thicknesses, it has been pointed out that precise analyses are meaningful only when the elastic scattering can be cleanly separated from inelastic events,[7] or when correction for the inelastic contribution can be made.[20]

[20] A. Isoya, H. E. Conzett, E. Hadjimichael, and E. Shield, in *Proceedings of the Third Conference on Reactions between Complex Nuclei*, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963), paper A9.

Quasifree Electron-Proton Scattering in H^3 and He^3†

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Measurements have been made of the cross sections of coincidences between scattered electrons and ejected protons, when targets of H^3 and He^3 are bombarded with 550-MeV electrons. The variation of the cross section with proton angle and proton energy has been studied for fixed electron angle and energy. The results are compared with theoretical calculations based on different three-body nuclear wave functions.

I. INTRODUCTION

R ECENTLY there has been considerable interest, both theoretical and experimental, in the study of the three-body nuclei.[1] In the present paper, we would like to report results of a scattering experiment which, in principle at least, is quite sensitive to the wave function of a proton inside the nucleus. High-energy electrons are used as incident particles and the experiment consists of the detection in coincidence of a scattered electron and a knockout proton, which has been given a comparatively large momentum by the electron. The fact that only protons of large momentum are considered means that the process can be viewed as a free collision inside the nucleus between an electron and a proton. The principal effect of the nuclear wave function is felt through the momentum distribution of the proton before the collision. This momentum distribution will reveal itself in the angular correlation distribution between the scattered electron and the proton. The energy required to break up the initial nucleus depends on the state of the two spectator particles. Thus in order to study the momentum distribution, i.e., the wave function, of protons coupled to various states of the other two nucleons one has to perform the experiment with good angular resolution as well as good energy resolution.

This experiment is similar to the quasifree scattering of protons on protons in various nuclei, a process which has been studied at several laboratories during the last few years.[2] It has been suggested by Jacob and Maris[3] that the same kind of study might be done using electrons as the incident particles rather than protons. The advantages would be that the electrons are far less distorted by the nuclear field than are the strongly interacting protons. The main drawback is the very low cross section in the electron case and the poor duty-cycle of existing electron accelerators. The present experiment is the first attempt to use electrons for such studies[4] and, besides offering an interesting study, the nuclei H^3 and He^3 have comparatively high cross sections and low background due to uncorrelated events.

II. THEORY

A calculation of the coincidence cross section as a function of proton angle when the electron energy and angle are kept fixed has been done by Griffy and Oakes[5] using the impulse approximation. We will only make a few remarks about the kinematics of the reactions.

We assume that the electron interacts only with the

[1] See, for instance, a. J. M. Blatt and L. M. Delves, Phys. Rev. Letters **12**, 544 (1964). b. L. I. Schiff, Phys. Rev. **133**, B802 (1964). c. J. S. Levinger and T. L. Chow, Bull. Am. Phys. Soc. **9**, 465 (1964). d. H. Collard, R. Hofstadter, A. Johansson, R. Parks *et al.*, Phys. Rev. Letters **11**, 132 (1963).

[2] See, for instance, G. Tibell, O. Sundberg, and P. U. Renberg, Arkiv Fysik **25**, 433 (1963), where further references are given.
[3] G. Jacob and Th. A. J. Maris, Nucl. Phys. **31**, 139 (1962); **31**, 152 (1962).
[4] M. Croissiaux [Phys. Rev. **127**, 613 (1962)] and D. Aitken [Ph.D. thesis, Stanford University, 1964 (unpublished)] have measured electron-proton coincidences from the reaction $e+d \rightarrow e'+p+n$ with the principal objective of determining the form factor of the bound proton.
[5] T. A. Griffy and R. J. Oakes, Phys. Rev. **135**, B1161 (1964).

proton that is knocked out and that the other nucleons are unaffected by the scattering process; in particular, their over-all momentum is unchanged. Conservation of momentum then gives

$$k_e + k_p \cos\theta_p = k_e{}' \cos\theta_e{}' + k_p{}' \cos\theta_p{}', \qquad (1)$$

$$k_p \sin\theta_p = k_e{}' \sin\theta_e{}' - k_p{}' \sin\theta_p{}', \qquad (2)$$

where k_e and k_p are the electron and proton momenta, respectively, before the collision. Primed symbols refer to quantities after the collision. It has been assumed that we are studying the coplanar reaction, i.e., the emerging particles move in the same plane as the incident electron, and that the proton and the electron scatter to opposite sides of the incident beam direction. If we assume further that the energy required to remove the proton from the other particles in this particular state is E_s, we get from conservation of energy

$$k_e = k_e{}' + T_p{}' + E_s + T_{\text{recoil}}{}', \qquad (3)$$

where T refers to the kinetic energy, and

$$T_{\text{recoil}}{}' = k_p{}^2 / 2M_{\text{recoil}}. \qquad (4)$$

For a bound final state we have $E_s = -Q$, and for an unbound state $E_s \geq -Q$. Units with $\hbar$ and c equal to 1 have been used everywhere.

It is now easily seen that if we fix k_e, $k_e{}'$, $\theta_e{}'$, and $\theta_p{}'$, we get definite values of E_s and k_p for each $k_p{}'$. The cross section for this reaction should then be proportional to the probability of finding the spectator nucleons in the state before the collision that leads to the separation energy E_s times the probability of finding at the same time the proton with a momentum k_p. Finally, the cross section should also be approximately proportional to the free electron-proton scattering cross section with the appropriate momenta. These intuitive remarks are borne out by the calculations by Griffy and Oakes,[5] whose results are quoted below. We are interested in the following three reactions:
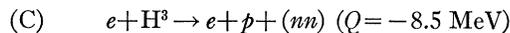
(A) $e + \text{He}^3 \rightarrow e + p + d$ $(Q = -5.5 \text{ MeV})$
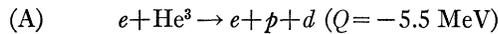
with a cross section

$$\frac{d^3\sigma}{d\Omega_e d\Omega_p dE_e{}'} = \tfrac{3}{2}\sigma_0 |I_1|^2; \qquad (5)$$

(B) $e + \text{He}^3 \rightarrow e + p + (pn)$ $(Q = -7.7 \text{ MeV})$
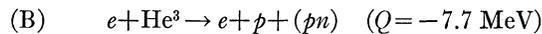
with a cross section

$$\frac{d^3\sigma}{d\Omega_e d\Omega_p dE_e{}'} = \tfrac{1}{2}\sigma_0 |I_0|^2; \qquad (6)$$

and

(C) $e + \text{H}^3 \rightarrow e + p + (nn)$ $(Q = -8.5 \text{ MeV})$

with a cross section

$$\frac{d^3\sigma}{d\Omega_e d\Omega_p dE_e{}'} = \sigma_0 |I_0|^2. \qquad (7)$$

The cross section σ_0 is determined almost exclusively from the electron scattering alone, while the nuclear matrix elements I_0 and I_1 are independent of the electron scattering process and express the product of the two probabilities mentioned above. It has been assumed that the ground state of a three-body nucleus is a completely symmetric $^2S_{1/2}$ state and that the unbound spectator nucleon pairs are in a singlet S state. Griffy and Oakes made numerical calculations only for reaction (A).

III. APPARATUS

The incident electrons were obtained from the Stanford Mark III linear accelerator. In order to achieve good energy resolution, the energy spread of the beam was limited to $\tfrac{1}{3}\%$ by slits, situated midway between the two bending magnets of the beam-transport system following the accelerator. The average beam intensity, which was continuously monitored by a Faraday cup and a secondary emission chamber, was then 0.3–0.5 μA under good operating conditions.

The targets were high-pressure gas cylinders, specially prepared by Los Alamos Scientific Laboratory and identical to those used in the elastic electron-scattering experiments from H³ and He³ at Stanford.[1d] The side and the spherical end walls were made of 20- and 10-mil stainless steel, respectively. All H³ results were obtained with a target filled to 3000 psi, but most of the He³ data were obtained with a 1500-psi target. An identical cylinder filled with ordinary hydrogen was found to be very useful for a number of checks and auxiliary measurements.

The emerging particles were analyzed in momentum by two 180° magnetic spectrometers, the electrons passing through the smaller magnet with a 36-in. radius of curvature, and the protons through the larger 72-in. magnet. This double spectrometer arrangement has been described by Hofstadter et al.[6] The electrons were detected by a $\tfrac{1}{2}$-in.-wide scintillation counter followed by a large Čerenkov counter, giving a momentum resolution of 0.37%. The protons were detected in an array of ten scintillation counters, each one inch wide corresponding to 0.36% momentum resolution.[7]

The fast output pulse from the coincidence unit of the electron detection system was "split" into 21 identical pulses, ten of which were used to form coincidences with the pulses from the proton detector array, ten other delayed so that the accidental coincidences could be counted at the same time and the last one used to count the number of electrons detected. Since the accidental coincidence rate could be as large as 70% of the total

[6] R. Hofstadter, F. Bumiller, B. R. Chambers, and M. Croissiaux, in *Proceedings of an International Conference on Instrumentation for High-Energy Physics* (Interscience Publishers, Inc., New York, 1961), pp. 310–315.

[7] D. Aitken, R. Hofstadter, E. B. Hughes, T. Janssens, and M. E. Yearian, in *Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), pp. 185–193.

rate, it was clearly important that the two systems counting total and accidental coincidences be closely similar or that their differences be known or made unimportant. This was achieved firstly, of course, by trying to make all circuits and pulses as nearly identical as possible and secondly by adjusting the scaler discriminators (i.e., the coincidence resolving times) so that the counting rate for a carbon target, for which more than 95% of the counts were accidental coincidences, was closely the same for all twenty scalers. Most of the remaining differences were made to cancel by frequently alternating the delays so that the two systems for total and accidental coincidences were interchanged.

In order to keep the accidental coincidence rate low, one would of course like to work with very short resolving times in the coincidence circuits. A lower limit is, however, set by the differences in time of flight for particles traveling different ways through the spectrometers, in particular the 72-in. spectrometer. For a vertical entrance slit opening of ±3° (which was mostly used in this experiment) the path difference for two 100-MeV protons traveling at the extreme outside and at the extreme inside corresponds to 4.5 nsec. Accordingly, the resolving times 2τ were chosen to be approximately 10 nsec. The efficiencies were constant over a sufficiently wide region, so that no true coincidences were lost due to time-of-flight differences.

IV. EXPERIMENTAL PROCEDURE

Initially, the energy of the incident electron beam and the angle of the 36-in. spectrometer were set at 550 MeV and 51.7°, respectively, and were kept at these values during the whole experiment.

The next step was to measure the energy spectrum of scattered electrons from the hydrogen-gas target. The position of the elastic peak provided a one-point recalibration of the 36-in. spectrometer energy and all subsequent energy settings were made relative to this peak. Since the elastic electron-proton cross section is quite well known, the area under the peak also provided a check on the effective solid angle and the electron detector efficiency. The solid angle could not be calculated from a knowledge of the entrance slits alone, since the targets were long and the solid angle seen from various points of the target was different and dependent on the size of the spectrometer vacuum tank. However, with these effects taken into account, the measured area was about 86% of that expected from first-order magnet theory. This is not unreasonable since the magnet was operated close to the saturation region.

With the electron detector at the peak of the electron spectrum, the field of the 72-in. spectrometer was adjusted so that the spectrum peak of recoil protons was centered in the 10-channel detector array. This provided the energy recalibration for this magnet and all subsequent energies were taken relative to this peak. The

maximum coincidence counting rate was found at 52.5°, which coincides with the calculated value for the recoil proton angle. The sum of coincidences for all proton energies agrees quite well with the number of detected electrons, if corrections are made for the fact that some of the electrons detected have recoil protons that are not able to pass through the 72-in. spectrometer. The final test of the 72-in. spectrometer was made with reversed field to detect scattered electrons from the same hydrogen-gas target. It was found that first-order magnet theory could well account for the observed counting rate.

After these tests, the energy of the 36-in. spectrometer was set at a certain value below the hydrogen peak, corresponding to the quasifree scattering of an electron from a bound proton at rest: 5.8 MeV lower in the case of He³ and 7.8 MeV for H³.[8] This energy setting of the 36-in. spectrometer is then maintained throughout measurements for that particular target. The proton angles[8] 51.7° and 51.5° correspond to knocking out a proton at rest from He³ and H³, respectively. For each target and proton angle, an energy spectrum was obtained which covered about 10 MeV. At regular intervals the checks with the hydrogen target were repeated.

V. EVALUATION OF CROSS SECTIONS

In order to compute absolute cross sections it is necessary to know the solid angles and the target thickness. Since for a long target the solid angles vary along the target axis (=the direction of the incident beam), the quantity required is an integral

$$\langle \Omega_p \Omega_e l \rangle = \int \Omega_p(z) \Omega_e(z) dz, \qquad (8)$$

where z is the coordinate along the target axis, and $\Omega_p(z)$ and $\Omega_e(z)$ are the solid angles for the proton and the electron, respectively. Unfortunately, this integral does not enter into the problem of evaluating cross sections from a hydrogen target, because of the exact correlation in angle between the scattered electron and the recoil proton in that case. As mentioned in Sec. IV, however, first-order magnet theory has been rather successful in accounting for the electron-proton coincidence rate as well as for the electron singles rate in both spectrometers when the hydrogen target was used. It should therefore be a good approximation to evaluate the above integral with $\Omega_p(z)$ and $\Omega_e(z)$ taken from that theory. This integral was computed numerically for the proton angles used. A correction for the 86% efficiency of the 36-in. spectrometer was also applied to the quasifree cross sections.

It was also necessary to correct the measured cross sections for radiative effects in the target and during

the scattering. This was achieved by a numerical calculation of the expected energy spectrum of knockout protons. The computation was done using an IBM-7090 computer and took into account the following contributions to the spectrum shape:

(1) The energy spectrum of the incident beam, assumed to have a square distribution with $\frac{1}{3}\%$ width.

(2) The finite energy resolution of the electron detector.

(3) The bremsstrahlung of the incident and scattered electrons in the target walls. This was the main effect and a simplified Bethe-Heitler formula was used,

$$P_1(E_0,E,t)dE = \frac{dE}{E_0}\frac{t}{\ln 2}\left(\frac{E_0-E}{E_0}\right)^{(t/\ln 2)-1},$$

for the probability that an incident electron of energy E_0 emerges with energy between E and $E-dE$ after the passage through a radiator of t radiation thicknesses.

(4) The radiative effects during the scattering. For this process an expression calculated by Atkinson[9] for the corresponding correction in $d(e,ep)n$ scattering was used in a somewhat simplified form:

$$P_2(E_p')dE_p' = \frac{dE_p'}{\bar{E}_e}\beta\left(\frac{E_2-E_p'}{\bar{E}_e}\right)^{\beta-1} e^{\delta}\sigma(E_e,E_e',E_2),$$

where $P_2(E_p')dE_p'$ is proportional to the probability that a proton emerges with energy between E_p' and $E_p'-dE_p'$,

$$\bar{E}_e = (E_e E_e')^{1/2},$$
$$\beta = (2\alpha/\pi)[-1+\ln(-q^2/m^2)],$$
$$\delta = (\alpha/\pi)[-(28/9)+(13/6)\ln(-q^2/m^2)],$$

and $\sigma(E_e,E_e',E_2)$ is proportional to the cross section when the incident energy E_e is divided so that the electron and the proton get E_e' and E_2, respectively. The quantity q^2 is the four-momentum transfer and m is the mass of the electron. The radiative correction was then

taken as the ratio between the areas of two proton spectra; one calculated with correct values for t, β, and δ, and the other with these quantities set equal to zero.

In using expression (8) for the product of solid angles and target thickness to find the quasifree cross sections, we have assumed that this cross section is approximately constant over the solid angles used. This, however, is not quite true and the final cross sections were corrected for the use of large solid angles by a numerical computation (with IBM 7090) of the fivefold integral of a theoretical cross section over the two solid angles and the coordinate along the target axis and comparing the result with the same integral of a constant cross section. The correction calculated in this way will, of course, depend on the choice of the theoretical cross section. The cross section actually used gave an excellent fit to the end results of this paper.

It is clear that because of these difficulties in the evaluation, the absolute values of the cross sections must be rather uncertain. It is conceivable that they are incorrect by as much as 20%. On the other hand, ratios should be quite well known since all uncertainties enter in approximately the same way for all angles and targets. The errors for the ratios between the cross sections are therefore mainly due to counting statistics.

The numerical values of all corrections are given in Table I.

VI. RESULTS

The experimental results are given in Table I and Figs. 1–4. Figure 1 shows the proton energy spectrum for He^3 at $\theta_p'=51.7°$. At this angle the cross section is close to a maximum because the scattering takes place from a proton almost at rest both for reaction (A) and reaction (B). The contribution from reaction (A) is clearly seen as a peak at 5.5-MeV separation energy, and there is also some indication of a second peak, due to reaction (B). The error bars shown are from counting statistics only, and it is clearly difficult to be specific about the relative contributions from the two reactions.

TABLE I. Final experimental cross sections and numerical values for parameters and corrections. Incident electron energy is 550 MeV. Electron scattering angle is 51.7°. Cross sections given for He^3 correspond to the sum for both reactions (A) and (B).

Target	Tritium			Helium-3			
θ_p'	44.2°	51.5°	62.0°	44.2°	51.7°	56.7°	62.0°
E_e' MeV	441	441	441	443	443	443	443
$10^5\langle\Omega_e\Omega_p t\rangle$(sr)^2 in.	1.32	1.28	1.22	1.32	1.28	1.25	1.22
Corrections:							
(a) Gaps in 10-channel array	1.08	1.08	1.08	1.08	1.08	1.08	1.08
(b) Optics in 36-in. spectrometer	1.16	1.16	1.16	1.16	1.16	1.16	1.16
(c) Radiative effects	1.63	1.63	1.63	1.65	1.65	1.65	1.65
(d) Angular resolution	1.02	1.14	1.19	1.00	1.17	1.23	1.23
$\left(\dfrac{d^3\sigma}{d\Omega_e d\Omega_p dE_e'}\right)10^{32}$ cm^2/(sr^2 MeV)	1.30	2.29	0.76	2.96	7.32	5.17	0.88
Standard deviation (in %) from counting statistics	±38	±11	±31	±15	±6.3	±11	±41

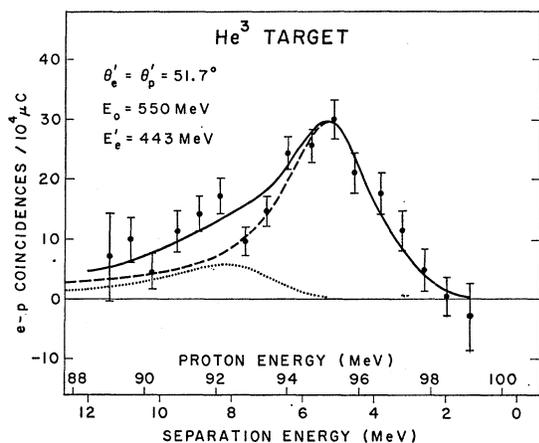[9] R. Atkinson, Ph.D. thesis, Stanford University, 1964 (unpublished).

FIG. 1. The energy spectrum of protons at 51.7° in coincidence with 443-MeV electrons at 51.7° from He³ (e,e'p). The curves are discussed in Sec. VI of the text.

Figure 2 shows the corresponding spectrum for the H³ target. There is a definite peak at about 8.7-MeV separation energy, and the cross section falls off quite rapidly with decreasing proton energy. This indicates that mostly the two neutrons come out of the reaction with very little relative energy and thus the assumption made by Griffy and Oakes[5] that they are in a singlet S state is very plausible.

According to Griffy and Oakes,[5] this peak in tritium should have a simple relation to the second peak in helium-3, given by the Eqs. (6) and (7). To test if this is consistent with the present experiment, we have indicated in Fig. 1 by a dotted curve the tritium spectrum of Fig. 2 divided by 2 and shifted in energy to make up for the difference in Q value between reactions (B) and (C). This difference would also mean a slightly different probability for the proton to be at rest in the two cases. This has been roughly taken into account by multiplying the tritium results by $(8.5/7.5)^{3/2}$. (It should be noted that the ordinate for He³ refers to a
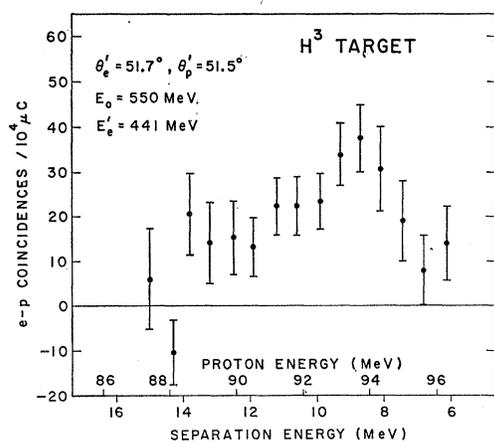

FIG. 2. The energy spectrum of protons at 51.5° in coincidence with 441-MeV electrons at 51.7° from H³ (e,e'p).

1500-psi target, while that for H³ refers to a 3000-psi target.) The dashed curve in Fig. 1 is a somewhat arbitrary curve, which is supposed to represent the contribution from reaction (A). It has been drawn so that it follows the experimental points at high proton energies and then on the low-energy side smoothly joins the radiative tail, calculated as described in Sec. V, in connection with the radiative corrections. The solid curve, finally, is the sum of the other two. It can be seen that there is a slight indication for the second peak in He³ to occur at a somewhat higher separation energy than assumed by the dotted curve. On the other hand, there is no serious discrepancy between the solid curve and the experimental points, in particular if one considers the fact that the data were obtained on several different occasions and during a total time of about 60 h with a considerable probability for small energy drifts. In fact, such drifts could be the explanation for the large
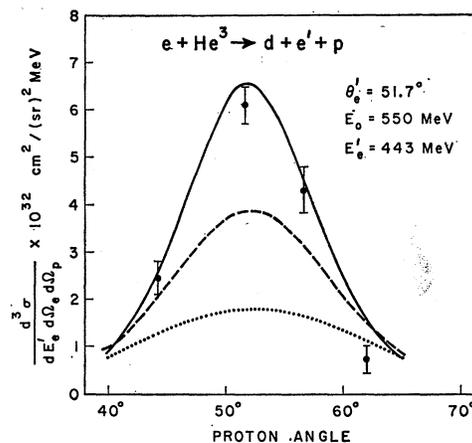

FIG. 3. The coincidence cross section of reaction (A) as a function of proton angle. The curves correspond to calculations using different wave functions and are explained in Sec. VI of the text.

width of the peak at 5.5 MeV (about 3 MeV instead of 2 MeV as expected from the calculation of the proton spectrum).

Since calculations of the cross sections only exist for reaction (A), it is of interest to be able to subtract out the contribution from reaction (B). This has been done by summing all counts in the proton spectrum down to a separation energy of 11 MeV and by multiplying the sum by 0.83 which is the ratio of the area under the dashed curve to that under the solid curve in Fig. 1. In Table I we give the cross sections corresponding to the sum for reactions (A) and (B).

In Fig. 3 are shown the cross sections for reaction (A) as a function of the proton angle together with three curves calculated by Griffy and Oakes.[5] The dotted curve is obtained when a Gaussian wave function is used; i.e.,

$$u_G \propto \exp[-\tfrac{1}{2}\alpha^2(r_{12}^2 + r_{23}^2 + r_{31}^2)];$$

the dashed curve comes from the use of an Irving wave function[10]:

$$u_I \propto \exp[-\tfrac{1}{2}\alpha(r_{12}{}^2+r_{23}{}^2+r_{31}{}^2)^{1/2}],$$

and the solid curve corresponds to an Irving-Gunn wave function[11]:

$$u_{IG} \propto u_I/(r_{12}{}^2+r_{23}{}^2+r_{31}{}^2)^{1/2}.$$

Only the Irving-Gunn wave function gives an adequate fit to the experimental cross sections. Furthermore, it was shown by Griffy and Oakes[5] that the data are inconsistent with a 4% admixture into the wave function of a $^2S_{1/2}$ state of mixed symmetry in coordinate space. Such an admixture was proposed by Schiff[1b] as a possible explanation for the small charge form factor of He³.[1d]

In Fig. 4 are shown the cross sections as a function of $\theta_p{}'$ for tritium. As there have been no calculations of this cross section, we have made a crude extrapolation of the Irving-Gunn (solid) curve in Fig. 3 to the case of tritium using Eqs. (5) and (7). It is assumed that $|I_0|^2$ and $|I_1|^2$ only depend on the probability of finding the proton in various states of motion, and that this probability in turn can be estimated simply from a wave function of the type

$$u \propto \frac{\exp[-r(2mB)^{1/2}]}{r},$$

where B is the binding energy (5.5 MeV for He³ and 8.5 MeV for H³). In order for this to be a good approximation it is necessary that the two neutrons in the tritium wave function have about the same overlap with the singlet S state as the spectator neutron-proton pair in the helium-3 wave function has with the deuteron. The result of this extrapolation is shown as the solid curve in Fig. 4. The agreement with experiment is excellent.

VII. CONCLUSIONS

It appears that the results of the present experiment can be explained quite well by the theory of Griffy and Oakes[5] if wave functions of the Irving-Gunn type are used in a completely symmetric $^2S_{1/2}$ state. There is,

[10] J. Irving, Phil. Mag. **42**, 338 (1951).
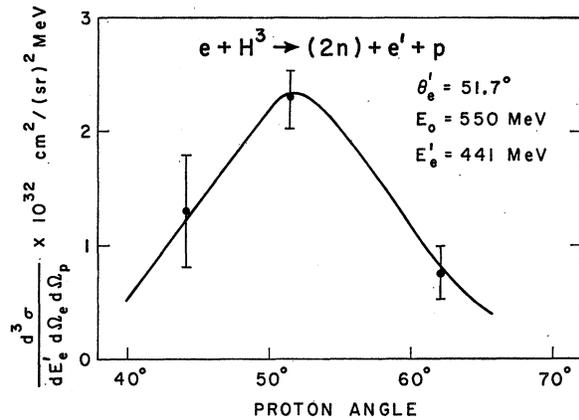[11] J. C. Gunn and J. Irving, Phil. Mag. **42**, 1353 (1951).

FIG. 4. The coincidence cross section of reaction (C) as a function of proton angle. The curve is explained in Sec. VI of the text.

however, evidence from several sources (of which elastic electron scattering is one) that there must be a considerable admixture of other states into the ground-state wave function, e.g., D states. These would make no direct contribution to the coincidence cross section at the peak in the angular distribution (other than caused by a renormalization of the symmetric S state), but could conceivably contribute significantly at other angles. For instance, the low cross section measured at 62° for reaction (A) (see Fig. 3) could be evidence for a D-state contribution or perhaps for a deviation from the Irving-Gunn wave function. At the present time, however, the experimental uncertainties are too large for any definite conclusion to be made.

ACKNOWLEDGMENTS