PHYSICAL REVIEW VOLUME 136, NUMBER 4B 23 NOVEMBER 1964

Model for Leptonic Interactions*

CHRISTIAN FRONSDAL†
Department of Physics, University of California, Los Angeles, California
(Received 26 February 1964)

There exists a theory of weak interactions, based on the Kemmer-Yukawa coupling of a scalar charged boson to the weak current, that is consistent with all known facts. We study the mathematical consistency of this theory and discover that the divergences of low-order perturbation amplitudes are much less serious than anticipated. Thus, (1) electromagnetic corrections of order $e^3(G)^{1/2}$ to the amplitude for weak photoproduction of the boson, (2) weak corrections of order eG to the magnetic moment of fermions, and (3) weak corrections of order e^2G to Compton scattering from charged fermions are all finite.

I. INTRODUCTION

IT has often been said, in order to explain our failures in strong interactions, that the outstanding success of perturbation theory in quantum electrodynamics is due to the smallness of the coupling constant. It is embarrassing to admit that the weak interactions have not yet yielded to this approach.

It has not yet been possible to show that the perturbation series in quantum electrodynamics is summable. Aside from this difficult question, however, quantum electrodynamics is a mathematically consistent theory that answers all questions relating to electrons and photons, with the exception of a finite and small number of parameters: viz., the electron mass and the fine structure constant. This theory is trivially extended to include the interactions between muons and photons.[1]

In view of the weakness of the weak interactions, it is legitimate to expect that the perturbation approach can be extended to include the "$V-A$" interactions between electrons, muons, and neutrinos.

In 1935, Yukawa proposed[2] the existence of the π meson to mediate nuclear forces, and since that time, the "Yukawa coupling" has been very much in vogue. Today, the existence of a Yukawa coupling between nucleons and pions is firmly established (what is not known is whether the entire pion-nucleon and nucleon-nucleon interaction can be generated by this simple coupling). Because the coupling between electrons and photons is of the same type, it is natural to attempt a construction of a Yukawa theory of all interactions between leptons.

In 1949, Yukawa proposed[3] a boson intermediary for weak interactions; one or more charged particles coupled linearly to pairs of leptons. He might have pointed out that there are at least two natural ways in which this may be done In perturbation theory, one starts with a Lagrangian for noninteracting particles, and introduces the interaction by adding a Yukawa coupling term to the Lagrangian. But the noninteracting theory may have several equivalent formulations, and what looks like a Yukawa coupling in one may appear quite different in another. To illustrate, consider the electromagnetic interactions of a boson without spin. The free boson may be described by the Klein-Gordon equation, which may be deduced from a suitable Lagrangian. The electromagnetic interaction, to be gauge invariant, can not be of the Yukawa type, but must include a term bilinear in the electromagnetic potential. Alternatively, the free boson may be described by a set of linear equations, the Kemmer equations,[4] which may also be deduced from a Lagrangian. In this case, the gauge-invariant interaction is of the Yukawa type. The two formulations are equivalent, but in one the coupling is pure Yukawa, in the other it is not.

The Yukawa coupling is preferred because of its simplicity, and this criterion is a very important one in physics. But it is prudent to keep in mind that, as a principle of selection, it is not unambiguous; what seems simple in one formulation is not always simple in another. I do not pretend to know that only two reasonable definitions of the Yukawa couplings exist, but I claim to be more open minded than those who admit only one.

The issue might be decided by showing that the Klein-Gordon equation is to be preferred to the linearized Kemmer equation, or vice versa. While it is true that the former is better known, first-order wave equations are simpler from the point of view expounded by Schwinger[5]; it is not accidental that two Harvard doctoral dissertations have been written on the use of the Kemmer formulation (by Klein on strong interactions,[6] and by Glashow on weak interactions[7]). On the other hand, *if we are to insist on Yukawa couplings throughout, then the Kemmer formulation must be chosen,* because in the Klein-Gordon formulation even the electromagnetic interactions are not Yukawa.

We estimate that these considerations make it worth while to study the following possibility: that the weak interactions between leptons are mediated by one or

* Work supported in part by the National Science Foundation.
† Present address: International Centre for Theoretical Physics, Trieste, Italy.
[1] The fact that the trivial extension accounts for what is observed is, of course, not trivial.
[2] H. Yukawa, Proc. Phys. Math. Soc. Japan **17**, 48 (1935).
[3] H. Yukawa, Rev. Mod. Phys. **21**, 474 (1949).
[4] N. Kemmer, Proc. Roy. Soc. (London) **A166**, 127 (1938).
[5] J. Schwinger, Phys. Rev. **91**, 713 (1953).
[6] A. Klein, Phys. Rev. **82**, 639 (1951).
[7] S. L. Glashow, thesis, Harvard University, 1958 (unpublished).

more spinless charged bosons, the coupling assuming the Yukawa form in the formulation that employs first-order wave equations. In order to avoid confusion, we shall use the term Kemmer-Yukawa coupling for an interaction that couples a meson field described by a Kemmer equation to two other fields.

It has already been pointed out that the above hypothesis is consistent with all weak interactions observed at low-momentum transfers.[7] This report attempts to give some support for hoping that it gives a theory which, when electromagnetic interactions are included, is a mathematically consistent extension of quantum electrodynamics to all leptonic interactions. By this we mean that, in spite of the impression received at first glance, this theory may be renormalizable.[8] A summary of the evidence is given in the last section.

II. NONINTERACTING FIELDS

We use the following notations for the single-particle wave functions[9]:

- ψ^-, a negative electron,
- ψ^+, a positive muon,
- ψ^0, a four component neutrino,[10]
- φ, a negative scalar boson in the Klein-Gordon formulation,
- χ, a negative scalar boson in the Kemmer formulation,
- A_μ, the electromagnetic potential.

The two neutrinos ν_e and $\bar{\nu}_\mu$ are the two projections $\frac{1}{2}(1\pm\gamma_5)\psi^0$, positive lepton number is carried by e^-, μ^+, ν_e, and $\bar{\nu}_\mu$.

Further conventions are:

$$ab=a_\mu b_\mu=a_0b_0-\mathbf{a}\cdot\mathbf{b},$$
$$\gamma_0^\dagger=\gamma_0,\quad \boldsymbol{\gamma}^\dagger=-\boldsymbol{\gamma},$$
$$\gamma_5=i\gamma_1\gamma_2\gamma_3\gamma_4.$$

The free field equation for φ is

$$(p^2-m^2)\varphi=0. \tag{1}$$

It is impossible for a Yukawa coupling between φ and the leptons to agree with the phenomenological $V-A$ coupling between leptons.

To linearize Eq. (1), write

$$m_1\varphi_\mu=p_\mu\varphi, \tag{2a}$$
$$m_2\varphi=p_\mu\varphi_\mu. \tag{2b}$$

This system of five first-order equations is equivalent to Eq. (1), provided

$$m_1m_2=m^2; \tag{3}$$

the value of m_1 fixes the normalization of φ_μ.

Equation (2) is the version of Eq. (1) that we shall use. It is convenient to write the five equations compactly as follows[4]:

$$(\beta p-m)\chi=0, \tag{4}$$

where

$$\chi=(m_1)^{1/2}\begin{pmatrix}\varphi_\mu\\ \varphi\end{pmatrix},\quad m=\begin{pmatrix}m_1 & \\ & m_2\end{pmatrix},$$
$$(\beta_\mu)_\alpha{}^\beta=g_{\alpha\mu}\delta_5{}^\beta+\delta_{\alpha5}\delta_\mu{}^\beta,\quad \mu=1,\cdots,4. \tag{5}$$

The 5×5 matrices β_μ satisfy the commutation relations

$$\beta_\mu\beta_\nu\beta_\lambda+\beta_\lambda\beta_\nu\beta_\mu=g_{\mu\nu}\beta_\lambda+g_{\nu\lambda}\beta_\mu.$$

The minimal electromagnetic coupling may be introduced into either Eq. (1) or Eq. (4), and the result is two equivalent formulations of scalar electrodynamics. The Lagrangian that describes the photon-lepton system without weak interactions, in the form that gives Eq. (4) as one of the field equations, is

$$L=L_++L_-+L_0+L_A+L_B, \tag{6}$$

where[10]

$$L_\pm=\bar{\psi}^\pm(p\gamma\pm eA\gamma-M^\pm)\psi^\pm,$$
$$L_0=\bar{\psi}^0(p\gamma-\mu)\psi^0,$$
$$L_A=\tfrac{1}{2}(p_\nu A_\mu)(p_\nu A_\mu),$$
$$L_B=\bar{\chi}(p\beta-eA\beta-m)\chi,$$

where $\bar{\chi}=(\varphi^{\mu*},\varphi^*)$.

Green's function for the field χ is defined by

$$S(p)=(p\beta-m)^{-1}=\frac{1}{m}\left[-1+\frac{(p\beta+m)p\beta}{p^2-m^2}\right]. \tag{7}$$

This formula shows that the scalar boson gives rise to a pole in the S-wave scattering amplitude only, but also that it may mediate a vector interaction locally. The term -1 may be thought of as the limit, as Λ tends to infinity, of $\Lambda^2/(p^2-\Lambda^2)$. With a finite value of Λ, the theory may then be interpreted as including a vector meson with mass Λ, which shows that a cancellation of divergences may be expected to the same extent as in the mixed meson theory of Beard and Bethe.[11] That theory is plagued by the existence of states with negative normalization, but this difficulty disappears in the limit of infinitely heavy mass.

[8] We were first led to suspect this possibility by the result of a now obsolete calculation, C. Fronsdal and S. L. Glashow, Phys. Rev. Letters **3**, 570 (1959).

[9] We define lepton number by assigning it the value +1 for the negative electron and for the positive muon. With this convention, the electron-neutrino and the muon-neutrino fit together in a single four-component Dirac spinor field.

[10] It is nowhere essential to our discussion that the neutrino mass μ be zero.

[11] D. B. Beard and H. A. Bethe, Phys. Rev. **83**, 1106 (1951).

III. THE "V-A" INTERACTION

A phenomenological interaction Lagrangian that accounts for all weak interactions between leptons at low-momentum transfer is

$$L_{\text{phen}}=(G/\sqrt{2})J_\mu{}^\dagger J_\mu\,, \tag{8}$$

where the weak current is

$$J_\mu=\bar{\psi}^0(1-\gamma_5)\gamma_\mu\psi^- - \bar{\psi}^+\gamma_\mu(1-\gamma_5)\psi^0. \tag{9}$$

It has been suggested repeatedly that the correct form is instead

$$L_{\text{V.M.}}=g(J_\mu{}^\dagger V_\mu+V_\mu{}^\dagger J_\mu)\,, \tag{10}$$

where V_μ is the field of a charged meson with spin 1. However, this interaction too is nonrenormalizable.

The interaction Lagrangian that we shall propose is the following:

$$L_{\text{S.M.}}=(G/\sqrt{2})J_\mu{}^\dagger J_\mu +[C_1J_\mu{}^\dagger(p_\mu-eA_\mu)\varphi+C_2J_5{}^\dagger\varphi+\text{H.c.}]\,, \tag{11}$$

where J_5 is obtained from (9) by putting $\mu=5$, and C_1, C_2 are complex constants. This combination of a local four-fermion coupling and a gradient coupling of the weak current to a scalar charged field has the following properties: (a) The coupling strength C_1 may be so adjusted that many divergencies cancel out in lowest order perturbation theory. (b) The remaining infinities in lowest order perturbation theory may be interpreted as renormalization of masses and coupling constants. (c) The coupling constant C_2 may be so adjusted (e.g., by setting it equal to zero) that $L_{\text{S.M.}}$ and L_{phen} describe the present experiment situation equally well.[7,12]

While all of these features are desirable, there is no reason to believe that the cancellation of divergences, brought about by design in lowest order perturbation theory, extend to the higher orders. Selecting the interaction on the basis of simplicity alone is much to be preferred, for then calcellations found to occur in the lowest orders are much less likely to be fortuitous.

We introduce a nonderivative Kemmer-Yukawa coupling between the five-component field χ and bilinear combinations of lepton fields by adding the following term to the Lagrangian:

$$L_{\text{weak}}=J^\dagger g\chi+\text{H.c.}\,, \tag{12}$$

with

$$J^\dagger=(J^{\mu*},J_5{}^*)\,,$$

$$g=\begin{pmatrix} g_1 & 0\\ 0 & g_2\end{pmatrix}.$$

It will immediately become obvious that the correct form of J_μ is that given by (9). There is no experimental basis for choosing J_5, but we shall show that much may be said for extending the range of μ in Eq. (9). First we note that the interaction (12) is equivalent to (11).

[12] This will become more obvious below, see especially Eq. (16).

The field equation for χ, with the interaction term (12) included in the Lagrangian, is

$$(p\beta-eA\beta-m)\text{p}+g^*J=0\,, \tag{13}$$

and in particular,

$$m_1\varphi_\mu=(p_\mu-eA_\mu)\varphi+(g_1{}^*/\sqrt{m_1})J_\mu.$$

Since there is no independent asymptotic field associated with φ_μ, we may eliminate it, with the result that

$$L_B+L_{\text{weak}}\Rightarrow\varphi^*[(p_\mu-eA_\mu)^2-m_1m_2]\varphi+L_{\text{S.M.}}\,, \tag{14}$$

with the following relations between the coupling constants,

$$G/\sqrt{2}=|g_1|^2/m_1\,,\; C_1=g_1/(m_1)^{1/2}\,,\; C_2=(\sqrt{m_1})^{1/2}g_2. \tag{15}$$

The effective Lagrangian for processes involving no absorption or emission of bosons, to lowest order in g, is obtained from (12) and (13) by eliminating all five components of χ. From (13),

$$\chi=-(p\beta-eA\beta-m)^{-1}g^\dagger J;$$

whence

$$L_{\text{weak}}{}^{\text{eff.}}=-J^\dagger(p\beta-eA\beta-m)^{-1}g^\dagger gJ.$$

Using (7), and ignoring the electromagnetic complications, we obtain

$$L_{\text{weak}}{}^{\text{eff.}}=\frac{G}{\sqrt{2}}J_\mu{}^\dagger J_\mu -\frac{1}{p^2-m^2}\left|\frac{g_1}{(\sqrt{m_1})^{1/2}}p_\mu J_\mu+(m_1)^{1/2}g_2J_5\right|^2. \tag{16}$$

This shows that the weak interaction is of the $V-A$ form except for momentum transfers near the boson "resonance"; the resonance peak is of the $S-P$ coupling form. (The g_2 term is small because g_2 is. The g_1 term is small because it contains a factor [(lepton mass)-(lepton mass)′]/boson mass.)

An observational excuse for J_5 does not exist, but if one insists on generality and supposes that g_2 is small but not zero, then there is a preferred form, namely,

$$J_5=\bar{\psi}^0(1-\gamma_5)\gamma_5\psi^- - \bar{\psi}^+\gamma_5(1-\gamma_5)\psi^0. \tag{17}$$

This form is preferred because it may be introduced without destroying the invariance of the theory under the transformations

$$\psi^\pm\rightarrow e^{\pm i\alpha}\psi^\pm\,,\quad \psi^0\rightarrow e^{-i\alpha\gamma_5}\psi^0. \tag{18}$$

In other terms, the current

$$F_\mu=\bar{\psi}^+\gamma_\mu\psi^+ - \bar{\psi}^-\gamma_\mu\psi^- - \bar{\psi}^0\gamma_\mu\gamma_5\psi^0\,, \tag{19}$$

which is always conserved when $g_2=0$, remains conserved when $g_2\neq0$, provided J_5 is of the form (17). In the two-neutrino formulation, this is simply the law of conservation of leptons. With the choice (17), the theory admits three conserved currents.

In order to interpret correctly the various renormalization terms encountered in this theory, it is important to take note of another invariance group. Of the four constants m_1, m_2, g_1, and g_2, only three independent combinations appear in the effective Lagrangian. To show that this is true to all orders, it is sufficient to notice that the total Lagrangian is invariant under the following one-parameter group of transformations:

$$\chi \to R^{-1}\chi, \quad m \to RmR, \quad g \to Rg, \tag{20}$$

where

$$R=\begin{pmatrix} r & 0 \\ 0 & r^{-1} \end{pmatrix}, \quad r \text{ real.}$$

This merely reflects the fact that the normalization of the "field" φ_μ is arbitrary. A convenient way of writing the matrix R is

$$R=e^{r\beta_5}, \quad \beta_5\equiv\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \tag{21}$$

the matrix β_5 anticommutes with β_μ.

IV. RENORMALIZATION, PART I

The test of the model proposed in the previous section must come from experiment, and there is reason to hope this will become available shortly. The relevance of high-energy neutrino experiments currently carried out at CERN will be discussed separately. At present, it seems reasonable to expend a limited amount of effort on investigating the mathematical consistency of the theory.

Renormalizability cannot be demonstrated by means of the simple counting of convergence factors used by Dyson.[13] The propagator for the boson does not seem to contain any convergence factor [see Eq. (7)], so that Dyson's method would give an infinite number of primitively divergent diagrams. Yet, it is known that the electromagnetic interaction is renormalizable, so that very strong cancellations must be taking place, and there is some evidence that this may be true of the weak interactions too.

Because a complete proof of renormalizability (or lack thereof) is beyond reach, we shall have to be content with a discussion of divergences in low orders of perturbation theory. In this section, we discuss electromagnetic corrections to weak interactions.

Mass renormalization to lowest order in e is given by the two Feynman diagrams[14] of Fig. 1. We find in the

FIG. 1. Electromagnetic mass renormalization of charged fermions (Σ_1) and of bosons (Σ_2).

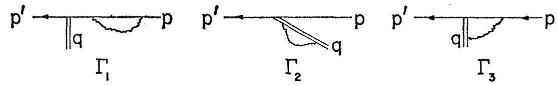

FIG. 2. Electromagnetic corrections to the weak vertex.

Feynman gauge[15]

$$\Sigma_1\approx e^2\int d^4k\, k^{-4}\{3M-(p\gamma-M)\}, \tag{22}$$

$$\Sigma_2\approx e^2\int d^4k\, k^{-4} \times\left\{\left[\tfrac{3}{2}\left(1-\frac{k^2}{m^2}\right)+\tfrac{3}{2}\beta_5\frac{k^2}{m^2}\right]m+\tfrac{1}{2}(q\beta-m)\right\}, \tag{23}$$

where the wavy conjunctions mean "equals up to finite parts." The first term in Σ_1 and the first term in Σ_2 are ordinary mass-renormalization terms of fermions and bosons, respectively, and the last terms are recognized as contributions to charge renormalization, to be discussed below. The second term in Σ_2 is not a renormalization of the observable mass, but may be interpreted as a coupling constant renormalization, because of the invariance discussed in the preceding section.

Electromagnetic renormalization of the weak vertex to lowest order in e consists of the three diagrams of Fig. 2. We find, exclusive of mass renormalization terms[15]:

$$\Gamma_1+\Gamma_2+\Gamma_3\approx e^2\int d^4k\, k^{-4} \times\left[\Gamma_0\left\{\tfrac{3}{2}+\tfrac{3}{4}\beta_5+\tfrac{3}{4}(\beta_5+1)\frac{1}{m}(q\beta-m)\right\} +\frac{3}{4}\frac{M}{m}\Gamma_{0\mu}\gamma_\mu\gamma_5\right], \tag{24}$$

where Γ_0 is the bare weak vertex.[16] Now, it is seen that the choice of the particular form of J_5 enables us to interpret the last term in (24) as a renormalization of g_2. (Because the scalar interaction is known to be small, it may be reasonable to try to put the bare g_2 equal to zero and to expect the renormalized g_2 to be of the order of magnitude $3e^2g_1M/m_1$.) We see that the electromagnetic renormalization preserves the universality of the important vector coupling constant g_1; by this we mean that if the bare g_1 is universal, then so is the

[13] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).

[14] A wavy line is a photon, a directed line represents a fermion with positive lepton number; a double line is a negative boson.

[15] The Feynman rules are: Propagators $(p\gamma-M^{\pm})^{-1}$, $(p\gamma-\mu)^{-1}$, $(\beta p-m)^{-1}$. Electromagnetic vertices $-e\gamma_\mu$, $+e\gamma_\mu$, $e\beta_\mu$ for positive fermion, negative fermion, and negative boson. Weak vertices $(\mp1-\gamma_5)(g\gamma)_\alpha$ if a lepton with charge $\pm e$ enters, and $(g\gamma)_\alpha \times(\mp1+\gamma_5)$ if it leaves the vertex. Here, $(g\gamma)_\alpha=(g_1\gamma_\mu, g_2\gamma_5)$.

[16] Reverse the order of matrix factors in (24) if the charged lepton leaves, rather than enters, the diagram.

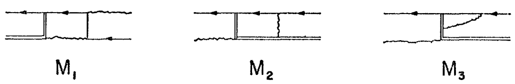

FIG. 3. Electromagnetic corrections to boson photoproduction.

renormalized g_1. This has a bearing on the possibility of comparing the lifetimes of the muon and of the neutron; we return to this point below.

Equation (24) is valid if any or all of the three particles are virtual, so that taking the fermions off the mass shell does not introduce new divergent parts. To study the new divergence which results from making the boson virtual, we must make the renormalized vertex a part of a diagram where the boson line is internal. The simplest way of doing this is to allow the boson to interact just once, either electromagnetically or weakly; we shall study both alternatives.

Let the boson in Fig. 2 absorb a photon; this gives the process in which a boson is produced in an external electromagnetic field. The divergent diagrams to the lowest order of electromagnetic corrections are shown in Fig. 3.[17] In the Feynman gauge, M_1 and all of the self-energy diagrams are finite; the divergent part of M_3 is obtained from the $(q\beta-m)$ term in Eq. (24),

$$M_3\approx e^2\int d^4k\, k^{-4}\Gamma_0\tfrac{3}{4}(\beta_5+1)\frac{1}{m}e\beta_\mu\,, \tag{25}$$

and this is precisely canceled by the divergent part of M_2.

Thus, the various corrections to boson photoproduction, though they are individually divergent, add up to a convergent expression in the lowest nontrivial order of the perturbation expansion.

Next, consider the other way of making the boson line in Fig. 2 virtual, by adding another weak interaction to it as in Fig. 4. In this case, which is the important four-fermion interaction, we obtain a very curious and not altogether encouraging result.

We shall compare the divergences arising in the calculation of M_4, M_5, and M_6 with the divergences in the local theory. The individual contributions of the graphs M_4, M_5, and M_6 are not gauge-invariant, and there is also a contribution from the boson self-energy diagram. We choose the Feynman gauge to describe the intermediate boson theory, for then M_4, M_5, and M_6 are the only divergent graphs (other than mass-renormalization and coupling-renormalization terms). The local theory contains a diagram similar to M_6, with

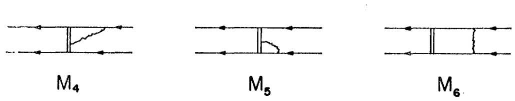

FIG. 4. Electromagnetic corrections to weak interactions involving electrons, muons, and neutrinos.

[17] There are also some well-known corrections to the electromagnetic vertex.

the boson line contracted to a point, and *in the Landau gauge* this diagram (call it $M_{\rm local}$) is the only divergent one. We therefore use the Landau gauge when referring to the local theory. This simplifies the comparison, because now $M_6\approx M_{\rm local}$ (that is, the divergent parts are equal).

We find that

$$M_6\approx M_{\rm local}\approx -\frac{3}{2}\frac{e^2}{m_1}\int d^4k\, k^{-4}[\Gamma_\mu\Gamma_\mu'+(\Gamma_\mu\gamma_5)(\Gamma_\mu\gamma_5)'],$$
$$=0\,, \tag{26}$$

$$M_4\approx M_5\approx \frac{3}{2}\frac{e^2}{m_1}\int d^4k\, k^{-4}\Gamma_\mu\Gamma_\mu\,, \tag{27}$$

where $\Gamma_{0\mu}$ is the four-vector part of the bare Yukawa vertex. Thus, there is an infinite renormalization in the meson theory but not in the local theory.

Let us digress for a moment to consider the renormalization in neutron decay. In this case, the $V-A$ nature of the nuclear matrix elements makes the two terms in M_6 add constructively rather than destructively, and M_6 does not vanish. Thus, electromagnetic corrections to neutron decay are infinite in the local theory, but in the scalar meson theory, M_6 is cancelled by M_4 and M_5, and the sum is *finite*.[18]

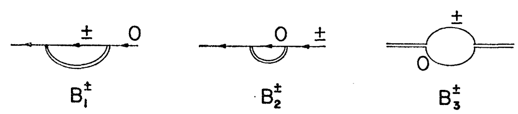

FIG. 5. Weak mass renormalization diagrams. The double signs refer to the electric charge.

It is very tempting to conjecture that the theory might be modified in such a manner that the vertex parts cancel for lepton interactions, but add for neutron decay (so that their sum is canceled by M_6). The theory under investigation, though not satisfactory when applied to μ decay, does nevertheless suggest an interesting possibility for achieving finite electromagnetic corrections to both μ decay and neutron decay.

V. RENORMALIZATION, PART II

We have studied electromagnetic damping of weak interactions, and turn now to the opposite situation—weak corrections to electromagnetic processes. Superficially, these would seem to be more divergent, but here we find the strongest cancellations.

Weak mass renormalization to lowest order in g consists of the three diagrams of Fig. 5. We find

$$B_1^{\pm}\approx\int d^4k\, k^{-4}\{I_1+(p\gamma-\mu)\bar{J}_1^{\pm}+J_1^{\pm}(p\gamma-\mu)$$
$$+(p\gamma-\mu)K_1^{\pm}(p\gamma-\mu)\}\,, \tag{28}$$

[18] We may add that the renormalization of g_2 represented by the last term in (24) also cancels in neutron decay.

and similar expressions for $B_2^{\pm}$ and $B_3^{\pm}$. Here I_1, I_2, and $I_3^{\pm}$ are constant matrices giving mass renormalization; the matrices $J_i^{\pm}$, $i=1, 2, 3$ contribute to coupling constant renormalization. The terms with $K_1^{\pm}$, $K_2^{\pm}$, and $K_3^{\pm}$ contribute only when the respective lines are internal. Most terms are quadratically divergent.

Weak renormalization of the charged fermion vertex to lowest order in g consists of three divergent diagrams, two of them self-energy diagrams; the third is shown in Fig. 6. We find

$$\Gamma_4^{\pm}\approx\mp e\int d^4k\, k^{-4}\{-\tfrac{1}{3}(q^2\gamma_\mu-q_\mu\gamma q)(1\mp\gamma_5)g_1^2m_1^{-1} +\gamma_\mu K_2^{\pm}(p')(\gamma p'-M^{\pm})+(\gamma p-M^{\pm})K_2^{\pm}(p)\gamma_\mu +\gamma_\mu\tilde{J}_2^{\pm}+J_2\gamma_\mu+(\gamma p-M^{\pm})\gamma_\mu(1\pm\gamma_5) \times(\gamma p'-M^{\pm})g_1^2m_1^{-1}\}. \quad (29)$$

The self-energy diagrams cancel out the terms $\gamma_\mu\tilde{J}_2$ and $J_2\gamma_\mu$. The first term vanishes when the photon is real, the remaining terms when the fermions are real. The magnetic moment correction is finite,[18a] but there is a renormalization of the first moment of the charge form factor (the first term).

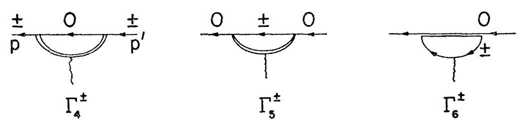

FIG. 6. Weak corrections to the electromagnetic vertices include these diagrams and self-energy diagrams.

To study the K_2 term, one of the fermions must be made virtual by adding one more interaction. The simplest way to do this is to study weak corrections to Compton scattering. The divergent diagrams are vertex corrections (the first term and the K_2 term in Γ_4), a propagator correction (the K_2 part of $B_2^{\pm}$), and finally the diagrams shown in Fig. 7 for which we find on the mass shell (for fermion)

$$\tfrac{1}{2}(M_7+M_7')^{\pm}\approx e^2\int d^4k\, k^{-4}\gamma_\mu K_2\gamma_\nu. \quad (30)$$

The bubble diagram M_8 gives a divergent contribution which is seen from (28) to be the same as (30). All this is then seen to be cancelled by the K_2 term in (29). The only remaining divergence is that associated with the first term of $\Gamma_4^{\pm}$. Hence, no new divergences appear in Compton scattering from charged fermions to this order.

The weak interactions give to the neutral fermion an electromagnetic structure. The diagrams, to lowest order in g are shown in Fig. 6 (Γ_5 and Γ_6). Again, the magnetic moment correction is finite, but there is a

[18a] The relative correction to the magnetic moment of the muon is 10^{-9}.

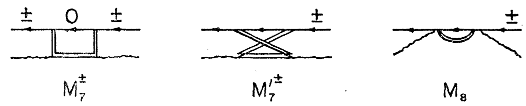

FIG. 7. Weak corrections to Compton scattering from charged fermions also include vertex corrections.

divergent renormalization of the first moment of the charge renormalization as found for charged fermions.

Compton scattering from neutral fermions is given by M_7, M_7', the two diagrams of Fig. 8, and the corresponding crossed diagrams. These have not yet been calculated.

The weak renormalization of the electromagnetic interactions of the boson has been calculated when the boson is on the mass shell only. In this case too, there is a divergent renormalization of the first moment of the charge form factor.

VI. CONCLUSIONS

The evidence which favors the possibility that the theory is renormalizable is the surprising number of cancellations of divergences that we have encountered so far, which cancellations produce finite results in the following cases: (a) Electromagnetic corrections to boson photoproduction. (b) Weak corrections to the magnetic moments of fermions. (c) Weak corrections to Compton scattering from charged fermions. (d) Electromagnetic corrections to neutron decay. The corrections to μ decay do not come out finite, but it is very tempting to conjecture that this difficulty might be overcome (see Sec. IV).

It is also encouraging thet electromagnetic renormalization of g_1 preserves the universality. The other coupling constant g_2 is known to be small experimentally; universality of g_2 is not preserved by electromagnetic damping, unless we postulate that g_2 is proportional to the mass of the charged member of the weak current. If the bare g_2 vanishes, we may expect an observed g_2 of the order of magnitude of $g_1(M/m_1)/100$.

Dyson's criterion does not establish the renormalizability of the theory, but this is true even for the electromagnetic interaction—which is known to be renormalizable. It may be possible to find an alternative formulation in which Dyson's criterion suffices.

We have stressed the evaluation of the theory from the point of view of mathematical consistency rather than comparison with experiment. When more data become available on high-energy neutrino scattering,

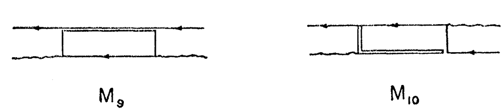

FIG. 8. Additional contributions to the Compton scattering from neutral fermions.

it may be possible to establish the existence and then the spin of a weak boson. We predict that double lepton events will proceed via an $S-P$, rather than a $V-A$ interaction; but this may not be a spectacular effect because neither the nuclear recoil nor the final neutrino momentum is measured.[19] Angular distributions of the "elastic" events may provide the distinguishing analyzer; a report on this will appear shortly. If the experiments should favor a scalar intermediary boson, then it will be worthwhile to study the mathematical structure of the theory in a more thorough fashion.

[19] I am indebted to Professor R. E. Norton for the following remark: If $g_2=0$, then the events in which a μe lepton pair is produced via the decay of a real scalar boson into $\nu+e$ will be suppressed relative to $\mu\mu$ events (by the decay of the scalar boson into $\nu+\mu$) to the same extent, and for the same reason, as $\pi\rightarrow\nu+e$ is suppressed relative to $\pi\rightarrow\nu+\mu$. Until the existence of an intermediary meson is definitely established and a selection of events proceeding through a *real* intermediary can be made, this test cannot be applied.

PHYSICAL REVIEW VOLUME 136, NUMBER 4B 23 NOVEMBER 1964

Inelastic Effects of the N^* and ρ on Pion-Nucleon Scattering*†

F. T. Meiere‡
Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 6 July 1964)

A formalism is developed for calculating the pion-nucleon scattering amplitude which conveniently includes the inelastic production of a (3,3) isobar, and is extended to include ρ-meson production. The exact unitarity relations are found to contain nondiagonal terms which, when simplified, are interpreted as the overlap between resonances. Calculations based on the N/D method are performed in which the unphysical discontinuities are evaluated from single-nucleon or pion-exchange diagrams. The numerical solution of the resulting integral equations is compared to simpler approximations and to experiment. The importance of both the isobar and the ρ is pointed out. For the $D_{3/2}$ channel, a resonance was found for $T=\frac{1}{2}$ but not for $T=\frac{3}{2}$. No resonance was found for either of the $F_{5/2}$ or $D_{5/2}$ channels.

I. INTRODUCTION

The importance of production channels in explaining the higher resonances of pion-nucleon scattering has been pointed out by several authors,[1–3] who concentrated primarily on the inelastic channel $\pi+\pi+N$ with the two pions resonating to form a ρ meson. We wish to calculate the scattering amplitude for the process $\pi+N\rightarrow\pi+N$ using partial-wave dispersion techniques to include the effects of production channels in which either a (3,3) pion-nucleon isobar, the N^*, or a ρ meson is produced. The formalism here also includes the overlap of these resonances. Although the threshold for N^* production is slightly further from the observed resonances than the threshold for ρ production, the higher spin of the N^* allows its effects to reach further than might otherwise be expected.

The detailed calculations are not designed as a fit to the data but proceed from known masses and three known coupling constants, so that no adjustable parameters are available.

Unitarity of the S matrix $S_{jk}=\delta_{jk}+iT_{jk}$, gives $T_{jk}(s_+)-T_{jk}(s_-)=i\sum_n T_{jn}(s_+)T_{nk}(s_-)$ for s real and greater than threshold. In the sum, we keep those two- and three-body states which can contribute to pion-nucleon scattering, namely $\pi+N$ and $\pi+\pi+N$ states. The three-body state increases the complexity of the problem considerably. However, using the observed resonances of the two-body pion-nucleon and pion-pion systems, several authors[1–4] have suggested reducing the complexity of the production channel by considering only those three-body states where two of the particles emerge in a resonant state and considering this two-body resonant state as a distinct, but unstable, particle. This would lead to processes $\pi+N\rightarrow\pi+N^*$ or π^*+N.

Federbush *et al.*[4] have shown a self-consistent treatment of unstable particles by coupling them to pions and nucleons as though they were stable elementary particles, and only using the properties of their decay to calculate the appropriate coupling constants from experiment. Cook and Lee[5] have developed an extended N/D formalism which imposes unitarity on the coupled $\pi+N$ and $\pi+\pi+N$ channels, while at the same time it easily permits the inclusion of a $\pi\pi$ resonance in the

* This work is supported in part through funds provided by the U. S. Atomic Energy Commission under Contract AT(30–1)–2098.
† A portion of this work was submitted to the Massachusetts Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
‡ Present address: Department of Physics, Purdue University, Lafayette, Indiana.

[1] L. F. Cook and B. W. Lee, Phys. Rev. **127**, 283 (1962).
[2] J. Ball, W. Frazer, and M. Nauenberg, Phys. Rev. **128**, 478 (1963).
[3] V. Teplitz, University of Maryland Technical Report 270, 1962 (unpublished).
[4] P. G. Federbush, M. Grisaru, and M. Tausner, Ann. Phys. (N. Y.) 18, 23 (1962).
[5] L. F. Cook and B. W. Lee, Phys. Rev. **127**, 297 (1962).