Spin Effects on the Long-Wavelength Oscillations of a Quantum Plasma in a Magnetic Field

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(Received 4 May 1964)

The results of our earlier work on the quantum theory of electron-gas plasma oscillations in a magnetic field are extended here to take account of the difference in masses associated with the orbital and spin parts of the individual electronic motions in the presence of a lattice, and allowance is made for an anomalous electronic g factor. Tractable expressions for the complete plasmon dispersion relation and damping constant (at arbitrary temperature and arbitrary magnetic field strength), which are obtained using a Green's-function formulation of the random-phase approximation, are reported. The low-wave-number (p) approximation of the dispersion relation is investigated in detail. For $p=0$ the usual result obtains, $1/\omega_p^2=\sin^2\theta/\Omega_0^2+\cos^2\theta/(\Omega_0^2-\omega_c^2)$, with two plasma modes; these are shifted to order p^2 by terms that are oscillatory in the de Haas–van Alphen (DHVA) sense in the degenerate case. Another plasma resonance in the vicinity of $2\omega_c$, which is active to order p^2, exhibits such DHVA oscillatory behavior, and the same may be said for a gap in the frequency spectrum for propagation perpendicular to the magnetic field in the interval $[\omega_c, 2\omega_c]$. The relative amplitudes for the plasma modes are reported also. The terms which are oscillatory in the DHVA sense are exhibited in terms of an appropriate Fourier series, with no restriction on temperature or magnetic field strength, save that $\zeta\beta\gg1$ (that is to say that no restriction is placed on $\hbar\omega_c\beta$). Moreover, the spectral composition of the DHVA oscillatory terms is thereby explicitly shown to be a sensitive function of $g(m/m_0)$, and this result may be useful for the experimental determination of the product of anomalous electronic g factor and effective mass m. A considerable improvement over other recent work on this subject has been achieved through a careful and correct determination of the role of the DHVA terms as a whole in the plasma oscillation spectrum.

I. INTRODUCTION

THE development of a quantum-theoretical plasmon dispersion relation for the electron gas in a magnetic field has been the object of considerable attention recently.[1–3] In a publication on this subject by Gartenhaus and Stranahan,[4] it was suggested that certain modifications are induced in the plasmon dispersion relation by recognizing a distinction between the spin part of the Hamiltonian and the part describing orbital motion due to the presence of a lattice. Specifically, this suggestion is based on the qualitative argument that the orbital part of the electronic motion in a magnetic field is coupled to the lattice so that one must at least introduce the effective mass m in the corresponding part of the Hamiltonian, whereas this is not the case for the spin part of the motion so that the ordinary mass m_0 is retained in the Pauli spin term of the Hamiltonian. In addition, allowance is made for an anomalous electronic g factor. A result of these considerations is that parts of the plasmon dispersion relation in the degenerate case which are oscillatory in the de Haas–van Alphen (DHVA) sense depend rather sensitively on the parameter $g(m/m_0)$.

The purpose of this paper is to report the results of extending our earlier work[5] on the plasmon dispersion relation to take account of the modifications indicated above. In particular a careful determination of the role of DHVA oscillatory terms as a whole in the plasma oscillation spectrum for the degenerate gas is presented. [These results are in disagreement with the corresponding conclusions of Gartenhaus and Stranahan.[4] Specifically, the latter authors obtain a zero-wave-number limit for the plasmon dispersion relation which is substantially different from the usual result, $1/\omega_p^2=\sin^2\theta/\Omega_0^2+\cos^2\theta/(\Omega_0^2-\omega_c^2)$, which we find to be valid within the scope of the random-phase approximation for the degenerate gas as well as the nondegenerate one, even with the indicated modifications.] In addition, the terms which are oscillatory in the DHVA sense are exhibited in terms of an appropriate Fourier series, with no restriction on temperature or magnetic field strength save that the condition of degeneracy be fulfilled, $\zeta\beta\gg1$ (that is to say that no restriction is placed on $\hbar\omega_c\beta$). The spectral composition of the DHVA oscillatory terms is thereby explicitly shown to be a sensitive function of $g(m/m_0)$.

II. PLASMONS IN A MAGNETIC FIELD

An analysis of the inverse longitudinal dielectric function for an electron gas in a magnetic field was presented by the author recently,[5] and the concomitant plasmon dispersion relation was studied within the scope of the random-phase approximation using the Green's-function method. There, a relatively tractable expression of the plasmon dispersion relation was obtained, and particular attention was given to the low-wave-number approximation of the dispersion relation, which involves terms that are oscillatory in the DHVA sense in the degenerate case, and involves quantum corrections through the parameter $\hbar\omega_c\beta$ in the non-

† Supported by the U. S. Air Force Office of Scientific Research.
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[2] M. J. Stephan, Phys. Rev. 129, 997 (1963).
[3] N. D. Mermin and E. Canel, Ann. Phys. (N. Y.) 26, 247 (1964).
[4] S. Gartenhaus and G. Stranahan, Phys. Rev. 133, A104 (1964).
[5] N. J. Horing, Ann. Phys. (to be published).

degenerate case.[6] Of course, the relative amplitudes of the various plasmon modes considered were calculated, and a thorough discussion of the natural damping was given.

There is no need to rederive the plasmon dispersion relation in detail here since the modifications contemplated in the introduction involve only slight changes in the formal structure of our earlier work. A thorough discussion of the calculational techniques is given there, and the interested reader will find detailed derivations of results which are analogs of all those which will be presented here. With this in mind, it is sufficient for our purposes here to note that the appropriate Green's function involved in the derivation of the plasmon dispersion relation satisfies the Eqs. (1a) and (1b) below,

$$\bar{G}(\mathbf{r},t;\,\mathbf{r}',t')=\exp[\tfrac{1}{2}ie\mathbf{r}\cdot\mathbf{H}\times\mathbf{r}'-i\varphi(\mathbf{r})+i\varphi(\mathbf{r}')]\times\bar{G}'(\mathbf{r}-\mathbf{r}',\,t-t'),\quad(1a)$$

$$[\nabla_R^2/2m-\tfrac{1}{8}m\omega_c^2(X^2+Y^2)+\tfrac{1}{4}\omega_cL_Z-\mu_0H\sigma_3+\zeta+i(\partial/\partial T)]\bar{G}'(\mathbf{R},T)=\delta(\mathbf{R})\delta(T).\quad(1b)$$

All gauge dependence φ and dependence on $\mathbf{r}+\mathbf{r}'$ is embodied in the factor

$$C(\mathbf{r},\mathbf{r}')=\exp[\tfrac{1}{2}ie\mathbf{r}\cdot\mathbf{H}\times\mathbf{r}'-i\varphi(\mathbf{r})+i\varphi(\mathbf{r}')].$$

[The quantity $L_Z\bar{G}'(\mathbf{R},T)=(L_z+L_{z'})\bar{G}'(\mathbf{r}-\mathbf{r}',\,t-t')$ measures the loss of orbital angular momentum in the direction of the magnetic field suffered by an electron as it propagates between the points $(\mathbf{r},t)$ and $(\mathbf{r}',t')$; since this component of orbital angular momentum is conserved, $L_Z\bar{G}'(\mathbf{R},T)=0$, and there is a considerable simplification of (1b).] A closed solution may be obtained for (1b), and the resulting grand-canonical ensemble averaged Green's function is given by

$$\begin{pmatrix}\bar{G}_>(\mathbf{r},\mathbf{r}',T)\\ \bar{G}_<(\mathbf{r},\mathbf{r}',T)\end{pmatrix}=e^{i\zeta T}C(\mathbf{r},\mathbf{r}')\int\frac{d\omega}{2\pi}\begin{pmatrix}-i[1-f_0(\omega)]\\ if_0(\omega)\end{pmatrix}e^{-i\omega T}\times\int_{-\infty}^{\infty}dT'e^{i\omega T'}\int\frac{d\mathbf{p}}{(2\pi)^3}e^{i\mathbf{p}\cdot\mathbf{R}}\exp\{-i[\mu_0H\sigma_3+(p_z^2/2m)]T'\}\times\sec(\tfrac{1}{2}\omega_cT')\exp\{-i[(p_x^2+p_y^2)/m\omega_c]\tan(\tfrac{1}{2}\omega_cT')\}.\quad(1c)$$

The notation of Ref. 5 is maintained here.[7] The only new notation arises from the fact that our considerations are somewhat extended here; g is the anomalous electronic g factor, m the effective electronic mass, and m_0 the ordinary electronic mass. These enter Eq. (1) in part explicitly and in part through the quantities

$$\omega_c=eH/mc;\quad\mu_0=g(e\hbar/2m_0c).$$

With the result (1c), a Green's-function formulation of the random-phase approximation leads to a tractable plasmon dispersion relation which takes the form

$$1-\frac{4\pi e^2}{\mathbf{p}^2}\frac{1}{\hbar^3}\,\mathrm{Im}I\left\{\hbar\mathbf{p},\frac{\tau}{\pi}[\hbar\Omega+i\epsilon]\right\}=0,\quad(2)$$

while the damping may be expressed as $\gamma=Z\Gamma$, where

$$\Gamma(\mathbf{p},\Omega)=\frac{4\pi e^2}{\mathbf{p}^2}\frac{1}{\hbar^3}\,2\mathrm{Re}I\left(\hbar\mathbf{p},\frac{\tau}{\pi}[\hbar\Omega+i\epsilon]\right),\quad(3)$$

and Z is the amplitude weight function which measures the relative importance of the various plasmon modes in

[6] The interested reader will also find (see Ref. 5) that the many plasmon resonances for propagation nearly perpendicular to the field were analyzed in detail in the semiclassical and classical limits, and their behavior in the asymptotic case $p^2(\zeta$ or $1/\beta)/m\omega_c^2\gg1$ was considered in order to achieve an understanding of the manner in which the nonuniform zero-field limit is attained.

[7] We briefly recount the notation of Ref. 5 which is relevant to the plasmon dispersion relation and damping constant [Eqs. (2), (3), (4), and (5)];

Ω=plasmon root; $\omega_p=(4\pi e^2\rho/m)^{1/2}$,

$\mathbf{p}=\text{wave vector}=\begin{pmatrix}\bar{p}=\begin{pmatrix}p_x\\p_y\end{pmatrix}\\p_z\end{pmatrix}$,

θ=angle between $\mathbf{p}$ and the plane perpendicular to $\mathbf{H}$.

The magnetic field is taken to be directed along the z axis and has strength H.

$\tau=-i\beta=-i/kT$; (k=Boltzmann constant, T=absolute temperature),

ζ=chemical potential,

$f_0(\omega)$=Fermi function$=[1+e^{(\omega-\zeta)\beta}]^{-1}$.

[Note that in Eq. (4b) the notation $\int_L dT$ is simply meant to indicate that $\int_L dTe^{-i(\Omega\pm\omega')T}=\frac{1}{i(\Omega\pm\omega')}$.]

[Note that in Eq. (4), $P\int$ indicates that the principal value of the integral is to be taken.]

response to excitation and may be calculated as the inverse of the Ω derivative of the dispersion relation. The modifications sought are embodied in the integral $I(\hbar\mathbf{p},(\tau/\pi)[\hbar\Omega+i\epsilon])$ whose real and imaginary parts may be calculated using the Green's function (1), with the result

$$\frac{1}{\hbar^3}\operatorname{Im}I\left(\hbar\mathbf{p},\frac{\tau}{\pi}[\hbar\Omega+i\epsilon]\right)=P\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}\frac{\omega'}{\Omega^2-\omega'^2}\frac{f_0(\omega)}{\hbar^3}R(\omega,\hbar\omega';\hbar\mathbf{p}) \tag{4a}$$

or

$$\frac{1}{\hbar^3}\operatorname{Im}I\left(\hbar\mathbf{p},\frac{\tau}{\pi}[\hbar\Omega+i\epsilon]\right)=\frac{P}{2i}\int_L dT\int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}(e^{-i(\Omega+\omega')T}-e^{-i(\Omega-\omega')T})[f_0(\omega)/\hbar^3]R(\omega,\hbar\omega';\hbar\mathbf{p})\,, \tag{4b}$$

and

$$\frac{1}{\hbar^3}\operatorname{Re}I\left(\hbar\mathbf{p},\frac{\tau}{\pi}[\hbar\Omega+i\epsilon]\right)=\tfrac{1}{2}\int\frac{d\omega}{2\pi}\frac{f_0(\omega)}{\hbar^3}R(\omega,\hbar\Omega;\hbar\mathbf{p})\,, \tag{4c}$$

where

$$R(\omega,\hbar\omega';\hbar\mathbf{p})=\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\,e^{i\omega x}\frac{2i\sin\frac{1}{2}\hbar\omega' x}{\hbar}e^{-\frac{1}{2}i\omega' y}\left(\frac{\pi^{3/2}}{(2\pi)^3}\right)\left(\frac{2m}{ix}\right)^{1/2}\frac{m\hbar\omega_c}{i\sin\frac{1}{2}\hbar\omega_c x/\cos\mu_0 Hx}$$
$$\times\exp\left(-i\frac{p_z^2}{2m}\frac{\hbar^2x^2-y^2}{4x}\right)\exp\left(-i\frac{\bar{p}^2}{2m}\frac{\cos\frac{1}{2}\omega_c y-\cos\frac{1}{2}\hbar\omega_c x}{(\omega_c/\hbar)\sin\frac{1}{2}\hbar\omega_c x}\right). \tag{5}$$

Note that the density is now expressed as

$$\rho=2\int\frac{d\omega}{2\pi}\frac{f_0(\omega)}{\hbar^3}\int dx\,e^{i\omega x}\frac{\pi^{3/2}}{(2\pi)^3}\left(\frac{2m}{ix}\right)^{1/2}\frac{m\hbar\omega_c}{i\sin\frac{1}{2}\hbar\omega_c x/\cos\mu_0 Hx}. \tag{6}$$

Here, a small negative imaginary part is associated with x, i.e., $x\to x-i\delta$, in the integrands of R and ρ. Therefore the x integration may be taken in the sense of an inverse Laplace transform.

The low-wave-number approximation of the dispersion relation may be written in terms of the quantities $s_1=\rho/\sigma$ and $s_2=\rho/\alpha$ as,

$$\frac{1}{\omega_p^2}=\frac{\sin^2\theta}{\Omega^2}+\frac{\cos^2\theta}{\Omega^2-\omega_c^2}+\frac{3\sin^4\theta p^2}{ms_2}\frac{1}{\Omega^4}+\frac{\cos^4\theta p^2}{ms_1\omega_c^2}\left(\frac{1}{\Omega^2-(2\omega_c)^2}-\frac{1}{\Omega^2-\omega_c^2}\right)$$
$$+\frac{\sin^2\theta\cos^2\theta p^2}{ms_1}\frac{3\Omega^2-\omega_c^2}{\Omega^2(\Omega^2-\omega_c^2)^2}+\frac{\sin^2\theta\cos^2\theta p^2}{ms_2}\frac{3\Omega^2+\omega_c^2}{(\Omega^2-\omega_c^2)^3}. \tag{7}$$

Here, σ and α are defined as

$$\sigma=\int_0^\infty d\omega\frac{f_0(\omega)}{\hbar^3}\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{\omega s}\frac{\pi^{3/2}}{(2\pi)^3}\left(\frac{2m}{s}\right)^{1/2}\frac{m(\hbar\omega_c)^2}{\tanh\frac{1}{2}\hbar\omega_c s\,\sinh\frac{1}{2}\hbar\omega_c s/\cosh\mu_0 Hs}, \tag{8a}$$

$$\alpha=\int_0^\infty d\omega\frac{f_0(\omega)}{\hbar^3}\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{\omega s}\frac{\pi^{3/2}}{(2\pi)^3}\left(\frac{2m}{s}\right)^{1/2}\frac{m\hbar\omega_c}{\sinh\frac{1}{2}\hbar\omega_c s/\cosh\mu_0 Hs}\frac{2}{s}, \tag{8b}$$

and ρ is the density which may be expressed as

$$\rho=2\int_0^\infty d\omega\frac{f_0(\omega)}{\hbar^3}\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{\omega s}\frac{\pi^{3/2}}{(2\pi)^3}\left(\frac{2m}{s}\right)^{1/2}\frac{m\hbar\omega_c}{\sinh\frac{1}{2}\hbar\omega_c s/\cosh\mu_0 Hs}. \tag{8c}$$

A direct physical interpretation of σ and α has been advanced in Ref. 3. Putting $T_{||}=1/2s_2=\alpha/2\rho$ and $W=1/s_1=\sigma/\rho$, one may take $T_{||}$ and W, respectively, as the mean kinetic energy of motion parallel to $\mathbf{H}$, and the mean oscillator "potential energy" per particle in equilibrium. This dispersion relation has just two roots in the limit of zero wave number, and they are given by,

$$\Omega_{0\gtrless}{}^2=\tfrac{1}{2}(\omega_p^2+\omega_c^2)\pm\tfrac{1}{2}[(\omega_p^2+\omega_c^2)^2-4\omega_p^2\omega_c^2\sin^2\theta]^{1/2}. \tag{9}$$

This result and the usual zero-wave-number dispersion relation from which it follows, $1/\omega_p^2=\sin^2\theta/\Omega_0^2+\cos^2\theta/(\Omega_0^2-\omega_c^2)$, are in sharp disagreement with the corresponding conclusions of Ref. 4. The substantially different results of the latter authors seem to be erroneous even in the simpler situation when $m=m_0$ and $g=1$. [This error may be traced to the incorrectness of Eq. (12) of Ref. 4.]

The p^2 terms of the dispersion relation (7) shift the roots (9), and to order p^2 this shift is described by the formula

$$\Omega_{\gtrless}{}^2=\Omega_{0\gtrless}{}^2+\tfrac{1}{2}p^2\omega_p{}^4F(\Omega_{0\gtrless})\pm p^2\frac{(\omega_p{}^2+\omega_c{}^2-2\omega_c{}^2\sin^2\theta)\omega_p{}^4F(\Omega_{0\gtrless})}{2[(\omega_p{}^2+\omega_c{}^2)^2-4\omega_p{}^2\omega_c{}^2\sin^2\theta]^{1/2}}, \tag{10}$$

where

$$F(\Omega)=\frac{3\sin^4\theta}{ms_2}\frac{1}{\Omega^4}+\frac{3\cos^4\theta}{ms_1}\frac{1}{(\Omega^2-(2\omega_c)^2)(\Omega^2-\omega_c{}^2)}+\frac{\sin^2\theta\cos^2\theta}{m}\left(\frac{1}{s_1}\frac{3\Omega^2-\omega_c{}^2}{\Omega^2(\Omega^2-\omega_c{}^2)^2}+\frac{1}{s_2}\frac{3\Omega^2+\omega_c{}^2}{(\Omega^2-\omega_c{}^2)^3}\right). \tag{11}$$

In addition to shifting the roots $\Omega_{\gtrless}{}^2$ the dispersion relation (7) dictates the presence of another root in the small interval $|\Omega^2-(2\omega_c)^2|\approx p^2/ms_1$. The location of this root is readily seen to be given by

$$\Omega_{(2\omega_c)}{}^2=(2\omega_c)^2+\frac{\cos^4\theta p^2}{ms_1}\times\frac{\omega_p{}^2}{\omega_c{}^2-\frac{1}{4}\omega_p{}^2\sin^2\theta-\frac{1}{3}\omega_p{}^2\cos^2\theta}. \tag{12}$$

Of the three plasmon modes $\Omega_<$, $\Omega_>$, $\Omega_{(2\omega_c)}$ discussed so far, one lies in the interval $[\omega_c,2\omega_c]$. It may be either $\Omega_>$ or $\Omega_{(2\omega_c)}$ and we will denote it by $\Omega_{[\omega_c,2\omega_c]}=\Omega_>$ or $\Omega_{(2\omega_c)}$. Part of the interval $[\omega_c,2\omega_c]$ is inaccessible to $\Omega_{[\omega_c,2\omega_c]}$, and this "gap" in the frequency spectrum may be calculated for $\theta=0$, with the result

$$2\omega_c-\Omega_{[\omega_c,2\omega_c]}>3p^2/4m\omega_cs_1. \tag{13}$$

The relative amplitudes of the plasmon modes $\Omega_>$ and $\Omega_<$ may be approximated as follows. For $\omega_p\gg\omega_c$ we have

$$Z(\Omega_>)=\tfrac{1}{2}\omega_p;\quad Z(\Omega_<)=(\omega_c{}^3/2\omega_p{}^2)\sin\theta\cos^2\theta, \tag{14}$$

whereas for $\omega_c\gg\omega_p$ we have

$$Z(\Omega_>)=(\omega_p{}^2/2\omega_c)\cos^2\theta;\quad Z(\Omega_<)=\tfrac{1}{2}\omega_p\sin\theta. \tag{15}$$

The root $\Omega_{(2\omega_c)}$ has corresponding weight

$$Z(\Omega_{(2\omega_c)})=\frac{p^2\omega_c\cos^4\theta}{4m\omega_p{}^2s_1}\left[\left(\frac{\omega_c{}^2}{\omega_p{}^2}-\tfrac{1}{4}\sin^2\theta-\tfrac{1}{3}\cos^2\theta\right)^2\right]^{-1}. \tag{16}$$

III. SPIN EFFECTS AND THE SPECTRAL COMPOSITION OF DHVA OSCILLATORY TERMS

The effects of distinguishing the spin part of the motion from the orbital part of the motion in the manner indicated above are transmitted to the description of low-wave-number plasmon propagation in a magnetic field through the integrals ρ, σ, α. In the nondegenerate case the relations

$$\sigma=(\tfrac{1}{2}\hbar\omega_c/\tanh\tfrac{1}{2}\hbar\omega_c\beta)\rho, \tag{17a}$$

$$\alpha=(1/\beta)\rho, \tag{17b}$$

are exactly preserved, and so there are no changes arising from the fact that $m\neq m^*$ and $g\neq 1$. The situation is quite different in the degenerate case. Separating ρ, σ, α into branch line and isolated pole contributions as usual,

$$(\rho,\sigma,\alpha)=(\rho_\Gamma,\sigma_\Gamma,\alpha_\Gamma)+\sum_n(\rho_{c_n},\sigma_{c_n},\alpha_{c_n}), \tag{18}$$

one finds that the branch line contributions are approximately,

$$\rho_\Gamma\cong\left(\frac{m}{2\pi}\right)^{3/2}\frac{2}{\Gamma(5/2)}\frac{\zeta^{3/2}}{\hbar^3}, \tag{19a}$$

$$\sigma_\Gamma\cong\left(\frac{m}{2\pi}\right)^{3/2}\frac{2}{\Gamma(7/2)}\frac{\zeta^{5/2}}{\hbar^3}, \tag{19b}$$

$$\alpha_\Gamma\cong\left(\frac{m}{2\pi}\right)^{3/2}\frac{2}{\Gamma(7/2)}\frac{\zeta^{5/2}}{\hbar^3}. \tag{19c}$$

[One should not confuse the subscript Γ on the left-hand side of (19) which refers to the branch line with the Γ function appearing on the right-hand side of (19).] Roughly speaking, this shows that $\sigma\approx\alpha\approx\zeta\rho$ in the degenerate case [whereas $\sigma\approx\alpha\approx(1/\beta)\rho$ in the nondegenerate case]. The isolated pole contributions yield terms that are oscillatory in the de Haas–van Alphen (DHVA) sense,

$$\sum_n\rho_{c_n}=\frac{m^{3/2}(\hbar\omega_c)^{1/2}}{\pi\beta\hbar^3}\sum_{n=1}^{\infty}(-1)^n\frac{\cos[\pi n(2\mu_0H/\hbar\omega_c)]\cos[(2\pi n/\hbar\omega_c)\zeta-3\pi/4]}{n^{1/2}\sinh(2\pi^2n/\hbar\omega_c\beta)}, \tag{20a}$$

$$\sum_n\sigma_{c_n}=\frac{m^{3/2}(\hbar\omega_c)^{3/2}}{2\pi^2\beta\hbar^3}\sum_{n=1}^{\infty}\frac{(-1)^n\cos[\pi n(2\mu_0H/\hbar\omega_c)]}{n^{3/2}\sinh(2\pi^2n/\hbar\omega_c\beta)}\left\{-\left(\tfrac{1}{2}+\frac{2\pi^2n}{\hbar\omega_c\beta}\bigg/\tanh\left(\frac{2\pi^2n}{\hbar\omega_c\beta}\right)\right)\cos\left(\frac{2\pi n}{\hbar\omega_c}\zeta-\frac{5\pi}{4}\right)\right.$$
$$\left.+2\pi n\frac{\zeta}{\hbar\omega_c}\cos\left(\frac{2\pi n}{\hbar\omega_c}\zeta-\frac{3\pi}{4}\right)+\pi n\frac{2\mu_0H}{\hbar\omega_c}\tan\left(\pi n\frac{2\mu_0H}{\hbar\omega_c}\right)\cos\left(\frac{2\pi n}{\hbar\omega_c}\zeta-\frac{\pi}{4}\right)\right\}, \tag{20b}$$

$$\sum_n\alpha_{c_n}=\frac{m^{3/2}(\hbar\omega_c)^{3/2}}{2\pi^2\beta\hbar^3}\sum_{n=1}^{\infty}(-1)^n\frac{\cos[\pi n(2\mu_0H/\hbar\omega_c)]\cos[(2\pi n/\hbar\omega_c)\zeta-5\pi/4]}{n^{3/2}\sinh(2\pi^2n/\hbar\omega_c\beta)}. \tag{20c}$$

These Fourier series representations of the DHVA oscillatory terms explicitly show that the spectral composition depends rather sensitively on the parameter $g(m/m_0)$ through factors such as $\cos[\pi n(2\mu_0 H/\hbar\omega_c)] = \cos[\pi ng(m/m_0)]$. In fact, ρ_{c_n} and α_{c_n} differ from their counterparts in the simpler situation (when $m=m_0$ and $g=1$) just by factors of $(-1)^n \cos[\pi ng(m/m_0)]$; the more complicated nature of σ_{c_n} derives from the fact that these terms arise from second-order poles in the s integration of (8a), whereas the poles responsible for ρ_{c_n} and α_{c_n} are simple poles. The sensitive dependence of DHVA oscillatory terms on the parameter $g(m/m_0)$ is not at all an unfamiliar phenomenon. An account of such behavior in the DHVA oscillatory terms of the magnetic susceptibility may be found in Wilson's book.[8] One may feel encouraged by the fact that such DHVA oscillatory terms should be observable under the same conditions of field strength as the de Haas–van Alphen effect itself is. However, the shortcomings[8] of a simple electron gas theory of the latter in explaining data (even with provision for taking account of effective mass in the orbital part of the motion and an anomalous g factor in the spin part of the motion) may be expected here too. Such shortcomings must be reckoned with in attempting to use the sensitivity of the DHVA oscillatory terms to variations in the parameter $g(m/m_0)$ as a mechanism for measuring the product of the anomalous electronic g factor and effective mass m.

The role of the DHVA oscillatory terms as a whole in the low-wave-number plasma oscillation spectrum is determined by the occurrence of the quantities $s_1=\rho/\sigma$ and $s_2=\rho/\alpha$ in the plasmon dispersion relation, and the ramifications of this with respect to the plasmon modes $\Omega_>$, $\Omega_<$ and $\Omega_{(2\omega_c)}$ have been discussed in the preceding section. It is appropriate at this point to give a few qualitative statements about the natural damping of these modes. (Detailed natural damping formulas and derivations may be found in Ref. 5, and will not be presented here.) The natural damping of $\Omega_>$ and $\Omega_<$ in the degenerate case is exponentially small in a manner that is formally similar to the nondegenerate natural damping expression of Landau, the latter being appropriately modified to take account of the presence of the magnetic field. The same may be said for the natural damping of the mode $\Omega_{(2\omega_c)}$ provided that the direction of propagation is confined to the angular interval $\theta \lesssim (p^2\zeta/m\omega_c^2)^{1/2}$. For propagation outside this angular interval, the natural damping of $\Omega_{(2\omega_c)}$ ceases to be exponentially small, and the increase in natural damping is accompanied by terms that are oscillatory in the DHVA sense, but with a modified DHVA oscillation frequency. [Although the natural damping formulas of Ref. 5 are given for the case $m=m_0$ and $g=1$, it is clear that the spectral composition of the DHVA oscillatory terms in the natural damping will depend sensitively on the parameter $g(m/m_0)$.] It should be noted that DHVA oscillatory terms are completely absent from the natural damping as long as it is exponentially small.

[8] A. H. Wilson, *Theory of Metals* (Cambridge University Press, Cambridge, England, 1953), 2nd ed., pp. 164–175.

IV. CONCLUSIONS

Tractable expressions for the complete dispersion relation [Eqs. (2), (4), and (5)] and damping constant [Eqs. (3), (4), and (5)] for plasmons in a magnetic field have been obtained using a Green's-function formulation of the random-phase approximation. These general results are valid for all magnetic field strengths as well as all values of temperature, and are applicable to both the degenerate and nondegenerate cases. Moreover, they are not encumbered by unwieldy summations over Landau eigenstates. The dispersion relation has been investigated in the long-wavelength approximation in detail, with special attention given to determining the role of de Haas–van Alphen (DHVA) oscillatory terms in the degenerate case. In the long-wavelength limit, ($p=0$), the usual result of two plasma modes is confirmed here exactly [Eq. (9)]. These two modes are shifted to order p^2 by terms that are oscillatory in the DHVA sense in the degenerate case [Eq. (10)]. Another plasma resonance in the vicinity of $2\omega_c$, which is active to order p^2 [Eq. (16)], exhibits such DHVA oscillatory behavior [Eq. (12)], and the same may be said for a gap in the frequency spectrum for propagation perpendicular to the magnetic field in the interval $[\omega_c, 2\omega_c]$ [Eq. (13)]. The distinction between the masses associated with the orbital and spin parts of the individual electronic motions in the presence of the lattice, and the anomalous electronic g factor, give rise to a special sensitivity of the DHVA oscillatory terms. A representation of the latter in terms of an appropriate Fourier series [which is valid for all magnetic field strengths and temperatures as long as the condition of degeneracy is fulfilled $\zeta\beta \gg 1$; that is to say that no restriction is placed on $\hbar\omega_c\beta$; Eq. (20)], explicitly shows that the spectral composition of the DHVA oscillatory terms is a sensitive function of $g(m/m_0)$. This is manifested by the appearance of factors such as $\cos[\pi ng(m/m_0)]$ in the Fourier coefficients, and the small shift in $g(m/m_0)$ from integral to half-integral values produces significant changes in the spectral composition.

It has already been pointed out[4] that one may hope to observe effects of the type discussed here in n-type InAs and InSb, since the properties of these materials are in accord with the conditions under which the present analysis is valid. Specifically, the considerations taken here are valid only as long as a "one band" description is reasonable. Therefore, it is at least necessary that the plasmon energy $\hbar\omega_p$ be too small to excite interband transitions, $\hbar\omega_p \ll$(energy gap). Moreover, the collision time τ must be large compared to ω_c^{-1} as well as ω_p^{-1} so that the free and collective aspects of the electronic motions, which are an integral part of the description given by the random-phase approximation, are not destroyed by collisions. In addition, it should be noted that the results given by Eq. (20) indicate that the DHVA oscillatory terms arising from the Landau quantization of orbits are most strongly felt when $\hbar\omega_c\beta \gtrsim 1$. Finally, one must bear in mind that the validity of the random-phase approximation is restricted to long wavelengths in the sense that $p^2\zeta/m\omega_p^2 \ll 1$, (degenerate case). The long-wavelength approximation as it is formulated here, in the presence of a magnetic field, also requires that $p^2\zeta/m\omega_c^2 \ll 1$.

The results here show that the suggestion of Gartenhaus and Stranahan,[4] concerning the possibility of determining the product of anomalous electronic g factor and effective mass m by using the sensitivity of DHVA oscillatory terms associated with plasmon phenomena to order p^0 (zero wave number), can be considered a meaningful scheme only if one takes account of plasmon phenomena to order p^2 rather than p^0. This will be the case for reflection experiments as well as any other plasmon excitation experiments (fast particle energy loss, etc.). The implication is that the effects will be small in the same sense that p^2 is small. In the presence of a magnetic field, "small p^2" has different meanings in connection with different phenomena, and while p^2 must be small in all possible senses it may be larger in some contexts than others. For example when $\omega_p > \omega_c$ the p^2 shifts of the roots $\Omega_{\gtrless}$, $(\Omega_{0>}{}^2 = \omega_p^2 + \cos^2\theta\omega_c^2$; $\Omega_{0<}{}^2 = \sin^2\theta\omega_c^2)$, may be estimated as

$$\left|\frac{\Omega_> - \Omega_{0>}}{\Omega_{0>}}\right| \approx \frac{p^2\zeta\omega_p^4}{m\Omega_{0>}{}^6} \approx \frac{p^2\zeta}{m\omega_p^2},$$

$$\left|\frac{\Omega_< - \Omega_{0<}}{\Omega_{0<}}\right| \approx \frac{p^2\zeta\omega_p^4}{m\Omega_{0<}{}^6}\frac{\omega_c^2}{\omega_p^2} \approx \frac{p^2\zeta}{m\omega_p^2}\frac{\omega_p^4}{\omega_c^4}.$$

While the shift of $\Omega_>$ is governed by the small parameter $p^2\zeta/m\omega_p^2$, the shift of $\Omega_<$ is governed by the parameter $(p^2\zeta/m\omega_p^2)(\omega_p^4/\omega_c^4)$ which must still be small but is larger than $p^2\zeta/m\omega_p^2$ by a factor $\omega_p^4/\omega_c^4 \sim 100$ for $\omega_p^2 \sim 10\omega_c^2$. (It is not advisable to consider magnetic field strengths for which ω_c is very small because the amplitude for exciting $\Omega_{0<}$ would then be very small; moreover, the magnetic field must be sufficiently large that $p^2\zeta/m\omega_c^2 < 1$.) The plasma resonance in the vicinity of $2\omega_c$ may be estimated as

$$\left|\frac{\Omega_{(2\omega_c)} - 2\omega_c}{2\omega_c}\right| \approx \frac{p^2\zeta}{m\omega_p^2}\frac{\omega_p^2}{\omega_c^2},$$

and the gap in the frequency interval $[\omega_c, 2\omega_c]$ for propagation perpendicular to the magnetic field may be estimated as

$$\left|\frac{\Omega_{[\omega_c, 2\omega_c]} - 2\omega_c}{2\omega_c}\right| \approx \frac{p^2\zeta}{m\omega_p^2}\frac{\omega_p^2}{\omega_c^2}.$$

Again, the "small p^2" parameter is somewhat larger than $p^2\zeta/m\omega_p^2$.

The formulas which we have presented here show that the oscillatory parts of ρ, σ, α are small compared to the branch line contributions, since the latter are given by

$$\rho_\Gamma \sim (m^{3/2}/\hbar^3)\zeta^{3/2},$$
$$\sigma_\Gamma \sim (m^{3/2}/\hbar^3)\zeta^{5/2},$$
$$\alpha_\Gamma \sim (m^{3/2}/\hbar^3)\zeta^{5/2},$$

whereas the oscillatory parts are given by $(\hbar\omega_c\beta \gg 1)$,

$$\rho_{c_n} \sim (m^{3/2}/\hbar^3)(\hbar\omega_c)^{3/2},$$
$$\sigma_{c_n} \sim (m^{3/2}/\hbar^3)(\hbar\omega_c)^{5/2}(1+\zeta/\hbar\omega_c),$$
$$\alpha_{c_n} \sim (m^{3/2}/\hbar^3)(\hbar\omega_c)^{5/2}.$$

These results for the branch-line contribution obtain under the condition $\hbar\omega_c \ll \zeta$, whereas the results for the oscillatory parts are free of this restriction. The smallness of oscillatory effects which was anticipated in Ref. 3, on the basis of the fact that the energies $T_{11} = 1/2s_2 = \alpha/2\rho$ and $W = 1/s_1 = \sigma/\rho$ contain terms which are independent of or linear in H (which do not appear in the susceptibility), is thus explicitly demonstrated under the condition $\hbar\omega_c \ll \zeta$. However, the oscillatory terms can be of greater importance in the physically realizable quantum strong field situation, when $\hbar\omega_c \sim \zeta$. The estimates for the oscillatory parts given above are still reasonable, but one must reconsider the branch-line contributions. An estimate of the latter may be obtained by inspecting the p^2 terms of Ref. 5, Eq. (AIII.6), [or equivalently the $n=0$ term of Ref. 3, Eq. (3.21)]. This results in

$$\rho_\Gamma \sim (m^{3/2}/\hbar^3)\hbar\omega_c\zeta^{1/2},$$
$$\sigma_\Gamma \sim (m^{3/2}/\hbar^3)(\hbar\omega_c)^2\zeta^{1/2},$$
$$\alpha_\Gamma \sim (m^{3/2}/\hbar^3)\hbar\omega_c\zeta^{3/2},$$

and in view of the fact that $\hbar\omega_c \sim \zeta$ the oscillatory parts are of comparable importance in the quantum strong field situation. This is simply to say that the presence of terms in the energies T_{11} and W which are independent of or linear in H cannot be taken to mean that the oscillatory parts are small when $\hbar\omega_c \sim \zeta$.