j'}V_j}{2}A_{jj'}{}^{2M\dagger}+(-1)^M A_{jj'}{}^{2-M}+(U_jU_{j'}-V_jV_{j'})\eta_{jj'}{}^{2M}\right], \qquad (2)$$

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with

$$A_{jj'}{}^{LM\dagger}=[\alpha_j^\dagger\alpha_{j'}^\dagger]_M{}^L(-1)^{1+l'}, \qquad (3)$$

$$\eta_{jj'}{}^{LM}=[\alpha_j^\dagger\alpha_{j'}]_M{}^L, \qquad (4)$$

being the vector coupled quasiparticle double creation operator and scattering operator of rank L, respectively. Only n-n or p-p terms are present in (3). The quasiparticles are particle-hole combinations, with $V_j{}^2(U_j{}^2)$ being the probability of a shell-model j level being occupied (unoccupied), E_j the quasiparticle energies (which are >1 MeV here), and χ the force strength of the quadrupole coupling of the quasiparticles (which is taken the same for the n-n, p-p, and n-p interactions). All of the parameters are approximately the same as those used in Ref. 3. All of the single-particle quantum numbers are implied by j in above equations.

The 2^+ first excited state of the even-even nucleus is described as a one-phonon state

$$\Psi(2^+)=B^\dagger\Psi_0, \qquad (5)$$

with the phonon creation operator B^+ given as

$$B^\dagger=\sum_{jj'}[a_{jj'}A_{jj'}{}^{\dagger 2M}+b_{jj'}(-1)^M A_{jj'}{}^{2-M}], \qquad (6a)$$

$$a_{jj'}=\frac{N_\omega\langle j\|Y^2\|j'\rangle(U_jV_{j'}+U_{j'}V_j)}{(\sqrt{5})(E_j+E_{j'}-\hbar\omega)}, \qquad (6b)$$

$$b_{jj'}=-\frac{N_\omega\langle j\|Y^2\|j'\rangle(U_jV_{j'}+U_{j'}V_j)}{(\sqrt{5})(E_j+E_{j'}+\hbar\omega)}, \qquad (6c)$$

$$N_\omega{}^2=5\left[8\hbar\omega\times\sum_{jj'}\frac{(E_j+E_{j'})\langle j\|r^2Y^2\|j'\rangle^2(U_jV_{j'}+U_{j'}V_j)^2}{[(E_j+E_{j'})^2-(\hbar\omega)^2]^2}\right]^{-1}. \qquad (6d)$$

The 0^+ ground state is the phonon vacuum

$$B\Psi_0=0. \qquad (7)$$

Equations (6) and (7) are true in the quasiparticle random-phase approximation which is described, and for which references are given, in Ref. 3. The approximate even-even ground state, in terms of quasiparticles, is

$$\Psi_0=N_\omega\left\{1-\sum_{j_1j_2j_3j_4}\frac{(\sqrt{5})b_{j_1j_2}b_{j_3j_4}}{4\sum_{jj'}a_{jj'}b_{jj'}}\times[A_{j_1j_2}{}^{2\dagger}A_{j_3j_4}{}^{2\dagger}]_0{}^0+\cdots\right\}\Psi_0{}^0$$

$$\equiv N_\omega\{1+\textstyle\sum_{1234}C_{1234}[A_{j_1j_2}{}^{2\dagger}A_{j_3j_4}{}^{2\dagger}]_0{}^0+\cdots\}\Psi_0{}^0, \qquad (8)$$

with $\Psi_0{}^0$ the quasiparticle vacuum, i.e.,

$$\alpha\Psi_0{}^0=0. \qquad (9)$$

The ground state of the odd-odd nucleus is given approximately as

$$\psi_J{}^{j_pj_n}=\textstyle\sum_{j_pj_n}C_{j_p00}{}^{j_p}C_{j_n00}{}^{j_n}(-1)^{1+l_n}A_{j_pj_n}{}^{J\dagger}\Psi_0'+\text{phonon component}, \qquad (10)$$

where $(C_{j00}{}^{j_i})^2$ is the probability that the quasiparticle j_i with no phonons is present in the state of spin j. In (10) the interaction between quasiparticles through the quadrupole interactions in the Hamiltonian (1) is neglected, so the $C_{j00}{}^j$ needed here can be taken from Ref. 3 (Appendix II). Note that if we include this effect the coefficient $C_{7/2\ 00}{}^{7/2}$ for the proton will be diminished. Since we shall treat these as pure quasiparticle states, our results are an overestimate of the magnitude of the matrix element. Also, one should observe that the 2^- state must come from the $g_{7/2}$ quasiproton, while for the 3^- state there might be a strong admixture of the $d_{5/2}$ quasiproton. Since the $d_{5/2}$ quasiproton does not enter into the β decay, as we have seen, this would cut down the resulting nuclear matrix element. The phonon parts enter in the second order and are not included here.

C. Beta-Decay Operators

Any one of the operators leading to the β decay can be expressed in terms of the quasiparticle creation and destruction operators

$$O_\beta{}^{LM}=\sum_{jj'}U_{j_2}V_{j_1}\frac{\langle j_2{}^{(p)}\|O_\beta{}^{-L}\|j_1{}^{(n)}\rangle}{(2L+1)^{1/2}}A_{j_2{}^{(p)}j_1{}^{(n)}}{}^{LM\dagger}+(-1)^{j_2+j_1+1+M}U_{j_1}V_{j_2}\frac{\langle j_2{}^{(p)}\|O_\beta{}^{-L}\|j_1{}^{(n)}\rangle}{(2L+1)^{1/2}}A_{j_1{}^{(n)}j_2{}^{(p)}}{}^{L-M}$$
$$+U_{j_2}U_{j_1}\frac{\langle j_2{}^{(p)}\|O_\beta{}^{-L}\|j_1{}^{(n)}\rangle}{(2L+1)^{1/2}}\eta_{j_2{}^{(p)}j_1{}^{(n)}}{}^{LM\dagger}+(-1)^{\tau+1}V_{j_2}V_{j_1}\frac{\langle j_1{}^{(n)}\|O_\beta{}^{-L}|j_2{}^{(p)}\rangle}{(2L+1)^{1/2}}\eta_{j_2{}^{(n)}j_1{}^{(p)}}{}^{LM}, \qquad (11)$$

where L and M are the tensor rank and z component of the operator and $(-1)^\tau$ is the time-reversal phase, determined by

$$\langle j'-m'|O^L|j-m\rangle_T=(-1)^\tau\langle jm|O^L|j'm'\rangle_T,$$

in which the time-reversed phases of the single-particle $|j-m\rangle$ states are used (see, e.g., Ref. 3), and $\langle j\|O_\beta{}^{-L}\|j'\rangle$ is the reduced single-particle matrix element. In the shell-model picture the first two terms are particle-hole pair creation and destruction terms (e.g., the first term creates a neutron hole and proton particle, respectively; the third term is a neutron particle to proton particle operator; and the fourth, a proton hole to neutron hole operator. The third term is the ordinary β-transition operator as the pairing force strength $G\rightarrow 0$.

III. NUCLEAR MATRIX ELEMENTS

Assuming that the initial state is given by the no-phonon part of (10) and the final state is the one-phonon state given by (5), (7), and (8), the general form for a β^--decay reduced matrix element follows from (11) as

$$\begin{aligned}\langle\Psi(2+)\|O_\beta{}^{-L}\|\Psi_J{}^{j_pj_n}\rangle = {} & -2N_\omega[5(2J+1)]^{1/2}\Big[\textstyle\sum_{j_2^{(p)}}(-1)^{l_p}U_{j_2^{(p)}}U_{j_n}a_{j_pj_2^{(p)}}\langle j_2^{(p)}\|O_\beta{}^{-L}\|j_n\rangle W(Lj_n2j_p;j_2^{(p)}J)\\ & +(-1)^{\tau+J}\textstyle\sum_{j_1^{(n)}}(-1)^{j_p+j_n+l_n}V_{j_p}V_{j_1^{(n)}}a_{j_nj_1^{(n)}}\langle j_1^{(n)}\|O_\beta{}^{-L}\|j_p\rangle W(Lj_p2j_n;j_1^{(n)}J)\\ & +\sum_{j_1^{(n)}j_2^{(p)}j_3^{(n)}}U_{j_2^{(p)}}U_{j_1^{(n)}}8(\sqrt{5})(-1)^{l_p+L+j_1^{(n)}+j_2^{(p)}}\langle j_2^{(p)}\|O_\beta{}^{-L}\|j_1^{(n)}\rangle W(Lj_2^{(p)}2j_3^{(n)};j_1^{(n)}J)\\ & \times W(j_3^{(n)}j_2^{(p)}j_nj_p;J2)b_{j_3^{(n)}j_1^{(n)}}C_{j_2^{(p)}j_pj_nj_3^{(n)}}+(-1)^\tau\sum_{j_1^{(n)}j_2^{(p)}j_4^{(p)}}(-1)^{L+j_2^{(p)}+j_4^{(p)}+J+1+l_n}\\ & \times V_{j_2^{(p)}}V_{j_1^{(n)}}8(\sqrt{5})\langle j_1^{(n)}\|O_\beta{}^{-L}\|j_2^{(p)}\rangle b_{j_4^{(p)}j_2^{(p)}}C_{j_4^{(p)}j_pj_nj_1^{(n)}}\\ & \times W(Lj_1^{(n)}2j_4^{(p)};j_2^{(p)}J)W(j_1^{(n)}j_4^{(p)}j_nj_p;J2)\Big]. \qquad (12)\end{aligned}$$

The $A^\dagger$ and A terms in the operator (11) do not contribute to the matrix element (12) in the approximations used here, since the various terms in the initial and final states differ by zero, four, etc., quasiparticles while $A^\dagger$ and A change the number of quasiparticles by two. The major part of the matrix element is contained in the first and second terms. The first term is a contribution due to a transition of the initially odd quasineutron into a quasiproton, which then couples with the initially odd quasiproton into a state of total spin 2. The second term is due to a transition of the initially odd quasiproton into a quasineutron, which then couples with the initially odd quasineutron into a state of total spin 2. As mentioned above, they correspond to a particle-to-particle and a hole-to-hole transition respectively. Both types of transitions become possible between these states due to the configuration mixing caused by the quadrupole interaction. The third and fourth terms contain additional C_{1234} and Racah coefficients and, therefore, are much smaller than the previous two terms. They give only a small correction to the major part contained in the previous terms unless the terms in the major part cancel to a very small value.

The matrix element for the transition to the ground state is

$$\langle\psi_0\|O_\beta{}^{-L}\|\psi_J{}^{j_pj_n}\rangle=N_\omega U_nV_p\langle p\|O_\beta{}^{-L}\|n\rangle+\text{corr.} \qquad (13)$$

The main effect is the smooth change in the statistical factor as in the odd-mass nuclei,[3,15] and the collective motions do not produce qualitative effects as above. These can help establish the magnitude of the single-particle matrix elements which are a major uncertainty (e.g., from configuration admixtures from other shells[3]).

It is not difficult to estimate the nuclear matrix elements for transitions to the two-phonon states. The matrix elements $\langle\psi_0|[BB]^IO_\beta{}^LA^\dagger{}_{j_pj_n}|\psi_0\rangle$ are easily calculated from (6a), (8), and (11) if one keeps only the largest terms [notice that in this case the particle-hole terms, i.e., the first two terms in Eq. (11), are the only ones which contribute]. However, this description of the two-phonon states is not as accurate as the one-phonon state, even though it might be of use in the region of isotopes being studied here, and we have not done numerical calculations for these cases.

Let us first consider as an example Sb^{124}. We take $j_p=g_{7/2}$ and $j_n=h_{11/2}$, since the spins of the ground states of Sb^{123} and Sb^{125} are both $\frac{7}{2}$. If we consider only those levels in the major shell, the $\langle j_2^{(p)}\|O^L\|j_n\rangle$ in the first term does not vanish only for $j_2^{(p)}=g_{7/2}$, and the $\langle j_1^{(n)}\|O^L\|j_p\rangle$ in the second term does not vanish only for $j_1^{(n)}=h_{11/2}$ which is the only odd-parity level in the major shell. It turns out that these two terms have magnitudes of the same order but with opposite signs and therefore there is a cancellation between these two major terms. Numerical calculations show that the second term cancels about 45% of the first term and the remaining terms contribute less than 5% to the net value of the previous terms. The final value for $|\int B_{ij}|/R$ is 0.34 which is still much larger than the experimental value. If the neighboring shells are taken into account the transition $g_{7/2}{}^{p(h)}\to f_{7/2}{}^{n(h)}$ may contribute to the second term, the magnitude of which is approximately of the order of the correction term in the major shell. The $h_{11/2}{}^n\to g_{9/2}{}^p$ transition is not possible because the $g_{9/2}{}^p$ level is filled. It is interesting to notice that the amount of cancellation is very sensitive to the U and V of the decaying neutron level so that the inaccuracies inherent in the methods being used could not predict the cancellation in this isolated case more accurately.

For the other three nuclear matrix elements, which are due to tensor operators of rank one with odd parity, none of the levels in the major shell contribute. Approximate orders of magnitudes of these matrix elements may be obtained by taking into account the neighboring shells which may contribute to the two major terms in (12). The only transition which contributes to these terms is the hole transition $g_{7/2}{}^{p(h)}\to f_{7/2}{}^{n(h)}$, and this

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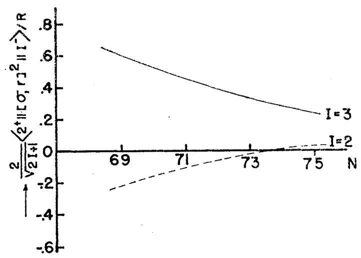

FIG. 1. Nuclear matrix elements $[2/(2I+1)^{1/2}]\langle 2^+\|[\boldsymbol{\sigma},\mathbf{r}]^2\|I^-\rangle/R$ for β^- decays of $_{51}$Sb isotopes. $|\int B_{ij}|/R$ is equal to the absolute value of this quantity.

is estimated simply by including this state in the vibration (i.e., configuration admixture due to the quadrupole interaction).

The numerical results are given in the table below with their experimental values as well as the theoretical value calculated from possible shell-model transitions. The calculated values for $\int \mathbf{r}, \int \boldsymbol{\alpha}$ and $\int(\boldsymbol{\sigma}\times\mathbf{r})$ simply show the orders of magnitudes of the largest term in (12) and are not meant to be quantitative. The factors

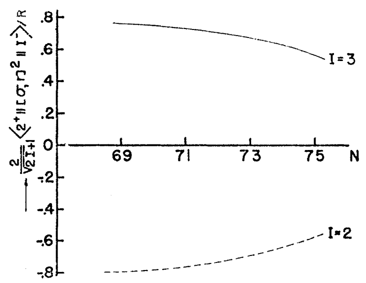

FIG. 2. Nuclear matrix elements $[2/(2I+1)^{1/2}]\langle 2^+\|[\boldsymbol{\sigma},\mathbf{r}]^2\|1^-\rangle/R$ for β^+ decays of Sb isotopes. $|\int B_{ij}|/R$ is equal to the absolute value of this quantity.

$c_{j00}{}^j$ are taken as unity. As has been mentioned above this certainly overestimates the $\int B_{ij}$ matrix element because of the neglect of the phonon admixtures; and for the $3^-\rightarrow 2+$ transition in Sb^{125}, for which any $d_{5/2}$ quasiparticle which is admixed will reduce the matrix element, there will be a further overestimate. However, the particle-hole cancellations still seem to be the main effect. (See Table I.)

In the six figures are plotted the $\int iB_{ij}$ matrix

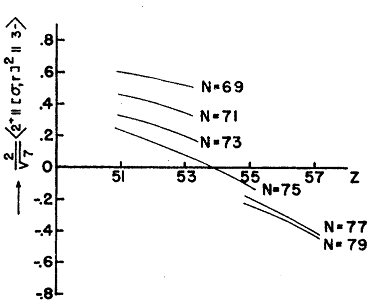

FIG. 3. Nuclear matrix elements $(2/\sqrt{7})\langle 2^+\|[\boldsymbol{\sigma},\mathbf{r}]^2\|3^-\rangle/R$ for β^- decays $3^-\rightarrow 2^+$.

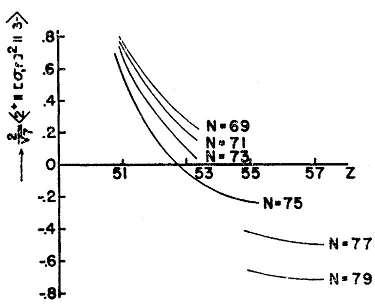

FIG. 4. Nuclear matrix elements $(2/\sqrt{7})\langle 2^+\|[\boldsymbol{\sigma},\mathbf{r}]^2\|3^-\rangle/R$ for β^+ decays $3^-\rightarrow 2^+$.

elements for β^- and β^+ transitions from the 2^- and 3^- states of the odd-odd isotopes in this region for which experiments are most likely. Note that results are given even when the transition is not energetically possible, since it is most important to try to discover the general trends in these phenomena for which it is difficult to gather systematic experimental data. The single-particle β-matrix elements are estimated using harmonic-oscillator wave functions. From Eq. (12) it

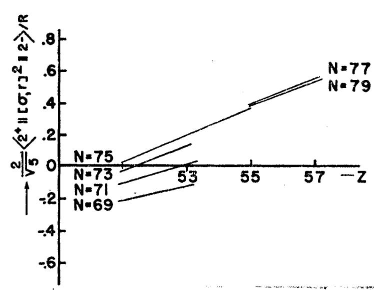

FIG. 5. Nuclear matrix elements $(2/\sqrt{5})\langle 2^+\|[\boldsymbol{\sigma},\mathbf{r}]^2\|2^-\rangle/R$ for β^- decays $2^-\rightarrow 2^+$.

can be seen that in most cases there will be two dominant terms and that they will always be of opposite sign. Figure 1 gives the results for the β^- decay from the Sb isotopes to Te isotopes. The complete cancellation for the 3^- transitions is predicted to occur for larger neutron number than that for the 2^- transition. For the isotopes lighter than Sb^{124} the lowest neutron quasiparticle is no longer $h_{11/2}$, so that it will be difficult to gain further information there; however, the results of the decay of the 2^- and 3^- states in Sb^{126} and Sb^{128}

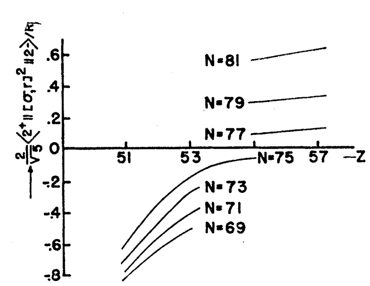

FIG. 6. Nuclear matrix elements $(2/\sqrt{5})\langle 2^+\|[\boldsymbol{\sigma},\mathbf{r}]^2\|2^-\rangle/R$ for β^+ decays $2^-\rightarrow 2^+$.

TABLE I. Matrix element of Sb^{124} decay.

Matrix element	Experimental[a]	Calculated Shell	Coll.
$\|\int B_{ij}\|/R$	$\sim 10^{-2}$	1.5	0.34
$\|\int \mathbf{r}\|/R$	$\sim 10^{-4}$	0	$\approx 10^{-3}$
$\|\int (\boldsymbol{\sigma}\times\mathbf{r})\|/R$	$\sim 10^{-4}$	0	$\approx 10^{-2}$
$\|\int \boldsymbol{\alpha}\|$	$\sim 10^{-4}$	0	$\approx 10^{-4}$

[a] These numbers are still uncertain, owing to the difficulty in interpreting the experimental data.

to the first 2^+ state would be quite significant if they could be obtained. Still, one can try to correlate the more isolated experimental results for the various elements by comparing the curves with different proton numbers. The extraction of the experimental values of the nuclear matrix elements is still subject to uncertainties so the experimental data has not been given.[16]

An important exception are decays in which the daughter isotope has only protons or only neutrons outside of the closed shells (single closed-shell nuclei). From Eq. (12) one can see that cancellation is possible only if both the proton and neutron involved in the single-particle β^- decay are important in the vibrational state of the daughter nucleus. Therefore, there is no cancellation in these cases. As an example, the β^+ decay of $_{51}$Sb to $_{50}$Sn would proceed only by the first-order UU term in the β^+ form of Eq. (12) (note that all of the correction terms, the terms involving the C_{1234}'s, are zero in this case). The results in Fig. 2 show that for $_{51}Sb^{122}$, where both the β^- and the β^+ decays to a vibrational state are energetically allowed, the $\int iB_{ij}$ should be some five times larger for the $Sb^{122} \xrightarrow{\beta^+} Sn^{122}$ than for the $Sb \xrightarrow{\beta^+} Te^{122}$. A measurement of the $\int iB_{ij}$ in Sn^{122} would thus be of great interest. Also, since there is no cancellation and since the correction terms are zero for decays to single-closed-shell nuclei, these decays could serve to determine the single-particle matrix elements, for which we have had to use theoretical shell-model estimates.

[16] See e.g., H. Frauenfelder and R. M. Steffen, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, to be published), 2nd ed.

IV. CONCLUSIONS

From the particle-hole correlations produced by the pairing force and expressed in terms of the quasiparticles there are alterations in the magnitude of the β-decay elements by the statistical factors representing the probability of occupation of the single-particle levels.[3,15] Additional and much more striking effects are seen to arise from the additional correlations associated with collective vibrational motion. Since the phonon modes can be described as strong quasiparticle pair and quasihole pair admixtures, an additional mechanism for the β decay is found, which generally will lead to an additional term for each shell model term that is allowed.

For the first-forbidden β decays in the region of proton number fifty to sixty it is found that for the transitions from the 2^- and 3^- states of odd-odd nuclei to the first excited (vibrational) states a cancellation always occurs between the largest terms leading to the $\int iB_{ij}$, except if the decay is to a single-closed-shell nucleus. Such cancellation effects should occur in many other situations for forbidden β decay to collective states, so that further measurements of the β-decay matrix elements, will offer a quantitative test of this important consequence of particle correlations. Because of the approximations, the quantitative results are necessarily poorer when cancellations are involved, so that systematic experimental results in each region are of great importance. Comparison with decays to single-closed-shell nuclei should be useful in estimating the magnitude of the single-particle matrix elements and more clearly establishing the process of cancellation. Rough estimates of the $\Delta J=1$ matrix elements, which are forbidden if only particles in the major shell are treated, have been made for Sb^{124} by including particles in the closed shell below mass number 50 in the vibration. The results seem reasonable.