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Neutrino Opacity I. Neutrino-Lepton Scattering*

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The contribution of neutrino-lepton scattering to the total neutrino opacity of matter is investigated; it is found that, contrary to previous beliefs, neutrino scattering dominates the neutrino opacity for many astrophysically important conditions. The rates for neutrino-electron scattering and antineutrino-electron scattering are given for a variety of conditions, including both degenerate and nondegenerate gases; the rates for some related reactions are also presented. Formulas are given for the mean scattering angle and the mean energy loss in neutrino and antineutrino scattering. Applications are made to the following problems: (a) the detection of solar neutrinos; (b) the escape of neutrinos from stars; (c) neutrino scattering in cosmology; and (d) energy deposition in supernova explosions.

I. INTRODUCTION

EXPERIMENTS[1,2] designed to detect solar neutrinos will soon provide crucial tests of the theory of stellar energy generation. Other neutrino experiments have been suggested as a test[3] of a possible mechanism for producing the high-energy electrons that are inferred to exist in strong radio sources and as a means[4] for studying the high-energy neutrinos emitted in the decay of cosmic-ray secondaries. From a theoretical standpoint, the production of neutrinos by electron-positron annihilation, or by related processes, has been shown[5,6] to play a pivotal role in the later stages of stellar evolution and in the formation of the elements near the iron peak. Moreover, neutrinos play an important role in a number of cosmological considerations, including the question of the energy density in the universe[7,8] and the problem of distinguishing between cosmological models.[9,10]

The processes by which neutrinos are emitted or absorbed have therefore been extensively discussed by many authors.[5,6,11,12] However, neutrino scattering has only been discussed for the special situation of electrons initially at rest.[13,14]

In this paper, we investigate the contribution of neutrino-lepton scattering to the total neutrino opacity of matter and show, contrary to previous beliefs, that neutrino-lepton scattering dominates the neutrino opacity for many astrophysically important conditions. Here, neutrino opacity is defined, analogously to photon opacity, as the inverse of the neutrino mean free path times the matter density [i.e., $K_\nu=(\lambda\rho)^{-1}$]. In a subsequent paper with Frautschi,[15] the contribution of neutrino-nucleon interactions to neutrino opacity is discussed.

As a basis for astrophysical applications, we give the rates, under a variety of conditions, for neutrino-electron scattering, i.e.,

$$\nu_\beta+e^-\rightarrow\nu_\beta'+e^{-\prime}, \tag{1}$$

and antineutrino-electron scattering, i.e.,

$$\bar{\nu}_\beta+e^-\rightarrow\bar{\nu}_\beta'+e^{-\prime}. \tag{2}$$

The formulas we present are derived from the conserved vector current theory.[13]

The cross sections for reactions (1) and (2) are identical, by PC invariance, with the cross sections for the interactions among the corresponding antiparticles, i.e.,

$$\bar{\nu}_\beta+e^+\rightarrow\bar{\nu}_\beta'+e^{+\prime}, \tag{1'}$$

and

$$\nu_\beta+e^+\rightarrow\nu_\beta'+e^{+\prime}. \tag{2'}$$

Hence, we discuss explicitly only reactions (1) and (2), although (1′) and (2′) also occur in a number of astronomical situations. The cross sections for neutrino-muon scattering can be obtained from the cross sections given in this paper by substituting, in all formulas, the

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[1] R. Davis, Jr., Phys. Rev. Letters **12**, 302 (1964); J. N. Bahcall, *ibid.* **12**, 300 (1964).

[2] F. Reines and W. R. Kropp, Phys. Rev. Letters **12**, 457 (1964).

[3] J. N. Bahcall and S. C. Frautschi, Phys. Rev. **135**, B788 (1964).

[4] K. Greisen, *Proceedings of the International Conference on High Energy Physics* (Interscience Publishers, Inc., New York, 1960), p. 209; T. D. Lee, H. Robinson, M. Schwartz, and R. Cool, Phys. Rev. **132**, 1297 (1963).

[5] W. A. Fowler and F. Hoyle, Astrophys. J. Suppl. **91**, 1 (1964). This article contains an extensive review of the role of most of the neutrino-emission processes as well as recent work on the abundance of the iron-peak elements.

[6] H. Y. Chiu and P. Morrison, Phys. Rev. Letters **5**, 573 (1960); H. Y. Chiu and R. Stabler, Phys. Rev. **122**, 1317 (1961). See also H. Y. Chiu, Ann. Phys. (N. Y.) **26**, 364 (1964) for recent work and references on the relation between supernovae, neutrinos, and neutron stars.

[7] B. Pontecorvo and Ya. Smorodinskii, Zh. Eksperim. i Teor. Fiz. **41**, 239 (1961) [English transl.: Soviet Phys.—JETP **14**, 173 (1962)].

[8] G. Marx, Nuovo Cimento **30**, 1555 (1963).

[9] S. Weinberg, Nuovo Cimento **25**, 15 (1962); S. Weinberg, Phys. Rev. **128**, 1457 (1962).

[10] J. V. Narlikar, Proc. Roy. Soc. (London) **270**, 553 (1962).

[11] R. N. Euwema, Phys. Rev. **133**, B1046 (1964).

[12] J. N. Bahcall, Phys. Rev. **135**, B137 (1964).

[13] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958). See in particular footnote seventeen for neutrino-electron scattering.

[14] Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **23**, 1117 (1960); S. M. Berman, *International Conference on Theoretical Aspects of Very High Energy Phenomena* (CERN, Geneva, 1961), p. 17.

[15] J. N. Bahcall and S. C. Frautschi, Phys. Rev. **135**, B788 (1964).

muon's mass for the electron's mass. In addition to the processes described by Eqs. (1)–(2′), we also discuss, briefly, a possible resonance in the $\bar{\nu}_\beta+e^-$ system, and some neutrino reactions that produce muons.

We begin by introducing, in Sec. II, the relevant notation and kinematics. We also present in Sec. II formulas giving the various differential and total cross sections for reaction (1) as well as the mean neutrino scattering angle and energy loss. In Sec. III, we present the corresponding formulae for reaction (2) and also discuss resonant scattering of antineutrinos, and the reactions $\bar{\nu}_\beta+e^-\to\bar{\nu}_\mu+\mu^-$ and $\nu_\mu+e^-\to\nu_\beta+\mu^-$. We then discuss in Sec. IV the effect of atomic binding on the proposed neutrino-electron scattering experiment of Reines and Kropp.[2] We present in Sec. V cross sections for reactions (1) and (2) when the target electrons constitute either a degenerate or a nondegenerate gas. We apply in Sec. VI the results of the previous sections to the following problems: (a) the detection of solar neutrinos; (b) the escape of neutrinos from stars; (c) neutrino scattering in cosmology; and (d) energy-deposition in supernovae explosions.

Readers who are primarily interested in astrophysical applications are advised to glance briefly at Secs. II–V and then turn to Sec. VI.

II. NEUTRINO-ELECTRON SCATTERING

A. Notation

Let $\mathbf{q}/c$ ($\mathbf{q}'/c$) be the momentum of the initial (final) neutrino, and let $\mathbf{p}_e$, W ($\mathbf{p}_e'$,W') be, respectively, the momentum and total energy of the initial (final) electron. It is convenient to introduce the following dimensionless variables:

$$\boldsymbol{\omega}\equiv\mathbf{q}/m_ec^2\,, \tag{3a}$$

$$\mathbf{p}\equiv\mathbf{p}_e/m_ec\,, \tag{3b}$$

$$\epsilon\equiv W/m_ec^2\,, \tag{3c}$$

$$\mu_\nu\equiv\mathbf{q}\cdot\mathbf{q}'/qq'\,, \tag{3d}$$

and

$$\mu_e\equiv\mathbf{q}\cdot\mathbf{p}'/qp'\,. \tag{3e}$$

Analogous variables, $\boldsymbol{\omega}'$, $\mathbf{p}'$ and ϵ', are defined by equations that are identical to Eqs. (3a), (3b), and (3c) except that primes, indicating final-state quantities, are inserted on both sides of the corresponding equations. It is also convenient to introduce the following dimensionless four-vectors: $\omega_\alpha\equiv(\omega,\boldsymbol{\omega})$, $\omega_\alpha'\equiv(\omega',\boldsymbol{\omega}')$, $p_\alpha=(\epsilon,\mathbf{p})$, $p_\alpha'=(\epsilon',\mathbf{p}')$, and $k_\alpha=p_\alpha+\omega_\alpha$. We use the metric in which $p_\alpha p_\alpha=p\cdot p=+1$.

The cross sections for neutrino-electron scattering are found to be proportional to

$$\sigma_0\equiv(4/\pi)(\hbar/m_ec)^{-4}(G^2/m_e^2c^4)\,, \tag{4a}$$

$$\cong 1.7\times10^{-44}\text{ cm}^2\,, \tag{4b}$$

where G is the coupling constant[16] for strangeness-conserving weak interactions. The corresponding value of σ_0 for muon-neutrino scattering processes is $\sigma_0(\nu_\mu-\mu^-)=(m_\mu/m_e)^2\sigma_0\cong7.3\times10^{-40}$ cm^2.

B. Kinematics

The energy and momentum conservation relations for electron-neutrino scattering are the same as for Compton scattering. Thus for electrons initially at rest one finds:

$$1\leq\epsilon'\leq1+2\omega^2/(1+2\omega)\,, \tag{5}$$

and

$$\omega/(1+2\omega)\leq\omega'\leq\omega\,. \tag{6}$$

The scattering is, of course, cylindrically symmetric about the initial neutrino momentum, q. The final electron or neutrino energies can be expressed in terms of the scattering angles as follows:

$$\omega'=\omega[1+\omega(1-\mu_\nu)]^{-1} \tag{7}$$

and

$$\epsilon'=1+(2\omega^2\mu_e^2)[(1+\omega)^2-\omega^2\mu_e^2]^{-1}\,. \tag{8}$$

The range of the scattering angles is given by

$$-1\leq\mu_\nu\leq+1\,, \tag{9}$$

and

$$0\leq\mu_e\leq+1\,. \tag{10}$$

C. Cross Sections

The conserved-vector-current theory[13] can be used to obtain the following expression for the invariant total cross sections for reaction (1):

$$\sigma=\frac{\sigma_0}{4\pi}\int\int\frac{d^3p'd^3\omega'}{\epsilon'\omega'}(p'\cdot\omega')\delta^{(4)}(p_\alpha+\omega_\alpha-p_\alpha'-\omega_\alpha')\,. \tag{11}$$

The differential cross sections obtained from Eq. (10) for an electron initially at rest $[p_\alpha=(1,\mathbf{0})]$ are

$$d\sigma/d\mu_\nu=\sigma_0\omega^2/2[1+\omega(1-\mu_\nu)]^2\,, \tag{12}$$

$$d\sigma/d\omega'=\sigma_0/2\,, \tag{13}$$

$$\frac{d\sigma}{d\mu_e}=\begin{cases}2\sigma_0[(1+\omega)/\omega]^2\mu_e[((1+\omega)/\omega)^2-\mu_e^2]^{-2}, & \mu_e\geq0 \quad (14a)\\ 0, & \mu_e\leq0 \quad (14b)\end{cases}$$

[16] R. K. Bardin, C. A. Barnes, W. A. Fowler, and P. A. Seeger, Phys. Rev. **127**, 583 (1962).

and

$$d\sigma/d\epsilon'=\sigma_0/2. \tag{15}$$

The total cross section for an electron initially at rest is[17]

$$\sigma=\sigma_0[\omega^2/(1+2\omega)]. \tag{16}$$

The following invariant expressions are needed when we discuss, in Sec. V, neutrino scattering by a gas of electrons:

$$d\sigma/d\omega'=\sigma_0(p\cdot\omega)/2K, \tag{17a}$$

where

$$\omega'_{\max\,(\min)}=(p\cdot\omega)[k_0\mp K]^{-1} \tag{17b}$$

and, in general,

$$\sigma=\sigma_0(p\cdot\omega)^2/(1+2p\cdot\omega). \tag{18}$$

In Eqs. (17a) and (17b), $k_0(K)$ is the magnitude of the time (space) component of the four-vector $k_\alpha=(p_\alpha+\omega_\alpha)$.

D. Mean Scattering Angle and Energy Loss

The mean scattering angle for neutrinos interacting with electrons at rest can easily be calculated using Eq. (12). We find

$$\langle 1-\mu_\nu\rangle\equiv\int\frac{d\sigma}{d\mu_\nu}(1-\mu_\nu)d\mu_\nu\Big/\int\frac{d\sigma}{d\mu_\nu}d\mu_\nu \tag{19a}$$

$$=(2\omega^2)^{-1}[(1+2\omega)\ln(1+2\omega)-2\omega]. \tag{19b}$$

The average fractional energy loss is given by

$$\frac{\langle\omega-\omega'\rangle}{\omega}=\int\frac{d\sigma}{d\omega'}(\omega-\omega')d\omega'\Big/\omega\int\frac{d\sigma}{d\omega'}d\omega' \tag{20a}$$

$$=\omega/(1+2\omega). \tag{20b}$$

III. ANTINEUTRINO-ELECTRON SCATTERING

A. Nonresonant Scattering

The notation and kinematics in this section are the same as in Sec. II. We find for the invariant total cross section for antineutrino-electron scattering:

$$\sigma=\frac{\sigma_0}{4\pi p\cdot\omega}\int\int\frac{d^3p'd^3\omega'}{\epsilon'\omega'}\times(p\cdot\omega')(p'\cdot\omega)\delta^{(4)}(p_\alpha+\omega_\alpha-p_\alpha'-\omega_\alpha'), \tag{21}$$

and, for electrons initially at rest:

$$\frac{d\sigma}{d\mu_{\bar\nu}}=\frac{\sigma_0}{2}\frac{\omega^2}{[1+\omega(1-\mu_{\bar\nu})]^4}, \tag{22}$$

$$\frac{d\sigma}{d\omega'}=\frac{\sigma_0}{2}\left(\frac{\omega'}{\omega}\right)^2, \tag{23}$$

$$\frac{d\sigma}{d\mu_e}=\begin{cases}2\sigma_0\left(\dfrac{1+\omega}{\omega}\right)^2\mu_e\left[\left(\dfrac{1+\omega}{\omega}\right)^2-\mu_e^2-2\mu_e^2\omega^{-1}\right]^2\Big/\left[\left(\dfrac{1+\omega}{\omega}\right)^2-\mu_e^2\right]^4, & \mu_e\geq 0 \quad (24a)\\ 0, & \mu_e\leq 0 \quad (24b)\end{cases}$$

$$\frac{d\sigma}{d\epsilon'}=\frac{\sigma_0}{2}\frac{(1+\omega-\epsilon)^2}{\epsilon^2}. \tag{25}$$

Note that:

$$\left(\frac{d\sigma}{d\mu_e}\right)_{\bar\nu-e^-}=\left(\frac{d\sigma}{d\mu_e}\right)_{\nu-e^-}\left[1-\frac{2\mu_e^2\omega^{-1}}{[(1+\omega)/\omega]^2-\mu_e^2}\right]^{+2}. \tag{26}$$

The total cross section for electrons initially at rest is

$$\sigma=(\sigma_0\omega/6)[1-(1+2\omega)^{-3}], \tag{27a}$$

and, in general,

$$\sigma=(\sigma_0p\cdot\omega/6)[1-(1+2p\cdot\omega)^{-3}]. \tag{27b}$$

B. Mean Scattering Angle and Energy Loss

The mean scattering angle and fractional energy lost for antineutrinos interacting with electrons initially at rest are defined, respectively, by Eqs. (19a) and (20a). We find:

$$\langle 1-\mu_{\bar\nu}\rangle=(3+2\omega)[3+6\omega+4\omega^2]^{-1}, \tag{28}$$

and

$$\langle(\omega-\omega')/\omega\rangle=\left\{1-\frac{3}{4}\left[\frac{1-(1+2\omega)^{-4}}{1-(1+2\omega)^{-3}}\right]\right\}. \tag{29}$$

[17] Equations (13), (15), and (16) agree with the expressions given originally by Feynman and Gell-Mann (Ref. 13) except for a multiplicative factor of two that occurs because neutrinos are completely polarized. The angular distributions for both neutrino-electron and antineutrino-electron scattering by electrons at rest have been given elsewhere, but are repeated here for completeness and because several mistakes exist in the formulas in the literature.

C. Resonant Scattering

Glashow[18] has pointed out that the usual[13,19] hypothesis of a charged intermediate boson to mediate the weak interactions leads to a resonance in antineutrino-electron scattering. Thus:

$$\bar{\nu}_\beta+e^- \to (W^-) \to \begin{cases} \bar{\nu}_\beta+e^- \\ \bar{\nu}_\mu+\mu^- \\ \pi^-+\pi^0\text{'s} \end{cases}. \quad (30)$$

The cross section for reactions (30) is[18]

$$\sigma_{\text{res}}=3\pi\lambda\!\!\!^{-2}\Gamma_{\text{in}}\sum_{\text{out}}\Gamma_{\text{out}}[(E-E_R)^2+(\Gamma_{\text{total}}/2)^2]^{-1}, \quad (31a)$$

where Γ_{in} and Γ_{out} are, respectively, the intrinsic laboratory full widths for a specific entrance or exit channel, and

$$E_R=(M_{W^-}/m_e)M_{W^-}c^2, \quad (31b)$$

$$\Gamma_{\text{tot}}=\sum_{\text{out}}\Gamma_{\text{out}}, \quad (31c)$$

and

$$\lambda\!\!\!^{-2}=(2m_ec^2/E_R)(\hbar/m_ec)^2. \quad (31d)$$

The intrinsic full widths for lepton emission are given approximately by

$$\Gamma(\bar{\nu}_\beta+e^-)\approx\Gamma(\bar{\nu}_\mu+\mu^-) \quad (31e)$$

$$\approx 1.6\times10^{-6}\left(\frac{M_{W^-}}{m_e}\right)\left(\frac{M_{W^-}}{M_N}\right)^2 M_{W^-}c^2, \quad (31f)$$

where M_N is the mass of the nucleon. The intrinsic width for pion emission depends somewhat[20] on the as-yet-unknown mass, M_{W^-}, of the intermediate boson. The pion width was taken to be approximately equal to the lepton width, Eq. (31f), by Bahcall and Frautschi.[3]

For electrons with an average velocity $\langle v/c\rangle$, the effective cross section averaged over the Doppler-broadened resonance is[3,18]

$$\sigma_{\text{eff}}\cong 6\pi\lambda\!\!\!^{-2}\Gamma_{\text{in}}(\langle v/c\rangle E_R)^{-1}. \quad (32)$$

The effective cross section σ_{eff} is, by Eqs. (31), independent of M_{W^-}.[3] For electrons in the earth's crust, σ_{eff} is approximately 10^{-30} cm^2, many orders of magnitude larger than the nonresonant scattering cross sections.[21] However, for the fashionable value $M_{W^-}\sim 1$ BeV, the resonant energy, E_R, is 10^{+12} eV. Thus resonant neutrino scattering is only expected to be important for the ultra-high-energy neutrinos that may come from strong radio sources[3] or the decay of cosmic-ray secondaries.[4]

[18] S. L. Glashow, Phys. Rev. **118**, 316 (1960).
[19] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).
[20] See, for example, M. A. B. Bég, J. M. Cornwall, and C-H. Woo, Phys. Rev. Letters **12**, 305 (1964).
[21] Professor J. J. Lord has kindly brought to our attention the interesting work of Y. Sekido, S. Yoshida, and Y. Kimaya, Phys. Rev. **113**, 1108 (1959), who state that they have detected point sources of cosmic rays. We have computed the probability for muon production in the atmosphere by neutrinos from strong radio sources using the fluxes given in Ref. 3. We find that the neutrino fluxes of Ref. 3 are far too small to account for the anisotropies claimed by Sekido *et al.*, even when the huge resonant cross sections discussed above are postulated.

D. $\bar{\nu}_\beta+e^- \to \bar{\nu}_\mu+\mu^-$ and $\nu_\mu+e^- \to \nu_\beta+\mu^-$.

The conserved vector current theory can also be used to calculate the cross section for the reaction $\bar{\nu}_\beta+e^- \to \bar{\nu}_\mu+\mu^-$. We find[22]

$$\sigma=\frac{\sigma_0}{24\pi}(1-\alpha)^2[k^2+2k\cdot p+\alpha(4k\cdot p-k^2)] \times\theta\left(k^2-\frac{m_\mu^2}{m_e^2}\right), \quad (33a)$$

where, as before, $k_\alpha=\omega_\alpha+p_\alpha$, σ_0 is given by Eqs. (4) and

$$\alpha=(m_\mu^2/m_e^2k^2). \quad (33b)$$

The function $\theta(x)$ is zero for $x\leq 0$ and unity for $x>0$. The threshold energy in the center of mass is $m_\mu c^2/2\cong 53$ MeV. For electrons initially at rest,

$$\sigma\cong\frac{\sigma_0\omega}{6}(1-q_{\min}/q)^2\left[1+\frac{q_{\min}}{2q}\right], \quad (34a)$$

and the minimum energy for muon production with electrons initially at rest is

$$q_{\min}=(m_\mu^2-m_e^2)/2m_e \quad (34b)$$

$$\cong 11\text{ BeV}. \quad (34c)$$

Similar formulas also apply, of course, to the reaction $\nu_\beta+e^+ \to \nu_\mu+\mu^+$.

The high threshold for muon production by neutrino-electron scattering implies that this reaction is of astrophysical significance only in extraordinary circumstances, e.g., in the presence of a dense Fermi gas of electrons with Fermi energy $\gtrsim m_\mu c^2/2(\sim 50$ MeV). The reaction $\bar{\nu}_\beta+e^- \to \bar{\nu}_\mu+\mu^-$ will also play only a minor role, if current estimates[4] are correct, in experiments designed to detect neutrinos from the decay of cosmic-ray secondaries.

The reaction $\nu_\mu+e^- \to \nu_\beta+\mu^-$ has a cross section given by $\sigma=\sigma_0(1-\alpha)^2k^2/4$ (ω_α now refers to ν_μ) and the same threshold as the reaction $\bar{\nu}_\beta+e^- \to \bar{\nu}_\mu+\mu^-$. Hence the above remarks regarding the probable unimportance of the reaction $\bar{\nu}_\beta+e^- \to \bar{\nu}_\mu+\mu^-$ for astrophysical applications pertain equally well to the reaction $\nu_\mu+e^- \to \nu_\beta+\mu^-$.

IV. EFFECT OF ATOMIC BINDING ON NEUTRINO-ELECTRON SCATTERING

The important experiment of Reines and Kropp[2] involving the scattering of solar neutrinos will be discussed in Sec. VI. In preparation for this discussion, we describe the effect of atomic binding upon neutrino-electron scattering. The simplest method for calculating the cross section for the inelastic process

$$\nu_\beta+e_{\text{bound}}^- \to \nu_\beta'+e_{\text{continuum}}^- \quad (35)$$

[22] A convenient formula for carrying out the covariant integrations needed here and in Secs. II and III is given in the Appendix of the paper by J. N. Bahcall and R. B. Curtis, Nuovo Cimento **21**, 442 (1961).

is to Fourier-analyze the wave function for the bound electron. Making this Fourier analysis, we find

$$\sigma=(\sigma_0/4)\int d^3p\,|g(\mathbf{p})|^2(k_b^2-1)^2/k_b^2\,, \tag{36}$$

where $g(\mathbf{p})$ is the Fourier transform of the bound electron's spatial wave function and the dimensionless four-vector k_b is $(\epsilon_b+\omega,\, \mathbf{p}+\boldsymbol{\omega})$. Here, ϵ_b is the total relativistic energy divided by m_ec^2 of the bound electron. Note that for a hydrogenic atom, $\epsilon_b \cong 1-\frac{1}{2}(\alpha Z/n)^2$.

For applications to the solar-neutrino experiment, we are primarily interested in the case $\omega \gg 1$. In this limit, Eq. (36) yields:

$$\sigma_{\text{bound}} \cong (\sigma_0\omega\epsilon_b)/2\,. \tag{37}$$

Thus the fractional correction to the free-electron total cross section due to atomic binding is of the order of $1-\epsilon_b$ and hence is rather small. Similarly, one can show that the fractional correction to the free-electron differential cross section, $(d\sigma/d\mu_e)$, is of the order of $\langle v^2/c^2\rangle_{\text{bound}}$. Note that for a hydrogenic atom, $\langle v^2/c^2\rangle \sim (\alpha Z/n)^2$.

Thus atomic binding does not have an important effect on neutrino-electron scattering in the Reines-Kropp experiment. This result is, in fact, rather obvious since the neutrinos being detected have energies much larger than the electron's binding energy.

V. SCATTERING BY A GAS

In this section, we investigate the effect of the motion of electrons in a gas upon the free-electron cross sections given in Secs. II and III.

A. Nondegenerate Gas

1. Cross Sections

For a nondegenerate gas of electrons, the total cross section averaged over the initial electron distribution is

$$\langle\sigma\rangle_{\omega,p}=[4\pi^3 n_e(\hbar/mc)^3]^{-1} \times\int d^3p[1+e^{-\nu+(W/kT)}]^{-1}\sigma(p_\alpha,\omega_\alpha)\,, \tag{38}$$

where $\sigma(p_\alpha,\omega_\alpha)$ is given by Eq. (18) for neutrino-electron scattering and by Eq. (27b) for antineutrino-electron scattering, and n_e is the number of electrons per unit volume. We treat separately the two temperature domains: (i) $kT\ll m_ec^2$ and (ii) $kT\gg m_ec^2$.

The equilibrium abundance of electron-positron pairs is negligible[23] for temperatures satisfying $kT\ll m_ec^2$ (i.e., $T\lesssim 10^{+9}$ °K). At such temperatures, $e^{-\nu}\gg 1$ for a nondegenerate gas. More explicitly:

$$e^{+\nu}\cong[\pi^2(\hbar/m_ec)^3 n_e\beta]/K_2(\beta)\,, \tag{39}$$

where $\beta=(m_ec^2/kT)$ and $K_2(\beta)$ is a Bessel function of the second kind.[24,25] Thus

$$\langle\sigma\rangle_{\omega,T}\cong\beta[4\pi K_2(\beta)]^{-1}\int d^3p\, e^{-W/kT}\sigma(p_\alpha,\omega_\alpha)\,, \tag{40}$$

for a nondegenerate gas. If $q\ll kT$, then

$$\langle\sigma\rangle_{\omega,T}\approx\sigma_0\omega^2\{1+[4K_3(\beta)/\beta K_2(\beta)]\}\,, \tag{41a}$$

for both neutrino-electron and antineutrino-electron scattering. Since $\beta\gg 1\,(kT\ll m_ec^2)$:

$$\langle\sigma\rangle_{\omega,T}\cong\sigma_0\omega^2[1+(4kT/m_ec^2)]\,. \tag{41b}$$

If $q\gg m_ec^2$, then

$$\langle\sigma\rangle_{\omega,T}\cong(\sigma_0\omega/2)\{[K_1(\beta)/K_2(\beta)]+(3/\beta)\}\,, \tag{42a}$$

for neutrino-electron scattering. Since $\beta\gg 1$

$$\langle\sigma\rangle_{\omega,T}\cong(\sigma_0\omega/2)[1+(3kT/2m_ec^2)]\,. \tag{42b}$$

For antineutrino-electron scattering, $(\sigma_0\omega/2)$ should be replaced by $(\sigma_0\omega/6)$ in Eqs. (42). Formulas (41) and (42) show, as expected, that thermal motion only changes the cross sections for electrons at rest by a small amount if $kT\ll m_ec^2$.

The situation is very different, however, in the important case[26] for which $kT\gg m_ec^2$. In this case, the thermal motion of the electrons produces a large center-of-mass energy and hence causes the cross section to greatly exceed the cross sections for electrons at rest. Also, the numerical preponderance of electron-positron pairs at high temperatures implies that the chemical potential for both electrons and positrons is zero.[23] Thus the appropriate high-temperature limit for the thermally averaged cross section is

$$\langle\sigma\rangle_{\omega,T}\cong[4\pi^3 n_\pm(\hbar/m_ec)^3]^{-1} \times\int d^3p[1+e^{+(W/kT)}]^{-1}\sigma(p_\alpha,\omega_\alpha)\,, \tag{43a}$$

where the equilibrium concentration of electrons or positrons is given by[23]

$$n_\pm=(1.80)\pi^{-2}[\beta(\hbar/m_ec)]^{-3}\,. \tag{43b}$$

If $\omega\ll 1$, one finds from Eqs. (18), (27b), and (43) that

$$\langle\sigma\rangle_{\omega,T}\cong 17(kT/m_ec^2)^2\sigma_0\omega^2 \tag{44}$$

for both neutrino-electron and antineutrino-electron

[23] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Permagon Press Ltd., London, 1958), pp. 325–326. For a discussion of the role of electron-positron pairs in stars, see D. H. Sampson, Astrophys. J. **135**, 261 (1962), and Ref. 5.

[24] G. N. Watson, *Treatise on the Theory of Bessell Functions* (The Macmillan Company, New York, 1944), 2nd ed., pp. 79, 181, and 202.

[25] S. Chandrasekhar, *Introduction to Stellar Structure* (Dover Publications, Inc., New York, 1957), pp. 395–397.

[26] S. A. Colgate and R. H. White, *Proceedings of the International Conference on Cosmic Rays, Jaipur, India, 1963* (to be published). See also S. A. Colgate and M. H. Johnson, Phys. Rev. Letters **5**, 235 (1960). A more complete exposition of this theory is being prepared for publication.

scattering. If $\omega \gg 1$, one finds that[27]

$$\langle\sigma\rangle_{\omega,T} \cong 3.2(kT/m_ec^2)(\sigma_0\omega/2), \tag{45}$$

for neutrino-electron scattering. For antineutrino-electron scattering, $(\sigma_0\omega/2)$ should be replaced in Eq. (45) by $(\sigma_0\omega/6)$.

Equations (45) and (46) show clearly that at high temperatures $\langle\sigma\rangle_{\omega,T}$ can greatly exceed the free-electron cross sections given in Secs. II and III. The effect of these increased cross sections is further enhanced by the large number of electron-positron pairs that are in equilibrium with the radiation field at high temperatures.[28]

2. *Neutrino Energy Loss*

In order to discuss (Sec. VID) neutrino energy-deposition in supernova explosions,[26] we need an expression for the fractional neutrino energy loss that is valid when both kT and q are large compared to m_ec^2. The appropriate expression can be obtained by inserting the invariant expression for $d\sigma/d\omega'$, Eq. (17a), in the defining relation:

$$\frac{\langle\omega-\omega'\rangle}{\omega} \equiv \frac{\int\int d^3p\,d^3\omega' e^{-\beta W}(d\sigma/d\omega')(\omega-\omega')}{\omega\int\int d^3p\,d^3\omega' e^{-\beta W}(d\sigma/d\omega')}. \tag{46}$$

We find

$$\langle\omega-\omega'\rangle/\omega \cong \tfrac{1}{2}[1-4kT/q], \tag{47}$$

when $q \gg m_ec^2$ and $kT \gg m_ec^2$. Note that at sufficiently high electron temperatures, neutrinos on the average gain more energy by collisions with electrons than they lose.

B. Degenerate Gas

For a degenerate gas, it is necessary to take account of (i) the distribution of initial electron momenta and (ii) the prior occupation of some of the final electron states. The general expression for neutrino-electron scattering is

$$\langle\sigma\rangle_\omega = (\sigma_0/4\pi)[4\pi^3 n_e(\hbar/m_ec)^3]^{-2} \times\int\int d^3p\frac{d^3p'\,d^3\omega'}{\epsilon'\omega'}S(\epsilon)[1-S(\epsilon')](p'\cdot\omega') \times\delta^{(4)}(p_\alpha+\omega_\alpha-p_\alpha'-\omega_\alpha'). \tag{48}$$

For a completely degenerate gas:

$$S(\epsilon)=\begin{cases}1, & \text{for } \epsilon<\epsilon_F \\ 0, & \text{for } \epsilon>\epsilon_F\end{cases}, \tag{49}$$

where ϵ_F is the total relativistic Fermi energy. One can, however, estimate $\langle\sigma\rangle_{\omega,\epsilon_F}$ as follows. Let

$$\langle\sigma\rangle_{\omega,\epsilon_F} = \langle\sigma\rangle_{\text{initial}} I(\omega,\epsilon_F), \tag{50}$$

where $\langle\sigma\rangle_{\text{initial}}$ is the scattering cross section averaged over the initial distribution of electron momenta and $I(\omega,\epsilon_F)$ is an inhibiting factor due to the exclusion principle. Then

$$\langle\sigma\rangle_{\text{initial}} \equiv \frac{\sigma_0\int d^3p[(p\cdot\omega)^2/(1+2p\cdot\omega)]}{\int d^3p}, \tag{51a}$$

$$\cong (9/8)\sigma_0\epsilon_F\omega, \tag{51b}$$

where we have assumed $\epsilon_F \gg 1$ and $\epsilon_F\omega \gg 1$. The quantity $I(\omega,\epsilon_F)$ can be estimated by methods that are described in Ref. 15. One finds:

$$I(\omega,\epsilon_F) \cong \begin{cases}\omega/\epsilon_F, & \omega\ll\epsilon_F \quad (52a)\\ 1, & \omega\gg\epsilon_F \quad (52b)\end{cases}.$$

Thus:

$$\langle\sigma\rangle_{\omega,\epsilon_F} \cong \begin{cases}\sigma_0\omega^2, & \omega\ll\epsilon_F \quad (53a)\\ \sigma_0\epsilon_F\omega, & \omega\gg\epsilon_F \quad (53b)\end{cases}.$$

Equations (53) should be multiplied by one-third for antineutrino-electron scattering.

VI. ASTROPHYSICAL APPLICATIONS

A. Solar Neutrinos

Reines and Kropp[2] proposed an experiment in which neutrinos emitted from the interior of the sun are to be detected on earth by observing electrons scattered elastically via reaction (1). The relatively high-energy neutrinos that make possible such an experiment are expected to come from the decay of B^8, which is formed in a rare mode of the hydrogen-burning fusion reactions in the sun.[1,29,30] Reines and Kropp proposed looking for the high-energy electron recoils ($W' \gtrsim 8$ MeV), pointing out that such a measurement is in principle capable of giving information regarding the neutrino energy spectrum.

We wish to point out an additional feature of neutrino-electron scattering that is readily apparent from the differential cross section given in Eq. (14), namely, that

[27] For very high temperatures, the average thermal energy of an electron is $3.2kT$. Thus, Eq. (45) has the same form as Eq. (37) for neutrino scattering by bound electrons, namely, $\sigma=(\sigma_0\omega/2)\epsilon_{\text{av}}$. The origin of this expression is simple. For large ω, $\sigma=\sigma_0(p\cdot\omega/2)$ and the spatial part of $p\cdot\omega$ averages to zero for isotropic distributions.

[28] The following reactions may also be significant at high temperatures: (i) $\nu+e^-+e^+\rightarrow\nu'+\gamma$ and (ii) $\bar{\nu}+e^-+e^+\rightarrow\bar{\nu}'+\gamma$.

[29] W. A. Fowler, Astrophys. J. **127**, 551 (1958). The role in stellar energy generation of the various hydrogen-burning fusion reactions has recently been reviewed by P. Parker, J. N. Bahcall, and W. A. Fowler, Astrophys. J. **139**, 602 (1964).

[30] R. L. Sears, Astrophys. J. **140**, 477 (1964); J. N. Bahcall, W. A. Fowler, I. Iben, Jr., and R. L. Sears, Astrophys. J. **137**, 344 (1963).

the recoil electrons are strongly peaked in the direction of the incident neutrinos. In fact, one can show by an elementary calculation using Eq. (8) that the maximum angle, $\theta_{\max}$, that a recoil electron makes with respect to the incident neutrino direction is given approximately by:

$$\theta_{\max}\cong\left[\frac{2(\omega_{\max}-\epsilon_{\min}')}{\omega_{\max}\epsilon_{\min}'}\right]^{+\frac{1}{2}}, \tag{54}$$

where $\omega_{\max}$ is the maximum incident neutrino energy and $\epsilon_{\min}'$ is the minimum energy that a recoil electron must have before it is counted. For the conditions suggested by Reines and Kropp, $\theta_{\max}\approx 10°$. Equation (14) shows, in fact, that most of the recoil electrons that are counted will actually lie inside $\theta_{\max}$.

Thus the use of reaction (1) to detect neutrinos can in principle enable one to locate the direction (presumably toward the sun) of an extraterrestrial neutrino signal.

For the experiment under consideration,[2] the angular distributions and total cross sections are affected only slightly by atomic binding of the initial electrons (see Sec. IV).

B. Neutrino Escape from Stars

It has usually been assumed that neutrinos escape without interaction from the interiors of stars; we investigate the validity of this assumption. The ratio of the neutrino mean free path to the stellar radius R is given by

$$(\lambda/R)\equiv(\kappa_\omega\rho R)_{\text{average}}^{-1}, \tag{55}$$

where κ_ω is the opacity of the stellar matter to a neutrino of energy ω and ρ is the density of the stellar matter. Recall that, according to the definition given in Sec. I,

$$\kappa_\omega\equiv\left(\textstyle\sum_i \sigma_i n_i\right)/\rho, \tag{56}$$

where σ_i is the neutrino interaction cross section for particles of number density n_i, and ρ is the stellar density. A crude but convenient numerical approximation to formula (56) is given by

$$(\lambda/R)\cong 10^{+9}\mu_e(1+2\omega)\omega^{-2}(R/R_\odot)^2(M_\odot/M), \tag{57}$$

where μ_e is the mean molecular weight per electron, M is the total stellar mass traversed, $R_\odot\cong 7\times 10^{+10}$ cm, and $M_\odot\cong 2\times 10^{+33}$ g. For antineutrinos, $(1+2\omega)$ should be replaced by $\omega[1-(1+2\omega)^{-3}]^{-1}$ in formula (57).

In making the transition from Eq. (55) to Eq. (57), we have neglected the effect of neutrino absorption by nucleons, the neutrino red shift, and the statistical effects discussed in Sec. V. Neutrino absorption by nucleons has been considered by Euwema[11] and is rediscussed in Ref. 15; the effect of the gravitational red shift can be estimated with the help of the relation[31] $(\Delta\nu/\nu)=-GM/RC^2$ and is not important for the order of magnitude estimates made here. Statistical effects, which were treated in Sec. V, should, of course, be included in any detailed investigation.

One can easily show with the help of Eq. (57) that the probability is only about 10^{-8} or 10^{-9} that a neutrino emitted from the center of the sun will be scattered before escaping from the surface of the sun. For massive stars in the range $10^{+6}M_\odot$ to $10^{+8}M_\odot$, which have been considered by Fowler and Hoyle,[32] (λ/R) does not equal unity until the massive stars have contracted well inside the Schwarzschild radius. Even for white dwarfs, for which[25,33] $(R/R_\odot)\gtrsim 10^{-3}$ and $(M/M_\odot\sim 1$, (λ/R) is much greater than unity and hence neutrino scattering is not important for white dwarfs.

However, for neutron stars,[34,35] $(R/R_\odot)\sim 10^{-4}$ to 10^{-5} and $(M/M_\odot)\sim 1$. Thus, (λ/R) can be much less than unity for neutron stars. Hence the neutrino opacity of neutron stars (including statistical effects and the mean angle of scattering) should be included in future models of these stars.

C. Neutrino Scattering in Cosmology

Neutrinos play a major role in a number of cosmological speculations.[7–10] It is therefore interesting to note that neutrinos (and antineutrinos) with energies less than a BeV have only negligible interactions with matter that is distributed according to current estimates of the large scale composition of the universe. For example, the mean free path of neutrinos with energies of the order of a few MeV to scattering (or absorption[15]) by matter at cosmological densities ($\sim 10^{-29}$ g) is about 10^{+20} times the "radius of the universe," i.e., the mean free path is about 10^{+48} cm.[36]

D. Neutrino Energy Deposition in Supernova Explosions

Colgate and White[26] suggested that neutrinos emitted from the core of a dense collapsing star can deposit sufficient energy by absorption in the mantle of such a star to produce a supernova explosion and blow off the mantle. The conditions under which neutrino energy deposition might explode the mantle depend critically

[31] In this relation, G is, of course, the gravitational coupling constant, *not* the weak interaction coupling constant.

[32] W. A. Fowler and F. Hoyle, Astrophys. J. **140**, 830 (1964); F. Hoyle and W. A. Fowler, Monthly Notices Roy. Astron. Soc. **125**, 169 (1963).

[33] M. Schwarzschild, *Structure and Evolution of the Stars* (Princeton University Press, Princeton, New Jersey, 1958).

[34] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).

[35] A. G. W. Cameron, Astrophys. J. **130**, 884 (1959); V. A. Ambartsumyan and G. S. Saakyan, Astron. Zh. **38**, 785 (1961) [English transl.: Soviet Astron.—AJ **5**, 601 (1962)]; H. Y. Chiu and E. E. Salpeter, Phys. Rev. Letters **12**, 413 (1964).

[36] This estimate also includes the possibility of resonant (see Sec. IIIC) antineutrino scattering by high-energy leptons in the primary cosmic radiation. A probable upper limit on the amount of resonant scattering that occurs can easily be estimated by assuming that the cosmic rays fill all space with the local spectrum and energy density (~1 eV/cc) and that the high-energy leptons constitute, as they do at lower energies, about 1% of the cosmic-ray energy density. The formulas given in Sec. IIIC then yield a mean free path of approximately 10^{+48} cm.

upon the neutrino mean free path as a function of the parameters describing the state of the stellar matter. For the densities ($\sim 10^{+10}-11^{+11}$ g/cc), temperatures (10–100 MeV), and neutrino energies ($\sim 10-100$ MeV) considered by Colgate and White, we find with the help of Eqs. (45) and (47) that energy deposition via neutrino-electron scattering is larger than energy deposition via neutrino absorption by nucleons. This result, which is contrary to what has been previously thought to be correct,[37] implies that neutrino scattering should be taken into account in future calculations of supernova explosions at high temperatures.[38]

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[37] Some previous workers have concluded that neutrino scattering is negligible compared to neutrino absorption for the conditions under consideration here. The basis for this erroneous conclusion was that the cross section for neutrino absorption contains an extra factor of ω, for large ω, compared to the cross section for neutrino scattering by electrons *at rest*. The results of Sec. IV show, however, that the cross sections for neutrino scattering by electrons in a gas can be much larger, due to an increased center of mass energy, than the scattering cross sections for electrons at rest. Moreover, for neutrino absorption by nucleons in a nucleus, a large fraction of the incident neutrino energy can be spent in overcoming the nucleon binding (Ref. 15).

[38] This is currently being done by S. A. Colgate (private communication).

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Backward Scattering in π-p Collisions at 3.15 and 4 BeV/c*

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On the basis of an analysis of the experimental data for forward scattering, both the upper limit and the lower limit of $d\sigma/dt$ for backward scattering are predicted. The experimental values of $d\sigma/dt$ in the backward direction lie within the region defined by two curves corresponding to the upper and lower limits of $d\sigma/dt$.

THE purpose of this paper is to predict the allowed region of $d\sigma/dt$ for backward scattering in π-p collisions at 3.15 and 4 BeV/c. The result can be derived from an analysis of the experimental data for $d\sigma/dt$ in the forward direction and from the unitarity condition of the S matrix.

In a previous paper[1] we adopted an empirical formula for the scattering amplitude $f(\theta)$ of π-p scattering,

$$f(\theta)=(ik/\sqrt{\pi})\{\exp\tfrac{1}{2}(A_0+A_1t)+C-\exp\tfrac{1}{2}[B_0+B_1(u-u_0)]\}(\text{mb})^{1/2}, \quad (1)$$

and pointed out that if there exists a pronounced backward peak, there must be a scattering angle in the region 90–180° at which the imaginary part of scattering amplitude turns out to be zero [hereafter such a scattering angle is referred to as a zero point of $f(\theta)$ or simply a zero point], where $t=-2k^2(1-\cos\theta)$, $u=(m^2-\mu^2)^2/s-2k^2(1+\cos\theta)$, s is the square of the total energy in the center-of-mass system and u_0 is the value of u at 180°. Recent experimental data[2] for π^+-p scattering at 4 BeV/c have shown a pronounced backward peak and seem to support our prediction[1] for the zero point of $f(\theta)$. Using formula (1), we study in this paper backward scattering at 3.15 and 4 BeV/c from a phenomenological point of view.

Needless to say, the values of A_0 and A_1 can be estimated from the experimental data[2,3] for $d\sigma/dt$ in a very small $|t|$ region. In an intermediate $|t|$ region, for instance $|t|=1.5$–2.0 (BeV/c)^2, the first term $\exp\frac{1}{2}(A_0+A_1t)$ in Eq. (1) turns out to be so small that it may be neglected compared with the second term C. This makes it possible to estimate the value of C. The values of A_0, A_1, and C thus obtained are as follows[4]:

$$A_0=3.7,\ A_1=7.79\ (\text{BeV}/c)^{-2},\ \text{and}\ C=0.3 \quad \text{for } \pi^-\text{-}p \text{ scattering at 3.15 BeV}/c, \quad (2)$$

$$A_0=3.7,\ A_1=7.34\ (\text{BeV}/c)^{-2}\ \text{and}\ C=0.2 \quad \text{for } \pi^+\text{-}p \text{ scattering at 4 BeV}/c. \quad (2')$$

Note that the values of A_1 for π^--p and π^+-p scattering

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[1] S. Minami, Phys. Rev. 133, B1581 (1964).

[2] M. Aderholz *et al.*, Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Letters 10, 248 (1964).

[3] M. L. Perl, L. W. Jones, and C. C. Ting, Phys. Rev. 132, 1252 (1963).

[4] If $f(\theta)$ is expressed approximately by $f(\theta)\approx(ik/\sqrt{\pi})\times[\exp\frac{1}{2}(A_0+A_1t)+C]$, it follows from the optical theorem $\text{Im}f(0^0)=(k/4\pi)\sigma_{\text{tot}}$ that $\sigma_{\text{tot}}(\pi^--p)$ at 3.15 BeV/$c\cong 29.4$ mb and $\sigma_{\text{tot}}(\pi^+-p)$ at 4 BeV/$c\approx 28.9$ mb.