1^- Assignment for the *B* Meson*

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The $\pi\pi$, $\pi\omega$ scattering amplitudes in the $I=J=1$ state are parameterized in terms of five parameters which are determined so as to give the ρ meson position and width, the ω width, and the B meson position and its $\pi\omega$ decay width under the assumption that B has quantum numbers 1^-. These amplitudes are then used to discuss $\pi\pi,\pi\omega$ production processes in terms of a simple peripheral model including π,ω exchanges. The model predicts a large $\pi^+\pi^-$ decay rate of B, with a cross section comparable to the experimental $\pi^+\pi^-$ resonant cross section (the f^0) observed at approximately the same position as the B meson. On the other hand, since the B is an $I=1$ particle, this implies that the branching ratio of the $2\pi^0$ decay to that of the $\pi^+\pi^-$ decay should be small. This is incompatible with the experimentally observed large branching ratio. Within the limitations of our model, it is therefore unlikely that the B meson is a 1^- particle.

I. INTRODUCTION

THE B meson,[1] being a $\pi\omega$ resonance and decaying strongly, has isotopic spin 1. Also $B^{\pm}$ is found not to decay to any considerable extent into the $\pi^{\pm}\pi^0$ state, which seems to exclude 1^-, 3^-, etc., assignment to its quantum numbers. However, Frazer, Patil, and Xuong[2] put forth a hypothesis that f^0 and B are two decay modes of the same 1^- particle, the ρ'. In particular, they considered the production processes

$$\pi^-+p\rightarrow\pi^-+\pi^++n\,, \tag{1}$$

$$\pi^{\pm}+p\rightarrow\pi^{\pm}+\pi^0+p\,, \tag{2}$$

$$\pi^{\pm}+p\rightarrow\pi^{\pm}+\omega+p\,, \tag{3}$$

with reference to a simple peripheral model based on π and ω exchanges for these processes. Process (1) involves only the π^+ exchange while in process (2) one can have both π^0 and ω exchanges. It was then shown that it is possible to explain the nonobservance of $\pi^{\pm}\pi^0$ decay of ρ' in terms of ω exchange, which makes this decay difficult to observe. They also proposed some experiments to determine the isospin of f^0. One of these was to compare the missing mass distribution M_x of the reaction

$$\pi^++d\rightarrow(p)+p+x\ \text{(neutrals)}$$

with the $\pi^+\pi^-$ effective-mass distribution of the reaction

$$\pi^++d\rightarrow(p)+p+\pi^++\pi^-.$$

If f^0 has isospin 0, the peak in the neutral decay due to the $2\pi^0$ mode would be one half the corresponding $\pi^+\pi^-$ mode, while with the ρ' hypothesis one would expect a ratio $\leq 1/10$ (due to $B^0\rightarrow\omega+\pi^0$ and then $\omega\rightarrow\pi^0+\gamma$ electromagnetically). The experimental value has now been found to be 0.60 ± 0.17.[3] Also, Sodickson *et al.*[4] have observed a resonant $2\pi^0$ decay mode of f^0 by detecting the four product gammas in a spark chamber. This indicates that f^0 is primarily an $I=0$ resonance, with spin 0, 2, 4, etc., and that the B meson does not have a large 2π decay mode. However, this does not exclude the possibility that B is still a 1^- particle and that the observed f^0 bump in reaction (1) is a superposition of a small 2π decay mode of B and the $I=0$ resonance.

The purpose of the present paper is to discuss the model of Frazer *et al.*[2] in greater detail. We will first describe a parameterization of the $\pi\pi$, $\pi\omega$ scattering amplitudes and then apply this to the production processes (1), (2), and (3). We will consider some of the experimental consequences of this model and see whether the 2π decay mode is small enough to be compatible with the experimental observation of f^0 being primarily an $I=0$ particle.

II. $\pi\pi$, $\pi\omega$ SCATTERING AMPLITUDES

Consider the following reactions in which ω is taken to be a stable particle:

$$\begin{aligned}\pi+\pi&\rightarrow\pi+\pi\,,\\ \pi+\pi&\rightarrow\pi+\omega\,,\\ \pi+\omega&\rightarrow\pi+\omega\,.\end{aligned} \tag{4}$$

Let the partial wave amplitudes for these processes in the $J=L=I=1$ state be designated by T_{11}, T_{12}, and T_{22} which satisfy the unitarity condition

$$\operatorname{Im}T_{ij}=T_{ik}{}^*T_{kj}\,,\quad \text{for } t\geq t_k \tag{5}$$

where

$$t_1=4m_\pi{}^2\,,\quad t_2=(m_\pi+m_\omega)^2\,.$$

In order to be able to use dispersion relations for calculating the amplitudes, we must first examine the analyticity properties of T_{ij} and factor out the kinematic singularities. This can be done in several ways. One way is to write down the simplest possible Lorentz-invariant amplitudes for reactions (4) in the $J=I=1$ state and relate these to T_{ij} by comparing the corre-

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[1] M. Abolins, R. L. Lander, W. A. W. Mehlhop, N.-H. Xuong, and P. M. Yager, Phys. Rev. Letters **11**, 381 (1963).

[2] W. R. Frazer, S. H. Patil, and N.-H. Xuong, Phys. Rev. Letters **12**, 178 (1964).

[3] N. Gelfand, G. Lütjens, M. Nussbaum, J. Steinberger *et al.*, Phys. Rev. Letters **12**, 567 (1964).

[4] L. Sodickson, M. Wahlig, I. Mannelli, D. Frisch *et al.*, Phys. Rev. Letters **12**, 485 (1964).

sponding cross sections. The Lorentz-invariant amplitudes referring to Fig. 1 are

$$\begin{aligned} B_{11}(t) &= (1/4)(q_1-q_2)^i(q_3-q_4)_i A_{11}(t)\,, \\ B_{12}(t) &= \epsilon_{ijkl} q_1{}^i q_2{}^j q_3{}^k a^l A_{12}(t)\,, \\ B_{22}(t) &= \epsilon_{ijkl} Q^i q_1{}^j a_1{}^k \epsilon^{mnrl} Q_m q_{3n} a_{2r} A_{22}(t)\,, \end{aligned} \tag{6}$$

where $Q=q_1+q_2$ and a, a_1, a_2 are the polarization vectors of the omega meson. It is assumed that the $A_{ij}(t)$ are free from all kinematic singularities. Cross sections for processes (4) are given in terms of $B_{ij}(t)$ by[5]

$$\sigma_{ij} = \frac{(2\pi)^2 q_{10} q_{20}}{[(q_{1\mu}q_2{}^\mu)^2 - m_1{}^2 m_2{}^2]^{1/2}} S_f \bar{S}_i \delta^4(p_f - p_i)\,|B_{ij}|^2, \tag{7}$$

where 1 and 2 refer to the incoming particles, p_f, p_i to total final and initial momenta. S_f denotes integration over momenta and sum over the polarization of the final particles, and $\bar{S}_i$ the average over the polarization of the initial particles. From (7) we get

$$\begin{aligned} \sigma_{11} &= q_i q_f{}^3 |A_{11}|^2/48\pi t\,, \\ \sigma_{12} &= q_i q_f{}^3 |A_{12}|^2/24\pi\,, \\ \sigma_{22} &= q_i q_f{}^3 t |A_{22}|^2/36\pi\,, \end{aligned} \tag{8}$$

where q_i, q_f are the spatial part of the initial and final momenta in the c.m. system and $t=(q_1+q_2)^2$. These expressions are compared with the cross sections in terms of T_{ij} which are given by Jacob and Wick[6] for the helicity amplitudes and the comparison yields

$$\begin{aligned} A_{11} &= 24\pi t^{1/2} T_{11}/q_i{}^3\,, \\ A_{12} &= 12\pi(2^{1/2}) T_{12}/q_i{}^{3/2} q_f{}^{3/2}\,, \\ A_{22} &= 12\pi T_{22}/q_f{}^3 t^{1/2}\,. \end{aligned} \tag{9}$$

It is conjectured that the amplitudes for which we should write dispersion relations are proportional to A_{ij}. We therefore define

$$M_{ij} = T_{ij}/\rho_i{}^{1/2}\rho_j{}^{1/2}\,, \tag{10}$$

where

$$\rho_1 = 2q_1{}^3/t^{1/2}\,, \quad \rho^2 = 2q_2{}^3 t^{1/2}\,,$$

q_1 and q_2 being the spatial part of the c.m. momenta in the two-pion and pi-omega states, respectively, as the proper amplitudes for writing dispersion relations. Another way to see that M_{ij} are the proper amplitudes for writing dispersion relations is to write down the Born terms for T_{ij} corresponding to an $I=J=1$ particle

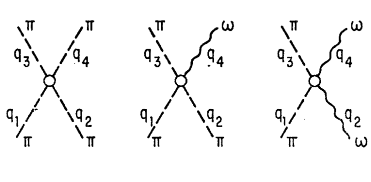

FIG. 1. Diagrams for reactions (4).

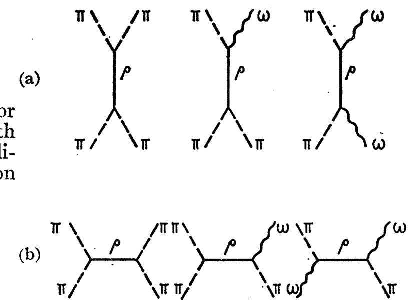

FIG. 2. Graphs for $\pi\pi$, $\pi\omega$ channels with (a) ρ meson intermediate state, (b) ρ meson exchange.

in the intermediate state such as ρ, and then separate out the kinematic singularities. The expressions for diagrams in Fig. 2(a) corresponding to a ρ meson intermediate state are of the form

$$\begin{aligned} T_{11} &= q_1{}^3 a_{11}/t^{1/2}(t-m_\rho{}^2)\,, \\ T_{12} &= q_1{}^{3/2} q_2{}^{3/2} a_{12}/(t-m_\rho{}^2)\,, \\ T_{22} &= q_2{}^3 t^{1/2} a_{22}/(t-m_\rho{}^2)\,, \end{aligned} \tag{11}$$

where a_{ij} are constants. In this consideration we have assumed ρ to be a stable particle. We can also calculate the (1^-) partial-wave projection of the diagrams in Fig. 2(b) where a ρ is exchanged. These are found to be

$$\begin{aligned} T_{11}{}^L(t) &= q_1{}^3/t^{1/2} A_{11}{}^L(t)\,, \\ T_{12}{}^L(t) &= q_1{}^{3/2} q_2{}^{3/2} A_{12}{}^L(t)\,, \\ T_{22}{}^L(t) &= q_2{}^3 t^{1/2} A_{22}{}^L(t)\,, \end{aligned} \tag{12}$$

where $A_{ij}{}^L(t)$ are well behaved at the origin and are free from kinematic singularities. Expressions (11) and (12) suggest that we define M_{ij} as in (10) for writing dispersion relations.

There is yet another guide to the choice of proper amplitudes for writing dispersion relations. One can calculate the width of ω for the decay process

$$\omega \longrightarrow \pi^0 + \gamma$$

described by $e\epsilon_{ijkl} a_\omega{}^i q_\omega{}^j a_\gamma{}^k q_\gamma{}^l f_\omega(q_\gamma{}^2)$. The unitarity condition allows us to relate the amplitude $f_\omega(q_\gamma{}^2)$ and the pion form factor to the amplitudes M_{ij}. Now, if a "wrong" choice is made for M_{ij} kinematics, one gets an absurd answer for the width. Thus if $\rho_2 = q_2{}^3/t^{1/2}$, it is found that

$$f_\omega(q_\gamma{}^2) \underset{q_\gamma{}^2 \to 0}{\longrightarrow} \infty\,.$$

We do find that $f_\omega(q_\gamma{}^2)$ is well-behaved at $q_\gamma{}^2=0$ if M_{ij} are chosen as in (10).

We will obtain the matrix elements M_{ij} using the well-known N/D method[7] in which we set $M=ND^{-1}$ where all quantities are matrices, N containing the left-hand singularities and D the right-hand singularities. Therefore, on the left we have

$$\text{Im} N_{ij} = (\text{Im} M_{ik}) D_{kj} \tag{13}$$

[5] J. Jauch and F. Rohrich, *Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1959), p. 167.

[6] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).

[7] J. D. Bjorken, Phys. Rev. Letters 4, 473 (1960).

and on the right

$$\mathrm{Im}D_{ij}=-\rho_i N_{ij}\theta(t-t_i). \qquad (14)$$

Dispersion relations are now written down for N and D with a single subtraction in D at $t=0$ and normalization $D_{ij}(0)=\delta_{ij}$.

$$D_{ij}(t)=\delta_{ij}-\frac{t}{\pi}\int_{t_i}^{\infty}\frac{\rho_i(t')N_{ij}(t')}{t'(t'-t)}dt', \qquad (15)$$

$$N_{ij}(t)=\frac{1}{\pi}\int_{\mathrm{left}}\frac{(\mathrm{Im}M_{ik}(t'))D_{kj}(t')}{t'-t}dt', \qquad (16)$$

where $t_1=4m_\pi^2$ and $t_2=(m_\pi+m_\omega)^2$.

We now parameterize the matrix elements M_{ij} by using the one-pole approximation for $N_{ij}(t)$ in (16). We set

$$N_{ij}(t)=t_0 n_{ij}/(t+t_0), \qquad (17)$$

where t_0, n_{ij} are constants and $n_{ij}=n_{ji}$ for $M_{ij}=M_{ji}$ to hold. For a better approximation one may take different pole positions for different $N_{ij}(t)$ elements at the expense of larger number of parameters. With the above approximation (17) we get

$$D_{ij}(t)=\delta_{ij}-t_0 n_{ij}K_i(t), \qquad (18)$$

where

$$K_i(t)=\frac{t}{\pi}\int_{t_i}^{\infty}\frac{\rho_i(t')}{t'(t'-t)(t'+t_0)}dt'.$$

However, it is observed that the integral in $K_2(t)$ is divergent. But in applying the unitarity condition, we have included only the first two terms of $\mathrm{Im}M_{ij}(t)$, i.e., those corresponding to two-pion and pi-omega states. For large t, the higher mass terms will become important and the imaginary part of $M_{ij}(t)$ can no longer be approximated by the first two terms. Explicitly (5) is expected to be a good approximation only for small values of t. For $t\gtrsim 4t_2$, contributions from higher mass states such as $\rho\rho$, $k\bar{k}$ will become important and should be taken into account. So in order to be consistent with the spirit of our approximation and avoid extraneous contributions, one should cut off the $\mathrm{Im}T_{ij}(t)$ for large values of t. This can be done in several ways. We have used a sharp cutoff for the K_2 integral. We also considered the case in which ρ_2 is modified so as to be more convergent, which while yielding similar results turned out to be interesting in itself and is discussed in the Appendix. With the sharp cutoff, we have

$$K_2(t)=\frac{t}{\pi}\int_{t_2}^{\Lambda}\frac{\rho_2(t')}{t'(t'-t)(t'+t_0)}dt'. \qquad (19)$$

Now we have $T_{ij}(t)$ as a function of five parameters; n_{11}, n_{12}, n_{22}, t_0, and Λ. Two of these are determined by requiring that the determinant of D has a zero to represent a ρ meson with the correct position and width. The position is fixed by requiring that

$$\mathrm{Re}[\det D(m_\rho^2)]=0 \qquad (20)$$

and the width by

$$\mathrm{Im}[\det D(m_\rho^2)]=m_\rho\Gamma_\rho\frac{d}{dt}\bigg|_{t=m_\rho^2}\mathrm{Re}[\det D(m_\rho^2)], \qquad (21)$$

where $\Gamma_\rho=$ width of ρ meson ≈ 100 MeV. Another parameter is determined so as to give the measured width of the ω meson. The ω width enters the problem via the $\pi\pi\rightarrow\pi\omega$ diagram in Fig. 1 described by $M_{12}(t)$, which is also the amplitude describing ω decay into 3π. The procedure for relating ω width to M_{12} is similar to that discussed in Ref. 8 except for slight kinematic modification. In the notation of this paper,

$$\Gamma_\omega=\frac{3}{\pi}\int_{4m_\pi^2}^{(m_\omega-m_\pi)^2}dt\left(\frac{q_1q_2}{m_\omega}\right)^3 t\int d(\cos\theta) \times\sin^2\theta|f(t,\cos\theta)|^2, \qquad (22)$$

where

$$f(t,\cos\theta)=\bar{M}_{12}(t)+\bar{M}_{12}(s)+\bar{M}_{12}(u),$$

with

$$s=2m_\pi^2-2q_{10}q_{20}+2q_1q_2\cos\theta,$$
$$u=2m_\pi^2-2q_{10}q_{20}-2q_1q_2\cos\theta,$$

and $\bar{M}_{12}$ denotes that the interaction pole at $-t_0$ has been subtracted out. The remaining two parameters are determined so as to fit the position and width of the B particle in the production process (3), using a π, ω exchange model which is the subject of discussion in the following section.

In an earlier paper, Frazer, Patil, and Watson[8] used a similar parameterization. However, they took $\rho_2=2q_2^3/t^{1/2}$ instead of $\rho_2=2q_2^3t^{1/2}$ and this did not require a cutoff in the K_2 integral. Of the four parameters, two were determined by the ρ-meson pole while the remaining two were determined by fitting the $\pi\pi\rightarrow\pi\omega$ cross section to the data obtained from the reaction (3) via Chew-Low extrapolation,[9] assuming a one-pion exchange model for the $\pi\omega$ production. Our present model treats the kinematics properly by taking $\rho_2=2q_2^3t^{1/2}$ at the expense of an extra parameter in the form of a cutoff, and also the production processes are analyzed taking into account both π and ω exchanges and off-mass-shell considerations as discussed in the next two sections. At this point we should also mention that one can parameterize our two-channel problem using the multichannel "effective-range" analysis of Ross and Shaw.[10] Their analysis also requires five parameters which can be discussed in connection with our parameterization. For the purpose of this discussion,

[8] W. R. Frazer, S. H. Patil, and H. L. Watson, Phys. Rev. Letters **11**, 231 (1963).
[9] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).
[10] M. Ross and G. Shaw, Ann. Phys. (N. Y.) **13**, 147 (1961).

it is convenient to use the form of N/D analysis in which N is diagonal[8] for which we have

$$N_{ij}(t)=\delta_{ij}/(t+t_0)\,,$$
$$D_{ij}(t)=C_{ij}-\delta_{ij}(t-m_\rho^2)\bar{K}_i(t)\,, \tag{23}$$

where

$$\bar{K}_i(t)=\frac{1}{\pi}\int_{t_i}^{\infty}\frac{\rho_i(t')dt'}{(t'-t)(t'-m_\rho^2)(t'+t_0)}.$$

A cutoff at Λ is introduced for $\bar{K}_2(t)$, so that we have five parameters: t_0, Λ, and $c_{ij}=c_{ji}$. Now if the principal part of the integral is approximated by a constant and the t dependence of the N_{ij} is neglected, which is permissible if t_0 is large, we can write $T=\bar{N}\bar{D}^{-1}$ where

$$\bar{N}_{ij}=\delta_{ij} \tag{24}$$

and

$$\bar{D}_{ij}(t)=\bar{c}_{ij}-\delta_{ij}R_i(t-m_\rho^2)-i\rho_i\delta_{ij}\theta(t-t_i)\,.$$

This form of T is identical to the one obtained by Ross and Shaw.[10] It seems, however, that our parameterization is perhaps more restrictive in that t_0 and Λ have to be positive and what is more, we are unable to obtain arbitrary omega widths from our parameters while satisfying the remaining four requirements discussed before. Also, we are using the N/D formalism with the intention of discussing other processes involving $\pi\pi$, $\pi\omega$ interaction such as $\omega\rightarrow\pi^0+\gamma$, nucleon form factors, etc., which can be simply related to D_{ij} elements of our analysis. It may be possible to use the Ross-Shaw solutions to define similar elements, but this seems to be more complicated.

III. π, ω EXCHANGE MODEL

Production experiments using high-energy $\pi^\pm$ beams with proton targets have been performed and information about reactions (1), (2), and (3) is available[1,11–15]

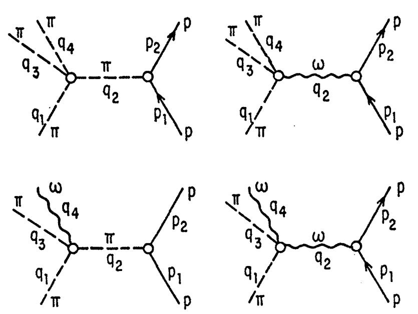

FIG. 3. Graphs for production processes (1), (2), (3) with π, ω exchanges.

[11] W. Selove, V. Hagopian, H. Brody, A. Baker, and E. Leboy, Phys. Rev. Letters **9**, 272 (1962).
[12] V. Hagopian and W. Selove, Phys. Rev. Letters **10**, 533 (1963).
[13] Z. Guiragossian, Phys. Rev. Letters **11**, 85 (1963).
[14] L. Bondar *et al.*, Phys. Letters **5**, 153 (1963).
[15] N.-H. Xuong, R. L. Lander, W. A. W. Mehlhop, and P. M. Yager, Phys. Rev. Letters **11**, 228 (1963); L. Bondar *et al.*, Phys. Letters **5**, 209 (1963).

for several values of the incident momentum. We will attempt to describe these processes in terms of a simple model based on π and ω exchanges.

Assume that the interaction in the reactions (2) and (3) is via exchange of a π^0 and an ω corresponding to the diagrams in Fig. 3. The $\pi\pi$ and $\pi\omega$ vertices are described by the elements A_{ij} defined in (6) and (9). Assuming point interaction, the $N\bar{N}\pi$ and $N\bar{N}\omega$ vertices are described by

$$g_\pi(\bar{u}_{p_2}|\gamma_5|u_{p_1})$$
$$g_\omega(\bar{u}_{p_2}|\gamma_\mu|u_{p_1})\,. \tag{25}$$

The differential cross sections for the processes (2) and (3) are

$$\frac{d^2\sigma_{\pi\pi}}{d\Delta^2dt}=\frac{1}{8\pi^2m^2q_{1L}^2}\left[\frac{g_\pi^2}{4\pi}\frac{\Delta^2}{(\Delta^2-m_\pi^2)^2}\frac{q_i^2q_f^3}{48}|A_{11}(t)|^2+\frac{g_\omega^2}{4\pi}\frac{(2\mathbf{p}_2^2\sin^2\theta'-\Delta^2)}{(\Delta^2-m_\omega^2)^2}\frac{q_i^2q_f^3}{24}|A_{12}(t)|^2\right], \tag{26}$$

$$\frac{d^2\sigma_{\pi\omega}}{d\Delta^2dt}=\frac{1}{8\pi^2m^2q_{1L}^2}\left[\frac{g_\pi^2}{4\pi}\frac{\Delta^2}{(\Delta^2-m_\pi^2)^2}\frac{q_i^2q_f^3}{24}|A_{12}(t)|^2+\frac{g_\omega^2}{4\pi}\frac{(2\mathbf{p}_2^2\sin^2\theta'-\Delta^2)}{(\Delta^2-m_\omega^2)^2}\frac{q_i^2q_f^3}{12}|A_{22}(t)|^2\right], \tag{27}$$

where m=nucleon mass, $\Delta^2=(p_1-p_2)^2$, q_{1L} is the magnitude of the incoming π momentum in the lab system, and q_i, q_f are the magnitudes of the incoming and outgoing pion momenta, θ' is the angle between $\mathbf{q}_1$ and $\mathbf{q}_2$, all of these being evaluated in the c.m. system of outgoing $\pi\pi$ or $\pi\omega$. All the kinematical quantities are evaluated with the exchanged particles off the mass shell.[16]

For g_ω, we use the value quoted by Scotti and Wong[17] from their NM scattering calculation, which is expected to be reliable within a factor of 2. There is some ambiguity in the use of this coupling constant. In our calculation we are including only the ω exchange and neglecting the φ exchange which, however, is known to be weakly coupled to pions. In the Scotti-Wong calculation of NN scattering, both the ω exchange and φ exchange are included and what is more, they find that neglecting the φ exchange increases the $N\bar{N}\omega$ coupling by about 50%. However, this ambiguity is taken care of to some extent by the introduction of a parameter in the "ω form factor" which is discussed in the next section. For $g_\omega^2/4\pi$ we use a value of 2.7 which is the value Scotti and Wong get when both ω and φ exchanges are included. At this point we must also mention that in our calculation, the magnetic coupling of the ω to $N\bar{N}$ is neglected. One may be tempted to justify this by pointing out the small isoscalar anomalous magnetic

[16] F. Selleri, Phys. Letters **3**, 76 (1962).
[17] A. Scotti and D. Wong, Phys. Rev. Letters **10**, 142 (1963).

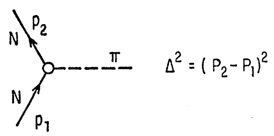

FIG. 4. The $NN\pi$ vertex.

moment of the nucleon. But it is possible that this is due to cancellation of the ω and φ contributions.

The formulas (24) and (25) as they stand, with point interaction at the $N\bar{N}$ vertex and the amplitudes $A_{ij}(t)$ evaluated for the exchanged particle on the mass shell, are not valid for off-shell calculations which are of physical interest in the production processes. In order to extend them to such calculations, we introduce form factors which will be the subject of the following discussion. It is to be noted that for the process (1), there is only the pion exchange (the exchange particle must be charged) and its differential cross section is twice the pion-exchange contribution to process (2).

IV. FORM FACTORS

First, consider the π exchange and a vertex at which the pion interacts with, say, $N\bar{N}$. This vertex will be described by an amplitude A which is a function of Δ^2. We write a dispersion relation in the variable Δ^2 for the diagram of Fig. 4. For the purpose of parameterizing the form factor, assume that the effective interaction is via a single particle state of mass m_1^2 in the intermediate state. Then we get

$$A(\Delta^2)\approx A(m_\pi^2)f_1(\Delta^2)\,, \tag{28}$$

where

$$f_1(\Delta^2)\approx(m_1^2-m_\pi^2)/(m_1^2-\Delta^2)\,.$$

For the vertex at which the pion interacts with $\pi\pi\omega$ or $\pi\pi\pi$, we make the assumption that the form factor is independent of the total mass of the outcoming particles (i.e., independent of $t^{1/2}$), and that it is the same as the one at the $N\bar{N}\pi$ vertex. Then the pion exchange is accompanied by an over-all form factor $\bar{f}_1(\Delta^2)$:

$$\bar{f}_1(\Delta^2)=|f_1(\Delta^2)|^2$$
$$\approx(\bar{m}_1^2-m_\pi^2)/(\bar{m}_1^2-\Delta^2)\,. \tag{29}$$

We will now attempt to estimate the value of the parameter $\bar{m}_1^2$. We look for a correlated state that has a small mass and the same quantum numbers as a single pion, to be the most important intermediate state. Such a state is $\pi\rho$. We further assume that this state interacts as a single-particle state of mass $(m_\pi+m_\rho)$. Hence for $|\Delta^2|<m_1^2$,

$$m_1\approx(m_\pi+m_\rho) \tag{30}$$

and

$$\bar{m}_1^2\approx 20m_\pi^2\,.$$

This value is of course only an order-of-magnitude estimate, but it is encouraging that it is close to the one in the phenomenological form factor obtained by Ferrari and Selleri[16] who get the result that for $|\Delta^2|<10m_\pi^2$,

$$\bar{f}_1(\Delta^2)\approx 12m_\pi^2/(13m_\pi^2-\Delta^2)\,. \tag{31}$$

For $|\Delta^2|>10m_\pi^2$, we expect the pole to shift to higher values since larger mass states become more important. For our calculations which in some cases extend as far as $|\Delta^2|=50m_\pi^2$, we should, of course, allow some variation of $\bar{m}_1^2$ about 13 m_π^2. Fortunately, this value of $\bar{m}_1^2$ is quite acceptable in our calculations. We find good fits for $11m_\pi^2<\bar{m}_1^2<15m_\pi^2$. The results given in this paper are for $\bar{m}_1^2=13m_\pi^2$.

A similar approach can be extended to the parameterization of the form factor for an omega exchange. But the estimation of the parameter in this case is more subtle. Firstly, g_ω, the $N\bar{N}\omega$ coupling constant given by the phenomenological calculation of Scotti and Wong,[17] is expected to be good for Δ^2 negative; i.e., it has already taken some of the form factor effect at the $N\bar{N}\omega$ vertex into account. So we need to consider the form factor coming from A_{ij} elements only. However, for the omega exchange, there are several correlated states which may be important; e.g. $k\bar{k}$, φ, $\rho\omega$ and possibly $\pi\rho'$. So we simply write

$$\bar{f}_2(\Delta^2)=(\bar{m}_2^2-m_\omega^2)/(\bar{m}_2^2-\Delta^2)\,, \tag{32}$$

where $\bar{m}_2^2>m_\omega^2$ and hope that this is not an unreasonable way to parameterize $\bar{f}_2(\Delta^2)$. We then find that the $\cos\theta$ distribution observed at the ρ meson in reaction (2) places a restriction $\bar{m}_2^2<70m_\pi^2$. We obtain a value of $\bar{m}_2^2\approx 43m_\pi^2$ so as to get a reasonable $\cos\theta$ distribution. This is discussed more fully in the following sections.

V. THE VARIATION OF PARAMETERS

Assuming that the form factors for the π,ω exchanges are known, we have altogether five parameters: Λ, s_0, n_{11}, n_{12}, and n_{22}. On account of the experimental uncertainties, the determination of the parameters is not unambiguous so that a certain amount of variation in the parameters is allowed. For information on the $B^\pm$ meson, we use the data given by Abolins *et al.*[1,18] for the $\pi\omega$ production at incident momentum of 3.5 BeV/c with the momentum transfer restricted to $|\Delta^2|\leq 30m_\pi^2$. We first take a certain value for the cutoff Λ and fix s_0 and two other parameters so as to give the ρ-meson position and width, and omega width of about 9 MeV. We then vary the remaining parameter α defined to be $(n_{12}^2-n_{11}n_{22})$, and examine the production cross section for reaction (3). Figure 5(a) shows the variation of the cross section as a function of α. We then take a different value of Λ and go through the same steps. In Fig. 5(b) we have plotted the "best" fits to the cross section as a function of the cutoff Λ. The dependence of these "best" fits on Λ is not very sensitive and the experimental data is not good enough to give us a unique value of Λ.

[18] The experimental data are due to private communication from N. Xuong.

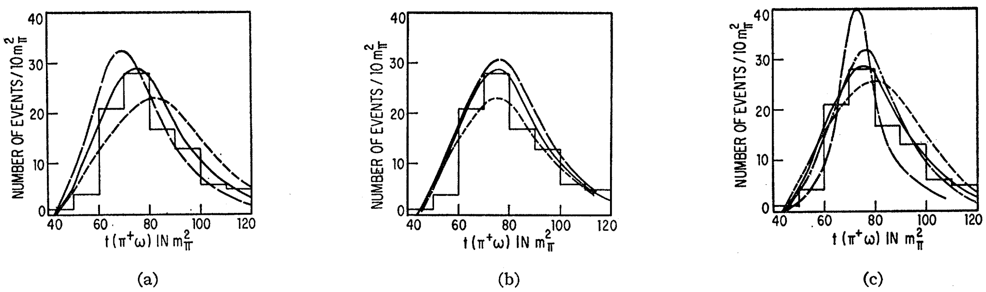

FIG. 5. (a) The $t(\pi\omega)$ plot for the reaction $\pi^{\mp}+p\to\pi^{\mp}+\omega+p$ for $\Lambda=175m_\pi^2$, $\Gamma_\omega=8$ MeV. --- for $\alpha=-6.95\times10^{-5}$; —— for $\alpha=-8.60\times10^{-5}$; — — for $\alpha=-9.94\times10^{-5}$. (b) The $t(\pi\omega)$ plot for the reaction $\pi^{\mp}+p\to\pi^{\mp}+\omega+p$: --- $\Lambda=200m_\pi^2$, $\Gamma_\omega=7.4$ MeV, $\alpha=-8.33\times10^{-5}$; —— $\Lambda=175m_\pi^2$, $\Gamma_\omega=8.0$ MeV, $\alpha=-8.60\times10^{-5}$; — — $\Lambda=160m_\pi^2$, $\Gamma_\omega=7.9$ MeV, $\alpha=-8.14\times10^{-5}$. (c) The $t(\pi\omega)$ plot for the reaction $\pi^{\mp}+p\to\pi^{\mp}+\omega+p$: - - - $\Lambda=320m_\pi^2$, $\Gamma_\omega=12.0$ MeV, $\alpha=-2.38\times10^{-5}$; —— $\Lambda=175m_\pi^2$, $\Gamma_\omega=8$ MeV, $\alpha=-8.60\times10^{-5}$; — — $\Lambda=140m_\pi^2$, $\Gamma_\omega=1.2$ MeV, $\alpha=-9.80\times10^{-5}$; ——— $\Lambda=160m_\pi^2$, $\Gamma_\omega=5.0$ MeV, $\alpha=-6.25\times10^{-5}$.

A value around $175m_\pi^2$ is found to be acceptable. We finally vary the omega width and examine how well we can fit the production cross section. We find acceptable fits only for widths between 2 and 12 MeV. Figure 5(c) shows our fits as a function of the omega width. The variation of the fits as a function of the omega width is similar to the one observed in Ref. 8.

VI. RESULTS AND DISCUSSION

Having used up all the parameters, we are now in a position to discuss the predictions of the model with reference to various experimental data. We will first fix the ρ-meson position at 750 MeV, ρ-meson width at 110 MeV and the ω width at 8.0 MeV and examine the predictions of the model in relation to experiments and then see if the predictions and the conclusions are changed on varying the ρ width and ω width within statistically allowed ranges.

Part A. The values of the parameters, giving ρ meson position at 750 MeV, width of 110 MeV and ω width of 8 MeV, are $t_0=5.5\times10^5m_\pi^2$, $\Lambda=1.75\times10^2m_\pi^2$, $n_{11}=3.06\times10^{-2}$, $n_{22}=4.30\times10^{-2}$, and $n_{12}=6.70\times10^{-3}$. For these values of the parameters we have the following predictions.

1. Process (2) at 1.6 BeV/c

The pion exchange is predominant with the omega contribution to the production cross section being about 8% at the ρ-meson position.[19] This is consistent with the analysis of Ferrari and Selleri[16] who considered the pion exchange only and found that the prediction of the cross section at the ρ position was about 10% smaller than the experimental value. It is to be noted that the contributions of π and ω exchanges to the cross section at the ρ position depend only on the ω width, ρ-meson position and width, and the π,ω form factors and are independent of the ρ' hypothesis.

[19] Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **25**, 365 (1962).

2. Processes (1) and (2) at 3.3 BeV/c

We use the data for the cosθ distribution of ρ^- production at this incident momentum for $|\Delta^2|<20m_\pi^2$ and $650<t^{1/2}<850$ MeV, given by Guiragossian,[13] to determine approximately what amount of ω contribution we need. The pion exchange gives a $\cos^2\theta$ distribution while the omega exchange gives a $\sin^2\theta$ distribution so that the resulting distribution for $\bar{m}_2^2=42.86m_\pi^2$ is approximately $1+3.5\cos^2\theta$. The distribution with the number of events properly normalized is shown in Fig. 6(a) along with the experimental distribution. It must be remembered that there is background which allows a certain amount of variation in $\bar{m}_2^2$. A variation of $\pm2m_\pi^2$ is found to be acceptable. The ω exchange contribution to the production cross section at $\bar{m}_2^2=42.86m_\pi^2$, near the ρ^- position, is about 30% of the total pi and omega contributions for $|\Delta^2|<20m_\pi^2$, but decreases rapidly with t on account of the factor $|\mathbf{p}_2|^2$ in the numerator of the second term in Eq. (24), which is large for small t but decreases rapidly with increasing t. The ratio of the number of events in the ρ^0 peak to that in the ρ^- peak, which depends only on the ρ position and width, ω width and the pi, omega form factors, and is independent of the ρ' hypothesis, is 1.5 compared to the experimental value of 1.3.

The Treiman-Yang distribution for the pion exchange is constant while that for omega exchange is $\sin^2\varphi$ so that for the contributions discussed above, we have approximately $1+0.75\sin^2\varphi$ distribution at the ρ^-. At this point, we should remark that the Treiman-Yang distribution provides another test for π,ω exchanges but unfortunately published data are not available at present for the $\pi\pi$ production at any of the incident momenta discussed in this paper.

The distributions at ρ^0 and ρ'^0 are of course due to the pion exchange only; i.e., $\cos^2\theta$ and constant Treiman-Yang distributions. At ρ'^- also, the omega-exchange contribution being small, the distributions are the same as those for ρ'^0 and this is found to be the case also at 3.0, 3.7 and 4.0 BeV/c incident momenta.

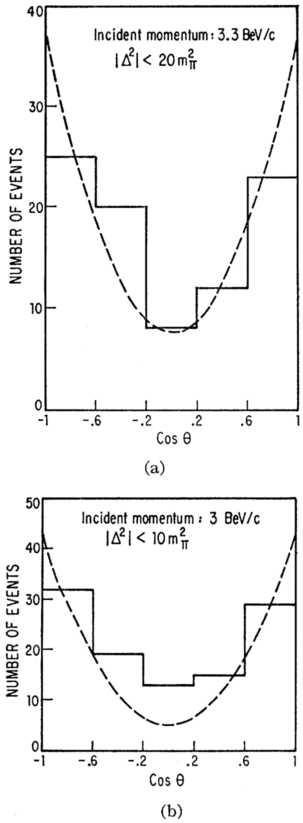

FIG. 6. Distributions in $\cos\theta$, the angle between the incoming and outgoing π^- in the barycentric system of the final pions for the reaction $\pi^-+p \to \pi^-+\pi^0+p$. (a) 700 MeV $< t^{1/2} <$ 850 MeV, $|\Delta^2|_{\min} < |\Delta^2| < 20 m_\pi^2$ at incident momentum 3.3 BeV/c. The experimental histogram is from Guiragossian (Ref. 13). (b) 650 MeV $< t^{1/2} <$ 850 MeV, $|\Delta^2|_{\min} < |\Delta^2| < 10 m_\pi^2 + |\Delta^2|_{\min}$ at incident momentum 3.0 BeV/c. The histogram is obtained from the $\cos\theta$ distribution given by Hagopian and Selove (Ref. 12).

3. Processes (1) and (2) at 3.0 BeV/c

Since the total cross section for processes (1) and (2) has been given by Selove *et al.*,[11] we can make direct comparisons of our theoretical predictions with the experimental data; we do not need to normalize our results and hence this should provide a more severe test to our theory. In Fig. 7, we have plotted the number of events predicted by our model along with the experimental values[11] as a function of t. We have added a properly normalized amount of phase space to the theoretical predictions as shown, to account for the background. It is to be noted that no information about the f^0 has been used as input in our model. Of course, the fact that the theoretical curve in Fig. 7(a) gives a good fit to the f^0 position and width is only a restatement of the fact that the mass and width of the f^0 and the B are the same within statistics. The fact that the height of the f^0 is given "correctly" by the model is, however, quite striking. The prediction in Fig. 7(b) of an $f^\pm$ bump, considerably reduced in apparent height compared to the f^0 bump, has been discussed in detail in Ref. 2.

We have also compared the $\cos\theta$ distribution at the ρ^-[12] for $|\Delta^2| < 10 m_\pi^2$ in Fig. 6(b), but here we have not added any phase space, since we do not know what kind of distribution the background may have. In Fig. 8 we have shown the Δ^2 distributions[11] for $20 m_\pi^2 < t < 35 m_\pi^2$.

It should once again be emphasized that in all comparisons at 3.0 BeV/c, the theoretical results are absolute predictions, not normalized to the data. The $\cos\theta$ distribution at ρ^- is approximately $1+3.0\cos^2\theta$ for $|\Delta^2| < 50 m_\pi^2$ while the Treiman-Yang distribution is $1+\sin^2\varphi$. The ratio of the number of events in the ρ^0 peak to that in the ρ^- peak is about 1.6 compared to an experimental ratio 1.5. As mentioned before, the differential cross sections and the various distributions at the ρ position are independent of the ρ' hypothesis or the quantum numbers of B and f^0, but depend upon the approximations of our model in which only π and ω exchanges are included.

4. Processes (1) and (2) at 3.7 and 4.0 BeV/c

The contribution to the cross section at the ρ^- meson from omega exchange is now nearly as large as that from pion exchange. The production cross section[14] at 3.7 BeV/c for process (1) is plotted in Fig. 9(b). The ratio of the number of ρ^0 events to that of ρ^- events is about 1.3 compared to the anomalously large experimental

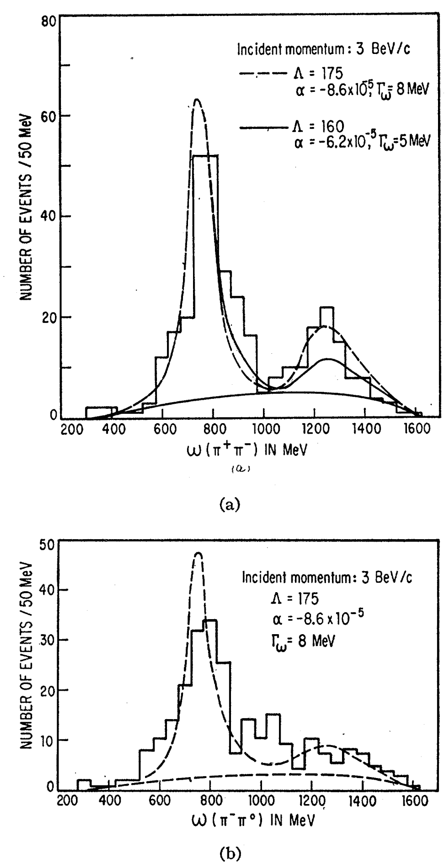

FIG. 7. (a) The $\pi-\pi$ mass plot for $\pi^-+p \to \pi^-+\pi^++n$; (b) $\pi-\pi$ mass plot for $\pi^-+p \to \pi^-+\pi^0+p$, both at incident momentum 3.0 BeV/c. The experimental data is from Selove *et al.* (Ref. 11). The theoretical curves are cut off at $|\Delta^2| = 50 m_\pi^2$ and are plotted after adding a properly normalized amount of phase space as shown.

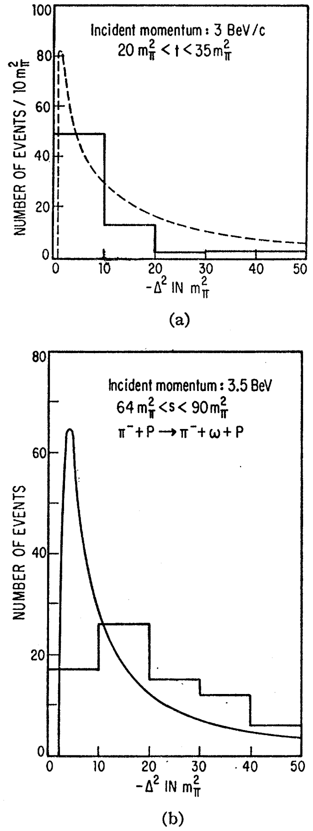

FIG. 8. The Δ^2 plots for incident momentum of 3.0 BeV/c and $20m_\pi^2 < t < 35m_\pi^2$ (a) for reaction $\pi^- + p \to \pi^- + \pi^0 + p$, and (b) for reaction $\pi^- + p \to \pi^- + \pi^+ + n$. The histograms are obtained from the Δ^2 distribution given by Selove *et al.* (Ref. 11).

number of about 2.5 obtained from the data of Lee *et al.* Figure 9(a) shows the plots of production cross sections at an incident momentum of 4.0 BeV/c.[20] The ratio of the number of ρ^0 events to that of ρ^- events is about 1.1 compared to the experimental value of 1.5. This seems to indicate that perhaps our simple peripheral model with only π and ω exchanges is not adequate to explain the data at higher incident momenta. At the incident momentum of 4.0 BeV/c, our model predicts a $\cos\theta$ distribution of $1+\cos^2\theta$ and Treiman-Yang distribution of $1+2.0\sin^2\varphi$, both at the ρ^- position.

5. Process (3) at 3.5 BeV/*c*

The contribution of ω exchange is about 20% of the total contribution which gives a $\cos\theta$ distribution of $1+3.5\sin^2\theta$, while the Treiman-Yang distribution is $1+0.2\cos^2\varphi$. We have plotted[18] the Δ^2 distribution in Fig. 10, and the $\cos\theta$ and Treiman-Yang distributions in Fig. 11 for $64m_\pi^2 \leq s \leq 90m_\pi^2$ taken as the B region. The comparison with the experimental data in this case is rather ambiguous. In order to avoid the $\pi^+ + p \to N^* + \omega$ events, the N^* events are subtracted from the mass distribution for $\pi^+ + p$, but in so doing, we are subtracting out some events of the reaction (3). It is also found that the subtraction of "N^* events" could make about 10% difference in the number of $\pi\omega$ events whether the range is taken to be 1120–1320 MeV or 1180–1380 MeV for the "N^* mass." The results shown are for 1120–1320-MeV range. In addition, the background for this process, i.e., process (3), is large and cannot be separated unambiguously from the $\pi\omega$ resonance.

Part B. In the above analysis we thus find that with the input information of the ρ-meson position and width, the ω width, and the position and width of the B meson taken to be a 1^- particle, we get out a large $\pi^+\pi^-$ decay mode of B which cannot be accommodated by f^0 since the f^0 peak has been shown to be associated with an $I=0$ resonance.[3] That the $I=0$ resonance has a differential cross section comparable to that of f^0 can be seen from the fact that ρ' with isospin 1 cannot decay into $2\pi^0$ while the experiment of Gelfand *et al.*[3] gives

$$R(f^0 \to \text{neutrals})/R(f^0 \to \pi^+ + \pi^-) = 0.60 \pm 0.17\,,$$

this ratio being $\frac{1}{2}$ if f^0 is an $I=0$ particle.

We will now vary the input information and see how the results discussed above, specifically the $\pi^+\pi^-$ production cross section of ρ', vary. Variation of ρ-meson width between 75–150 MeV produces only small changes

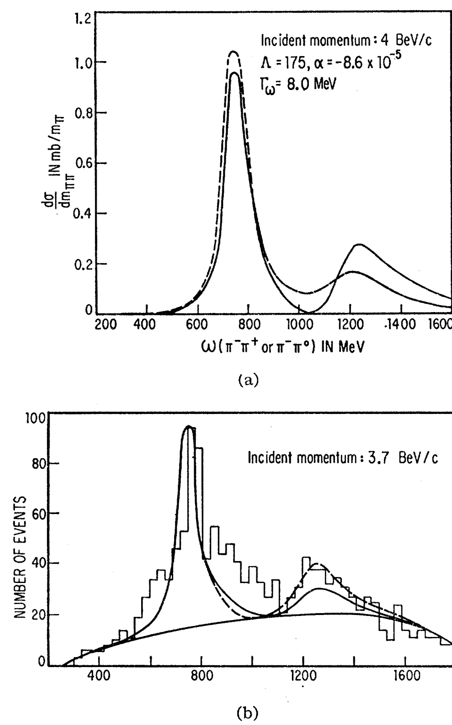

FIG. 9. (a). The $\pi-\pi$ mass plot at incident momentum 4.0 BeV/c. The dashed line is for the reaction $\pi^- + p \to \pi^- + \pi^0 + p$ and the solid line is for the reaction $\pi^- + p \to \pi^- + \pi^+ + n$. The plots are for $|\Delta^2| < 50m_\pi^2$. (b) The $\pi-\pi$ mass plot at incident momentum of 3.7 BeV/c for the reaction $\pi^- + p \to \pi^- + \pi^+ + n$ with $|\Delta^2| < 50m_\pi^2$. The dashed line is for $\Gamma_\omega = 8.0$ MeV, $\Lambda = 175m_\pi^2$, $\alpha = -8.6\times10^{-5}$ and the solid line for $\Gamma_\omega = 5.0$ MeV, $\Lambda = 160m_\pi^2$, $\alpha = -6.25\times10^{-5}$.

[20] Y. Y. Lee, B. P. Roe, D. Sinclair, and J. C. Vander Velde, Phys. Rev. Letters 12, 342 (1964).

in the $\pi^+\pi^-$ production rate of ρ'. This production rate is also quite insensitive to the width of the B, the height of the width of the B, the height of the cross section decreasing about 10% when the width of B is increased by 50%. However, we find that the production cross section at ρ' is reduced by about 40% if we decrease the omega width to 5.1 MeV in which case the omega width is already off from the experimental width of ω by about 2 standard deviations. The $\pi^+\pi^-$ production cross section at 3 BeV/c with the omega width of 5.1 MeV is plotted in Fig. 7(a) along with the experimental data of Selove *et al.*[11] A similar plot at 3.7 BeV/c along with the data of Lee *et al.*[20] is given in Fig. 9(b).

Even with this reduction in the $\rho' \to \pi^+ + \pi^-$ production rate assuming the remaining cross section of f^0 to be due to an $I=0$ resonance, we have

$$R(f^0 \to 2\pi^0)/R(f^0, \rho' \to \pi^+ + \pi^-) \approx 0.2$$

which is about 2.5 standard deviations off from the experimental value of 0.60 ± 0.17.

The above results seem to indicate that within the limitations of our parameterization of $\pi\pi$, $\pi\omega$ amplitudes and the peripheral model including only the π and ω exchanges, the apparent agreement of the predicted ρ' peak with the observed f^0 peak which led to the ρ' hypothesis[2] is only accidental and irrelevant, and that it is unlikely that the B is a 1^- particle.

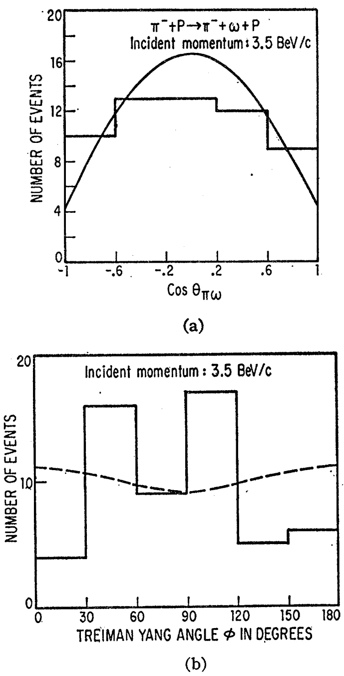

FIG. 11. (a) Distribution in $\cos\theta$, the angle between the incoming and outgoing π^- in the barycentric system of the outgoing $\pi\omega$. (b) Distribution in Treiman-Yang angle γ for the reaction $\pi^- + p \to \pi^- + \omega + p$ at an incident momentum of 3.5 BeV/c. The plots are for $|\Delta^2| < 30m_\pi^2$ and $64m_\pi^2 < s < 90m_\pi^2$.

VII. CONCLUSIONS

The apparent absence of a bump in $\pi^\pm\pi^0$ production reactions at the B-meson position seems to indicate that B is not a 1^- particle. But the question arises as to whether this is a sufficient reason to exclude the 1^- assignment. Our analysis based on a simple peripheral model including π,ω exchanges shows that it is possible that the apparent bump at the B position in $\pi^\pm\pi^0$ production reactions is considerably reduced. However, it also predicts a strong resonance in $\pi^+\pi^-$ decay of B^0, a resonance whose cross section is comparable to that of f^0. This is hard to accommodate in view of the fact that f^0 is shown to be primarily an $I=0$ resonance[3] of cross section approximately equal to that of f^0. Thus the calculations within the framework of our simple model support the conclusion that an observed large $2\pi^0$ decay of f^0 is an evidence against the 1^- model for the B meson.

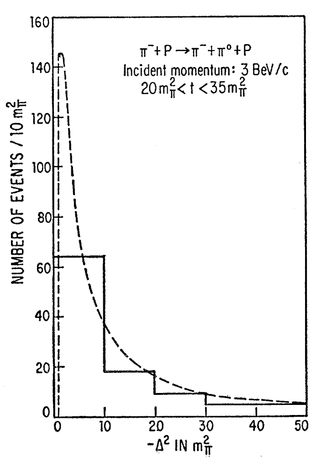

FIG. 10. The Δ^2 plot for the reaction $\pi^- + p \to \pi^- + \omega + p$ at incident momentum of 3.5 BeV/c for $64m_\pi^2 \leq s \leq 90m_\pi^2$.

There are some results of our analysis, such as the differential cross section at the ρ-meson position, momentum transfer, and various angular distributions at the ρ, which are independent of the quantum numbers assigned to the B meson, which it seems one can explain by the addition of the ω-exchange terms to the one-pion exchange model for incident momenta of 1.6, 3.0, and 3.3 BeV/c.

ACKNOWLEDGMENTS

This work would not have been possible without the guidance and encouragement of and discussions with Professor William R. Frazer. Many helpful suggestions by Professor David Y. Wong and the advance communications of experimental results by D. Carmony, R. Lander, N. Xuong, and P. Yager are gratefully acknowledged. Many thanks are due to Mrs. Maude Olsen for the careful preparation of the preprint.

APPENDIX

Instead of using a sharp cutoff for the second integral $K_2(t)$, one may modify the M_{ij} amplitudes so as to make $K_2(t)$ a convergent integral. For example, we define

$$M_{ij} = \alpha_i \alpha_j M_{ij}', \tag{33}$$

where $\alpha_1 = 1$, $\alpha_2 = \Lambda/(t+\Lambda)$, where Λ is a positive real constant, and write dispersion relations for M_{ij}' instead. With this convergence factor, the modified unitarity condition is

$$\mathrm{Im} M_{ij}' = M_{ik}' \rho_k' M_{kj}', \tag{34}$$

where $\rho_1'=\rho_1$, $\rho_2'=\rho_2\Lambda^2/(t+\Lambda)^2$. One can now go ahead and calculate M_{ij}' by using N/D method. An insight into the significance of the cutoff is obtained if one considers a one-channel calculation. In that case, before the cutoff is introduced,

$$T=N\bigg/\left(1-\frac{t}{\pi}\int\frac{N\rho}{t'(t'-t)}dt'\right) \qquad (35)$$

and after the cutoff is introduced, we have

$$T=\bar{N}\bigg/\left(1-\frac{t}{\pi}\int\frac{\bar{N}\rho}{t'(t'-t)}dt'\right), \qquad (36)$$

where $\bar{N}=N\Lambda^2/(t+\Lambda)^2$. Thus, introducing a cutoff is equivalent to modifying N to make it more convergent; also, $\bar{N}$ contains a "greater" amount of information; i.e., we are introducing additional interaction to make the integral convergent. In the two-channel calculation, $M_{12}=M_{21}=M_{12}'\Lambda/(t+\Lambda)$ and $M_{22}=M_{22}'\Lambda^2/(t+\Lambda)^2$. Again, we are introducing additional "interaction" in the form of a first-order pole at $t=-\Lambda$ for M_{12} and M_{21} and a second-order pole for M_{22}.

The calculations now proceed along the same lines as for the case of a sharp cutoff. The results from this are similar to the ones discussed in the main body of the paper.

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Self-Energy of the Electron*

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A perturbation theory is developed within the usual formalism of quantum electrodynamics which yields a finite unrenormalized electron Green's function and a finite value for the electron's electromagnetic self-mass in each order. This is subject only to the qualification in this paper, that the vacuum polarization is also obtained without divergences. Furthermore, the bare mass of the electron must vanish; the electron mass must be totally dynamical in origin.

I. INTRODUCTION

THE empirical success of the renormalized perturbation solution of quantum electrodynamics has produced the hope that relativistic field theory can provide an adequate description of the physics of elementary particles. On the other hand, the infinities which are present in the perturbation expression for the unrenormalized quantities have made one cautious about taking the theory too seriously.

In this work we will show that these infinities are not intrinsic to the theory but are due to the inadequacy of the usual perturbation method. We will attempt to develop an alternate perturbation approach to quantum electrodynamics which yields finite results for the basic unrenormalized Green's functions. In addition, in the weak-coupling limit, we will give explicit expressions for these functions in the region far off the mass shell where ordinary perturbation theory fails.

This method will work only for a spin-$\frac{1}{2}$ fermion field coupled with a conserved current to a neutral vector field. Hence the results of this work will not be applicable to a general relativistic field theory.

In quantum electrodynamics there are only three divergences (the minimum) in the ordinary perturbation treatment and they are all "weak" in the sense of being only logarithmically dependent on cutoffs. They are summarized by the constants δm, $Z_1(=Z_2)$, Z_3. The divergence of the self-mass δm is just the analog of the classical electromagnetic mass divergence. The divergence of the wave-function renormalization constant Z_2 represents an incompatibility of the perturbation treatment of the interaction with the canonical commutation rule for the electron field. The divergence of the charge renormalization Z_3 represents a similar

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