# Electronic *g* Factor of Rubidium

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Frequencies corresponding to Zeeman transitions in an optically pumped rubidium-87 sample have been measured relative to the free-precession frequency of protons in a spherical water sample. Observations were made at 12 and 6 G in a compensated solenoid. On the basis of accepted theory for the separation of energy levels in a magnetic field, the ratio of *g* factors for rubidium electrons and protons was found to be 658.2323 $\pm$0.0007. It follows that the electronic *g* factor for rubidium is 25$\pm$1 parts per million higher than the *g* factor for the electron in the hydrogen ground state. This result is combined with ratios of alkali *g* factors measured in atomic beam experiments to give the *g* factors for other alkalis.

## 1. INTRODUCTION

THE Zeeman frequencies corresponding to transitions between the ground-state hyperfine sublevels of rubidium-87 have been measured relative to the free-precession frequency of protons in a magnetic field of 12 G. The magnetic field was produced by passing a current, which was known relative to present NBS electrical standards, through the compensated solenoid[1,2] used earlier in the determination of the proton gyromagnetic ratio. The solenoid axis was placed in the nominal E-W magnetic direction and the off-axis components of the earth's field were cancelled by 5-ft-diam Helmholtz coils. The effect of stray earth's field along the solenoid direction was eliminated by averaging the results of observations made with solenoid current in the normal and reversed directions.

The prevailing 60-Hz stray field of about 1-mG peak to peak along the solenoid axis was reduced to negligible value ($\sim$0.01-mG rms) by means of a sensing coil, amplifier and drive coil operating in a closed servo loop.[3]

The proton and rubidium samples were exchanged at the center of the coil and their relative frequencies measured. Except for the proton pickup coil and thin plastic tube used to transport the water sample, all neighboring materials and equipment that might affect the value of the magnetic field after the exchange of samples was left in place to assure the accuracy of these relative measurements. Local field variations caused by vehicles and laboratory equipment were avoided by making the observations between 2 and 5 a.m.

The combined solenoid and compensating windings were designed to provide a field uniformity of 3 parts in $10^7$ over the volume of a 2-cm-diam sphere at the center of the solenoid. Experience with this coil system at the Fredericksburg Magnetic Observatory leads one to believe that, under ideal local conditions, the compensating windings can be adjusted to realize the design uniformity. This uniformity could not be realized in Room 15 of the East Building, where these measurements were made, on account of the local earth's magnetic field inhomogeneities. It was found possible, however, to adjust the compensating coils to give the maximum homogeneity consistent with equal uniformity for both directions of the solenoid field thus leaving the ambient inhomogeneities unchanged. When the coils were adjusted in this manner, the decay time of the proton free-precession signal from a 2-cm-diam water sample was about 1.5 sec (sample relaxation time $\sim$4 sec); and the linewidth in a 4.5-cm-diam rubidium vapor sample buffered with 3-cm neon was about 50 cycles.

## 2. RUBIDIUM-87 ZEEMAN TRANSITION FREQUENCIES

The Zeeman transitions in an optically pumped rubidium-87 sample were observed by Dehmelt's method.[4] The intensity of circularly polarized Rb D-1 radiation (7947 Å) transmitted along the solenoid field $H_0$ through the sample was observed as a function of the frequency of a weak rf field perpendicular to $H_0$. At the 12-G level, the four $F=2$ lines are spread over a range of about 64 kHz. According to the Breit-Rabi formula,[5] the mean frequency of the four $F=2$ lines is strictly linear with the field $H_0 \pm \Delta H$. If necessary, one might observe all four lines with the field in each of the normal and reversed directions in order to eliminate the effect of the ever present stray field $\Delta H$ along the solenoid direction. It will be shown that it is sufficient to observe the frequencies of the strongest lines found in the normal and reversed directions of $H_0$ for the purpose of obtaining a proper average. If the normal field direction and the sense of the circularly polarized radiation is such that the $(F=2;\ M_F=2) \rightarrow (F=2,\ M_F=1)$ line is the strongest, the situation, when the direction of $H_0$ is reversed and the strongest line measured again, is the same as if $\Delta H$ were reversed and $\nu_{-2,-1}$ observed instead of $\nu_{2,1}$.

From the Breit-Rabi formula, writing all *g* factors

[1] R. L. Driscoll and P. L. Bender, Phys. Rev. Letters **1**, 413 (1958).
[2] P. L. Bender and R. L. Driscoll, IRE Trans. Instr. **7**, 176 (1958).
[3] L. A. Marzetta, Rev. Sci. Instr. **32**, 1192 (1961).
[4] H. G. Dehmelt, Phys. Rev. **105**, 1487 (1957).
[5] J. M. B. Kellogg and S. Millman, Rev. Mod. Phys. **18**, 323 (1946); see p. 343.

TABLE I. Summary of experimental results.

| Date | $H_0$ gauss | $\bar{\nu}$ MHz | $\Delta\nu$ kHz | $\nu_c$ MHz | $\nu_c f/H_0$ MHz | $\nu_p'/H_0$ kHz | $4\nu_c f/\nu_p'$ |
|---|---|---|---|---|---|---|---|
| Mar. 17, 1960 | 12.331295 | 8.626874 | 32.796 | 8.644053 | 0.7009783 | 4.257643 | 658.5599 |
| Nov. 22, 1960 | 12.331309 | 8.626870 | 32.295 | 8.644049 | 75 | 43 | 5591 |
| May 10, 1961 | 12.331289 | 8.626873 | 32.704 | 8.644052 | 87 | 43 | 5602 |
| May 18, 1961 | 12.331225 | 8.626821 | 31.962 | 8.644000 | 85 | 43 | 5601 |
| Oct. 7, 1961* | 6.164967 | 4.312935 | 8.647 | 4.321524 | 90 | 51 | 5595 |
| Oct. 26, 1961* | 12.331299 | 8.626863 | 32.691 | 8.644042 | 72 | 40 | 5593 |
| Nov. 1, 1961* | 12.331292 | 8.626860 | 32.058 | 8.644039 | 78 | 43 | 5595 |
| Jan. 17, 1962* | 6.165750 | 4.313470 | 7.123 | 4.322060 | 77 | 42 | 5595 |
| Mar. 9, 1962 | 6.165710 | 4.313444 | 8.488 | 4.322033 | 71 | 43 | 5588 |
| Mar. 21, 1962 | 12.331191 | 8.626794 | 33.372 | 8.643973 | 74 | 43 | 5591 |
| July 11, 1962 | 12.331242 | 8.626828 | 31.140 | 8.644008 | 86 | 43 | 5602 |
| Dec. 19, 1962 | 12.331385 | 8.626903 | 30.170 | 8.644082 | 0.7009772 | 4.257643 | 658.5589 |
| | | | | | | | 658.5595 |

used as positive,

$$\nu_{87}(2,1)=-g_{87}\mu_0H_+/h +\tfrac{1}{2}\nu_H[(1+X_+)-(1+X_++X_+^2)^{1/2}], \quad (1)$$

$$\nu_{87}(-2,-1)=-g_{87}\mu_0H_-/h +\tfrac{1}{2}\nu_H[(1-X_-+X_-^2)^{1/2}-(1-X_-)], \quad (2)$$

where

$$X_\pm=(1/\nu_H)(g_{\rm Rb}+g_{87})\mu_0(H_\pm/h), \text{ and } H_\pm=H_0\pm\Delta H.$$

Here $g_{\rm Rb}$ and $g_{87}$ are the $g$ factors for the rubidium electron and nucleus (uncorrected for diamagnetism); $\mu_0$ is the Bohr magneton, $h$ is Planck's constant, $\nu_H=6.83468261\times10^9$ cps is the rubidium-87 ground-state hyperfine splitting,[6,7] and $\Delta H$ is the stray earth's field along the solenoid axis. In order to obtain the wanted quantity $X_0=\frac{1}{2}(X_++X_-)$, from the observed frequencies and the known solenoid field, one must solve Eqs. (1) and (2). It has been found convenient to make the following definitions: $\nu_+=\nu_{2,1}(H_0+\Delta H)$, $\nu_-=\nu_{-2,-1}(H_0-\Delta H)$, $\bar{\nu}=\frac{1}{2}(\nu_++\nu_-)$, $\Delta\nu=\frac{1}{2}(\nu_+-\nu_-)$, $\nu_c=\bar{\nu}+g_{87}\mu_0(H_0/h)$, and $a=(\Delta\nu+g_{87}\mu_0(\Delta H/h))$. From Eq. (1),

$$X_+=\frac{4}{\nu_H}(\nu_c+a)\left[\frac{1-(\nu_c+a)/\nu_H}{1-4(\nu_c+a)/\nu_H}\right],$$

and from Eq. (2),

$$X_-=\frac{4}{\nu_H}(\nu_c-a)\left[\frac{1+(\nu_c-a)/\nu_H}{1+4(\nu_c-a)/\nu_H}\right].$$

On averaging

$$X_0=\frac{4\nu_c}{\nu_H}\left[1+6\frac{\Delta\nu}{\nu_H}+12\left(\frac{\nu_c}{\nu_H}\right)^2\right]\equiv4\frac{\nu_c}{\nu_H}f, \quad (3)$$

or $(4\nu_c/H_0)f=(g_{\rm Rb}+g_{87})(\mu_0/h)$.

To be consistent with the definitions given above, $\Delta\nu$ must be negative. Terms neglected in the approximation of $X_0$ are less than 2 parts in $10^8$ for $H_0=12$ G and $\Delta H=10^{-2}$ G.

The required frequencies were supplied by a manually adjusted signal generator. The generator frequency was measured with a commercial counter supplied with the NBS 100-kHz frequency standard. With continuous duty, the drift rate of the generator was about 1 count per sec. The generator output was attenuated and chopped at 10 Hz before application to the transverse field producing coils. The transmitted rubidium (D-1) radiation was thus modulated. The resulting signal available from the silicon photodetector was amplified and passed through a narrow band 10-Hz filter before display on an oscilloscope. On the strongest line, the ratio of signal to off-resonance electronic noise was generally better than 200 to 1. With a linewidth of 40 Hz and frequencies around 8.6 MHz the generator frequency could be adjusted for maximum signal to better than 1 part in $10^6$. It was the practice in a run to take 10 frequency readings with the field direction alternated between normal and reversed directions, when each of four different standard cells was used to control the solenoid current. Any drift in the standards or solenoid constant during a run was checked by returning to the first cell and repeating a set of 10 readings. Each entry in Table I is thus the average of 50 observations. As an indication of over-all precision realized, the mean of 10 frequency observations repeated with a given standard cell was found to check to about 3 parts in $10^7$.

### 3. PROTON-PRECESSION FREQUENCY

The proton-precession frequency was measured using a free-precession technique[1,2] in which a 2-cm-diam spherical water sample is prepolarized in a strong field, transported by pneumatic means to the center of the solenoid, given a transverse rf pulse to produce the desired $\frac{1}{2}\pi$ angle with the solenoid axis, and the resulting signal induced in a surrounding pickup coil is observed.

[6] L. Essen, E. G. Hope, and D. Sutcliffe, Nature **189**, 298 (1961).
[7] S. Penselin, T. Moran, V. W. Cohen, and G. Winkler, Phys. Rev. **127**, 524 (1962).

The proton precession signal ($\sim$52.5 kHz at 12 G) was amplified and passed through a circuit which produced a pulse each time the signal passed through zero with a positive slope. Two of the pulses separated by a preset number were used to start and stop a commercial electronic timer which was supplied with the NBS 100-kHz frequency standard. The accuracy of this frequency measuring system was checked by coupling into the pickup coil a signal with known frequency and any desired signal-to-noise ratio. The electronics system was found to be accurate to about 3 parts in $10^7$ when the signal-to-noise ratio was greater than 10 to 1. The signal-to-noise ratio generally realized in the experiment was about 25 to 1.

Since the proton Larmor frequency is linear with the magnetic field, the mean value, after observing with both field directions to eliminate $\Delta H$, is given by

$$\nu_p' = g_p' \frac{\mu_0}{h}\left[\frac{H_+ + H_-}{2}\right] = g_p' \frac{\mu_0}{h} H_0 , \tag{4}$$

where $g_p'$ is the proton $g$ factor for a spherical water sample.

As with rubidium frequency measurements, the procedure involved a series of 10 observations with the field in alternate directions when using each of four standard cells; and the mean of 10 readings would generally repeat to about 3 parts in $10^7$ on a given cell.

## 4. RESULTS

The fluctuations in the experimental results shown in Table I are generally less than 1 ppm. On the date marked * both rubidium and proton frequencies were measured in order; on the other dates $(\nu_c/H_0)f$ was measured and $\nu_p'/H_0$ was obtained via the relation $\nu_p'/H_0 = \gamma_p'/2\pi$, where $\gamma_p'$ is the value of the proton gyromagnetic ratio established at the same place with the same apparatus.

Combining Eqs. (3) and (4), one gets

$$\frac{4\nu_c f}{H_0} \cdot \frac{H_0}{\nu_p'} = \frac{g_{\mathrm{Rb}} + g_{87}}{g_p'} . \tag{5}$$

Substituting[7,8] $g_{87}/g_p' = 0.3272$ and $4\nu_c f/\nu_p' = 658.5595$ from Table I, the final result is

$$g_{\mathrm{Rb}}/g_p' = 658.2323 .$$

The only known source of systematic error in $g_{\mathrm{Rb}}/g_p'$ is the possible effect of changing the rubidium and water samples, and the presence or absence of the proton signal pickup coil and transport tube. Checks made with the quantity of these materials increased lead one to believe that the error from this cause is less than 1 part in $10^6$. Earth's field variations could cause significant errors but during magnetically quiet periods, these appear to be largely averaged out by the method of reversal. Items such as the solenoid constant and current value entering the computation of $H_0$ could be held practically constant during a frequency comparison. As shown in Table I, the fluctuations among the runs where both rubidium and proton frequencies were measured are indeed small. The uncertainty in the value here found is estimated to be 1 ppm with even chance confidence, or $(g_{\mathrm{Rb}}/g_p')_{\mathrm{exp}} = 658.2323 \pm 0.0007$. This is in agreement with Bender's value[9] $(g_{\mathrm{Rb}}/g_p')_{\mathrm{exp}} = 658.234 \pm 0.004$ determined in the earth's magnetic field.

When the result of this experiment is combined with Lambe's value[10] of $(g_{\mathrm{H}}/g_p') = 658.21591 \pm 0.00004$ obtained at about 3000 G, where $g_{\mathrm{H}}$ is the electron $g$ factor for the ground state of hydrogen, the result is

$$(g_{\mathrm{Rb}}/g_{\mathrm{H}})_{\mathrm{exp}} = 1 + (25 \pm 1) \times 10^{-6} .$$

This ratio can be multiplied by the theoretical value of 2.0022838 for $g_{\mathrm{H}}$[11,12] to give

$$g_{\mathrm{Rb}} = 2.002334 \pm 0.000002 .$$

We can also use $(g_{\mathrm{Rb}}/g_{\mathrm{H}})_{\mathrm{exp}}$ to calibrate other experiments in which ratios of alkali $g$ factors have been measured. Recent preliminary results by the atomic beams group at Heidelberg give[13]

$$(g_{\mathrm{Rb}}/g_{\mathrm{K}})_{\mathrm{exp}} = 1 + (17.9 \pm 4.0) \times 10^{-6} ,$$
$$(g_{\mathrm{Cs}}/g_{\mathrm{K}})_{\mathrm{exp}} = 1 + (124.9 \pm 4.0) \times 10^{-6} .$$

When these values are combined with the rubidium result, we find

$$(g_{\mathrm{K}}/g_{\mathrm{H}})_{\mathrm{exp}} = 1 + (7 \pm 4) \times 10^{-6} ,$$
$$(g_{\mathrm{Cs}}/g_{\mathrm{H}})_{\mathrm{exp}} = 1 + (132 \pm 4) \times 10^{-6} .$$

The theoretical situation concerning alkali $g$ factors has been reviewed by Hughes[11] and by Kusch and Hughes.[12] Using values of the Margenau[14] and Lambe,[15] corrections as listed by Perl,[16] which are presumably accurate to about 1 part in $10^6$

$$(g_{\mathrm{K}}/g_{\mathrm{H}})_{\mathrm{M+L}} = 1 + 5 \cdot 8 \times 10^{-6} ,$$
$$(g_{\mathrm{Rb}}/g_{\mathrm{H}})_{\mathrm{M+L}} = 1 + 5 \cdot 8 \times 10^{-6} ,$$
$$(g_{\mathrm{Cs}}/g_{\mathrm{H}})_{\mathrm{M+L}} = 1 + 6 \cdot 8 \times 10^{-6} .$$

If we assume that the difference between these values and the experimental ones are due to a small admixture of excited core states with different $g$ factors as sug-

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[9] P. L. Bender, Phys. Rev. **128**, 2218 (1962).

[10] E. B. D. Lambe, thesis, Princeton University, 1959 (unpublished).

[11] V. W. Hughes, *Recent Research in Molecular Beams*, edited by I. Estermann (Academic Press Inc., New York, 1959), pp. 65–92.

[12] P. Kusch and V. W. Hughes, *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1959), Vol. 3, Part 1, pp. 115–120.

[13] S. Penselin (private communication).

[14] H. Margenau, Phys. Rev. **57**, 383 (1940).

[15] W. E. Lambe, Phys. Rev. **60**, 817 (1941).

[16] W. Perl, Phys. Rev. **91**, 852 (1953); see also Ref. 11, p. 85.

gested by Phillips,[17] we find the size of the required correction to be $(1\pm4)$, $(19\pm1)$, and $(125\pm4)\times10^{-6}$ for K, Rb, and Cs, respectively. These corrections are in reasonable agreement with the estimates made by Phillips,[17] although the Cs correction is somewhat higher than expected.

Previous experiments relating to the K, Rb, and Cs $g$ factors have been done by Franken and Koenig[18] and by Kusch and Taub.[19] The results are

$$(g_K/g_H)_{F+K}=1+(16\pm4)\times10^{-6},$$
$$(g_{Rb}/g_{Na})_{K+T}=1+(50\pm10)\times10^{-6},$$
$$(g_{Cs}/g_{Na})_{K+T}=1+(134\pm7)\times10^{-6}.$$

Since the Margenau and Lambe corrections give $(g_{Na}/g_H)=1+(4.8)\times10^{-6}$ and the effect of excited core states is expected to be even smaller for sodium than for potassium, we find

$$(g_{Rb}/g_H)_{K+T} \quad 1+(55\pm10)\times10^{-6},$$
$$(g_{Cs}/g_H)_{K+T} \quad 1+(139\pm7)\times10^{-6}.$$

Looked at in this way, the previous results were somewhat high for potassium and rubidium but were in agreement with recent results for cesium.

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# Emission Spectrum of ZnIn

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The spectrum of ZnIn shows bands which can be analyzed into four systems. Two narrow systems at λ4102 and λ4511 appeared continuous. Systems at λ5308 ($B-A$) and λ5625 ($D-C$) were photographed with a dispersion of 5 Å/mm. A possible vibrational analysis has been made for them. These systems exhibit an isotope effect which shows ZnIn as the emitter. Probable electronic states are $^2\Pi-{}^2\Sigma$ for λ5308 and $^2\Sigma-{}^2\Sigma$ for λ5625. Constants obtained are as follows: for $B^2\Pi_{\frac{1}{2}}-A^2\Sigma$ subsystem, $\nu_e=18810.8$, $\omega_e'=201.2$, $\omega_e'x_e'=0.6$, $\omega_e''=146.7$, $\omega_e''x_e''=0.7$; for $B^2\Pi_{\frac{3}{2}}-A^2\Sigma$ subsystem, $\nu_e=18831.2$, $\omega_e'=193.9$, $\omega_e'x_e'=0.6$ and for $D-C$ system, $\nu_e=17732.0$, $\omega_e'=107.0$, $\omega_e'x_e'=1.0$, $\omega_e''=56.1$, and $\omega_e''x_e''=0.1$.

## 1. INTRODUCTION

THE band spectrum of HgTl and HgIn have been studied by Winans, Pearce, Davis, Leitzke, and Purbrick.[1–3] Both of these molecules show band systems of two general types. One type is a short-wavelength range system on the long-wavelength side of Tl or In lines. The other type is a group of bands with sharp heads showing overlapping sequences. An attempt to observe spectra from similar molecules has led to the present study of the spectrum of ZnIn.

## 2. EXPERIMENTAL ARRANGEMENTS

Spectra were excited in a quartz tube of length about 5 cm and diameter 1 cm, with a flat window on one end. Tubes were degassed at less than $10^{-6}$ mm and zinc and indium inserted by distillation. Tubes were usually sealed off containing argon at about 3 mm pressure. Argon, however, is not necessary to obtain the ZnIn spectrum.

A 200-W oscillator at a frequency of about 100 Mc/sec served for excitation with external electrodes. The heat necessary to maintain the required pressures of indium and zinc was provided by a blow torch. Spectra were photographed with Bausch and Lomb medium quartz and quartz Littrow spectrographs and with a 3.4-m Jaco Ebert Plane grating spectrograph of dispersion 5 Å/mm in the first order. Exposures of 5 h with Kodak IIF plates were sufficient.

## 3. RESULTS

Fig. 1(a) shows the band systems photographed with the medium quartz spectrograph. The spectrum shows two continuous bands near the In lines λ4102 and λ4511 and two groups of bands with intensity maxima at

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