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Periodic Discontinuities in the Drift Velocity of Ions in Liquid Helium II*

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The drift velocity of positive and negative ions in liquid helium around 1°K has been studied by the time-of-flight method as a function of the applied field. Sharp and periodic discontinuities have been detected at values which are integer multiples of $\langle v_c \rangle = 5.2 \pm 0.2$ m/sec for positive ions, and $\langle v_c \rangle = 2.4 \pm 0.1$ m/sec for negative ones. Metastable mobility levels induced by bath disturbances have also been detected. Analysis of these results, together with the ones obtained by the heat-flush method, shows this process to be due to single ions which suffer additional interaction with the roton gas when their drift velocity relative to the background fluid reaches the observed critical velocities. A tentative model is proposed which explains these results by assuming vortex rings of different quantum numbers of circulation to be closely bound to the ion in motion, which interact with the roton gas via their velocity field.

1. INTRODUCTION

THE mobility of positive and negative charges (in the following briefly called "ions")[1] in liquid helium II has been investigated by several authors over a wide range of temperature[2] and pressure.[3] Their results are essentially consistent and can be well expressed in terms of the mean free path of the ions in the gas of excitations.

However, about two years ago, a careful study[4] of the field dependence of the mobility of moderately hot ions was performed in this laboratory, and some unexpected sharp discontinuities were detected at about 1°K. This first study was made by the time-of-flight method, but a further study by the heat-flush method gave quite similar results.[5] It was difficult to account for this behavior in the framework of the Landau picture of liquid helium, and therefore the time-of-flight method has been extended to negative ions and to a wider range of experimental conditions. The results of this experimental study are reported below in Sec. 3 and are discussed in a phenomenological way in Sec. 4. A tentative explanation of the observed phenomena is finally given in Sec. 5, using the same arguments and concepts as are currently widely used to explain the critical velocities in liquid helium. In this tentative explanation we will also make use of the experimental results obtained by the heat-flush method.[5]

2. EXPERIMENTAL TECHNIQUE

The apparatus to measure the drift velocity makes use of a time-of-flight method, which has been already described by one of us.[6] The drift velocity is measured by the experimental determination of the time t taken by the ions to traverse a distance d from a grid to a receiving electrode.

Different ionic sources have been employed, which gave different ionic densities of the beam in the range 7×10^4–5×10^5 ion/cm³. Different electrode materials (but usually silver or gold-plated silver) have been used. The grid-to-collector distance d was usually 6 mm, and was varied sometimes to 8 mm without affecting the results. A large number of cryogenic details were

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[1] For a general review of this topic see, for instance, G. Careri, in *Progress in Low-Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1961), Vol. 3, pp. 58–79.

[2] F. Reif and L. Meyer, Phys. Rev. **119**, 1164 (1960); L. Meyer and F. Reif, Phys. Rev. Letters **5**, 1 (1960).

[3] S. Cunsolo and P. Mazzoldi, Nuovo Cimento **20**, 949 (1961). L. Meyer and F. Reif, Phys. Rev. **123**, 727 (1961)

[4] G. Careri, S. Cunsolo, and P. Mazzoldi, Phys. Rev. Letters **7**, 151 (1961) later referred to as (I).

[5] G. Careri, S. Cunsolo, and M. Vicentini-Missoni, in *Proceedings of the Eighth International Conference on Low-Temperature Physics, London, 1962* (Butterworths Scientific Publications, Ltd., London, 1963), p. 218.

[6] S. Cunsolo, Nuovo Cimento **21**, 76 (1961).

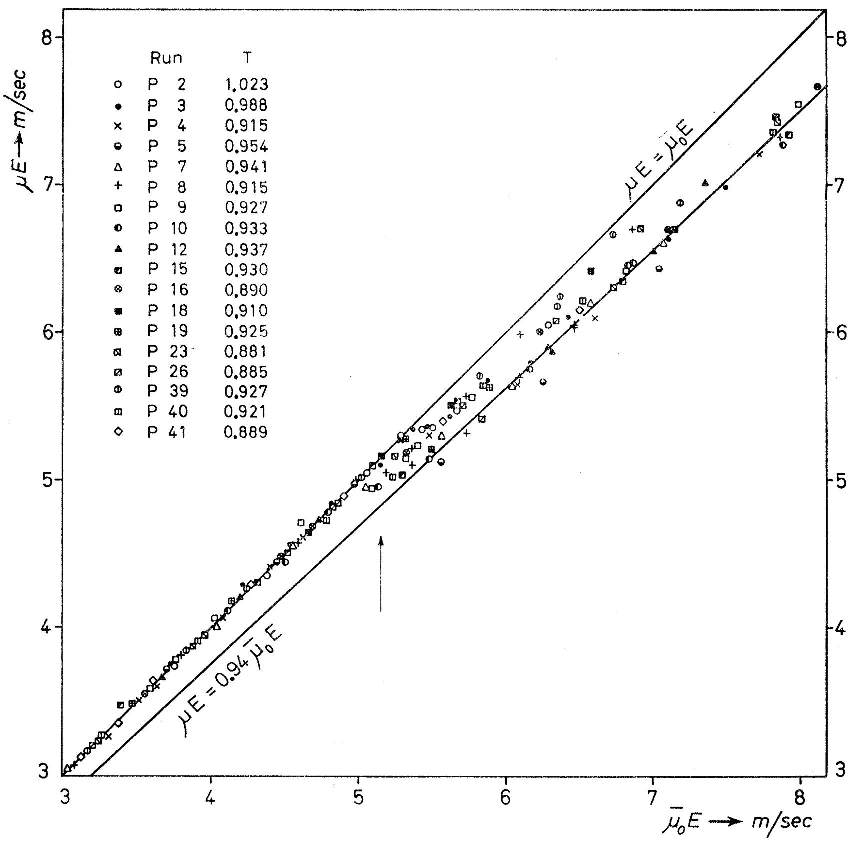

FIG. 1. A plot of the measured drift velocity μE versus the value $\mu_0 E$, averaged in each run for the low-field data. Different symbols refer to the different runs at the temperature indicated. From this plot the first discontinuity is estimated at $V_c = 5.2 \pm 0.2$ m/sec. The higher field data clusters around the lower mobility $\bar{\mu}_1 = 0.94\,\bar{\mu}$ with the exception of some data affected by metastability or taken at the highest temperature. The average value of the discontinuity is indicated by the arrow.

changed from time to time, but none was found to be of importance with the exception of the thermal disturbances in the helium bath. This influence will be reported later in Sec. 3.

Usually an entire helium run was needed to study the field dependence of the drift velocity at a single temperature. Since the mobility changes to be detected were less than 5–10%, it was necessary to keep the temperature constant within 10^{-3}°K during the whole run in order to have an accuracy of $\pm 0.5\%$ in the data taken in that particular run. This was achieved by an electronic thermoregulator, built in this laboratory, which is very similar to the one described by Edwards.[7]

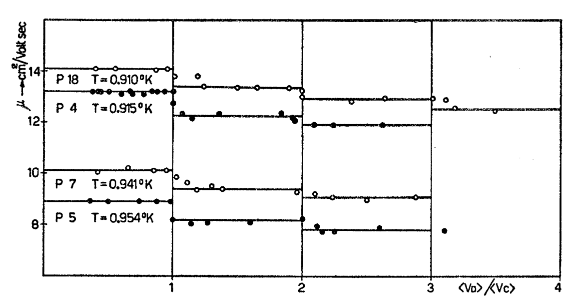

FIG. 2. Plot of the mobility μ of the positive ions versus the reduced drift velocity $\langle V\rangle/\langle V_0\rangle$ in four different runs, to show the periodic character of the discontinuities. Full lines are the supposed mobility levels.

Temperature fluctuations and other statistical errors affected one single drift-velocity determination by no more than $\pm 2\%$; usually more experimental determinations were made to reduce this uncertainty to $\pm 1\%$ by proper statistical analysis. The number of data taken in every run is a compromise between the necessity to have accurate data to determine the position of the discontinuities and the desire to explore a wider range of electric fields.

Systematic sources of error in the absolute value of the drift velocity are due to uncertainties in the time-of-flight measurement, and in the value of the electrode separation, which is known with an accuracy of $\pm 1.5\%$. The time is obtained by extrapolating the current-versus-square-wave-frequency plot[6] to zero, the frequency itself being directly read on the knob of the pulse generator. In the first runs reported in I a Tektronix Type 161 pulse generator was used, but a later comparison by means of a Tektronix Type 545 oscilloscope and a Philips GM 2314 pulse generator showed that its frequency-scale calibration was wrong. As a result of this systematic error, all the drift velocity and mobility data reported in I must be multiplied by a

[7] D. O. Edwards, thesis, University of Oxford, 1959 (unpublished).

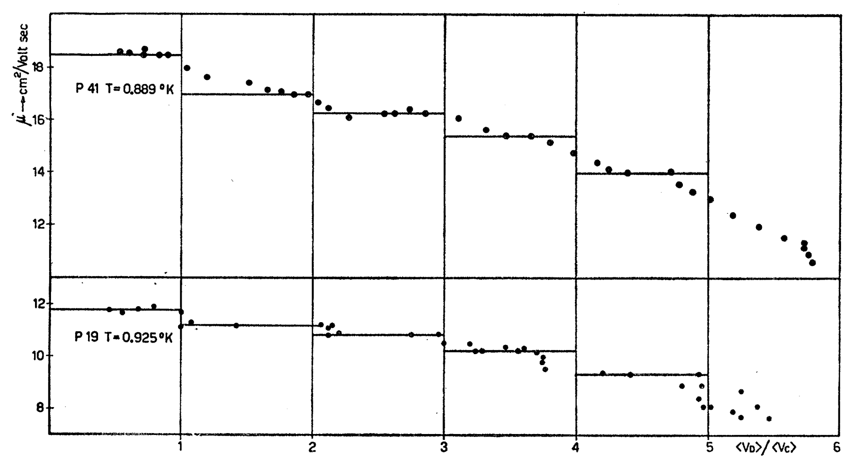

FIG. 3. A plot similar to the one of Fig. 2 for two runs in an extended velocity range. Note the impossibility of defining the mobility level after the fifth discontinuity.

constant factor 1.10. At present we believe the absolute values of the frequency to be known to better than $\pm 0.5\%$, so that the total systematic error in the absolute value of the drift velocity is not larger than $\pm 2\%$.

A further systematic error is due to the uncertainty in the absolute value of the amplitude of the square wave, but this error only affects the electric field and the mobility. During the measurements the wave shape was continuously monitored by an oscilloscope, and its amplitude was measured by means of a Brüel-Kjaer type 2409 electronic voltmeter with an accuracy of about 1%. The total systematic error in the electric-field determination thus rises to 2.5%.

The total systematic error affecting the mobility is therefore evaluated as 4.5%. Within this range our absolute mobility values were reproducible in different apparatuses and by different techniques[1,6]; they were also in good agreement with the data of Reif and Meyer.[2]

It must be emphasized, however, that these systematic errors affect by a common factor all the mobility data taken in the same run, and therefore they do not affect the mobility changes induced by the electronic field in a single run, which is the important feature to be studied. Moreover, when the mobility is field-independent over some field range (its value in this range will be later referred to as the "mobility level"), the error within each level can be reduced by a proper statistical analysis. Thus, the narrow gap $\Delta\mu$ between two mobility levels can be obtained with greater accuracy. Finally, we stress that the relative values of the gap are also not affected by systematic errors in a single run.

3. EXPERIMENTAL RESULTS

In the following we give the results of about 40 runs. Each run, as explained in Sec. 2, was devoted to studying the field dependence of the drift velocity at one particular temperature, which was kept constant during that run. Several common features have been noticed in these runs, and their significance will be discussed later in Secs. 4 and 5. In the following we merely list the important features observed:

(a) As already mentioned in I, the most striking aspect of the mobility-versus-field curves is the abrupt change of the mobility from one level to a lower one at a particular value of the drift velocity.

Let us initially examine the first of these discontinuities that appears on increasing the electric field E, and denote by μ, and μ_2, respectively, the two mobility levels before and after the discontinuity. A good example of this behavior was given in Fig. 1 of I, where μ was plotted versus E. Here we report in Fig. 1 the results of a great number of runs to show the consistency of all data taken in different apparatuses and at different temperatures. The coordinates in Fig. 1 have been chosen to reduce the effect of accidental errors on the drift velocity $\langle V_D \rangle$. The presence of two mobility levels is certain; however, the value of the lower mobility μ_1 seems to vary somewhat from run to run, as will be seen later in more detail. The absolute value of the critical velocity $\langle V_c \rangle$ from this plot is estimated to be $\langle V_c \rangle = 5.15 \pm 0.20$ m/sec.

Although the value of the first critical drift velocity is temperature-independent, it matters in practice what the absolute temperature is, since from Fig. 1 it is evident that at increasing temperatures it becomes progressively harder to detect the decreasing gap between the mobility levels. Furthermore, at low temperatures (less than 0.900°K) the flat portion of the mobility level after the transition is less well defined, as shown later in Fig. 5 for positive ions. Therefore, our study has been mainly concentrated in the temperature range 0.88–1.0°K where the phenomenon is easier to observe.

(b) In some runs the electric field was raised to higher values, and new mobility levels appeared at drift velocities which were nearly multiples of the first one. Figure 2 of I represented a first example of this behavior, and Figs. 2 and 3 of this paper show this periodicity in six runs.

An interesting question is the behavior of the drift

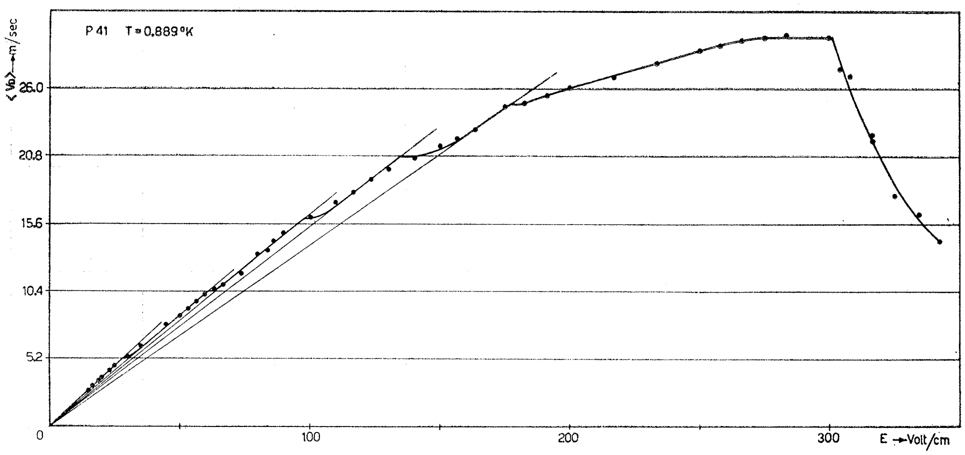

FIG. 4. The drift velocity $\langle V_D \rangle$ of positive ions plotted versus the electric field E in one single run (partly shown also in Fig. 3). Solid line is the best fit curve through data; straight lines indicate the different mobility levels. Note the new behavior of the drift velocity at the highest fields.

velocity at extremely high fields. This work is still in progress in this laboratory, and in Fig. 4 we report only the result of one run. From this run it seems that after the fifth discontinuity a different process sets in abruptly, in which the ions display a negative resistance. Since at the present time this phenomenon seems not to be related to the periodic character of the discontinuities, it will not be further considered in this paper.

In Fig. 5 we plot also two runs which apparently contradict the simple periodicity law expressed above. These runs, together with other apparent anomalies, will be considered in the following.

(c) We want now to mention a feature that was sometimes present at drift velocities larger than the critical one, and that we call "metastability." By this word we mean that sometimes, after the critical field was exceeded, the mobility remained for a while at a value corresponding to the previous level, as if the process that normally lowers the mobility had a kind of hindrance to its initiation.

In one single case this metastability lasted only for a few minutes and was not due to any apparent experimental reason. Usually the metastability was connected with the presence of thermal disturbances in the helium bath and persisted for a much longer time.

Because of the occurrence of this metastability, the precise point of the transition in these cases was more difficult to detect; nevertheless none of these points has been rejected in Figs. 1–5. As an example we show in Fig. 6 the region around the first discontinuity in run P39. The circles are the measurements taken during a period of bath disturbances due to some wrong operation of the thermoregulator, and the black points are the measurements taken after the thermoregulator had been switched off and the temperature was well controlled by hand. There is clear evidence for the presence of metastability (which lasted for about one hour) which only affected the measurements above the critical field, and which is certainly due to large bath disturbances.

We conclude by saying that thermal disturbances induce a long-lifetime metastability, and that most probably uncontrolled disturbances induce the shorter-lifetime metastability observed, sometimes without any

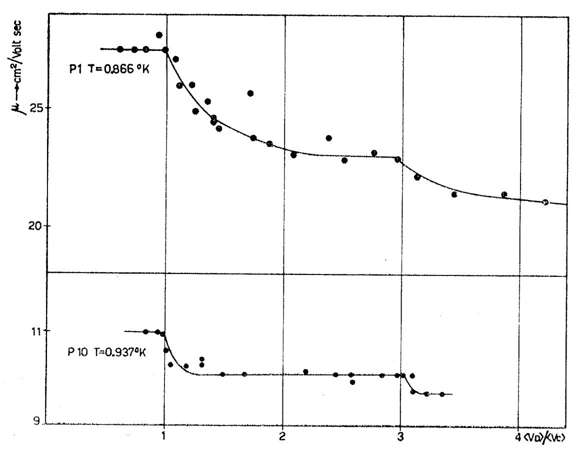

FIG. 5. A plot similar to the one of Fig. 2 for two runs which do not display the second discontinuity. Note also the different shape of the two curves, the one at lower temperature being considerably smoother.

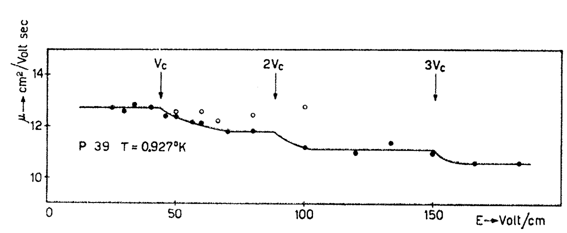

FIG. 6. The mobility μ of positive ions plotted versus the electric field E in one run. White circles represent the measurements taken during bath disturbances, which point to the existence of an upper metastable level. The curve which best fits the stable condition is indicated, and the discontinuities are shown by arrows.

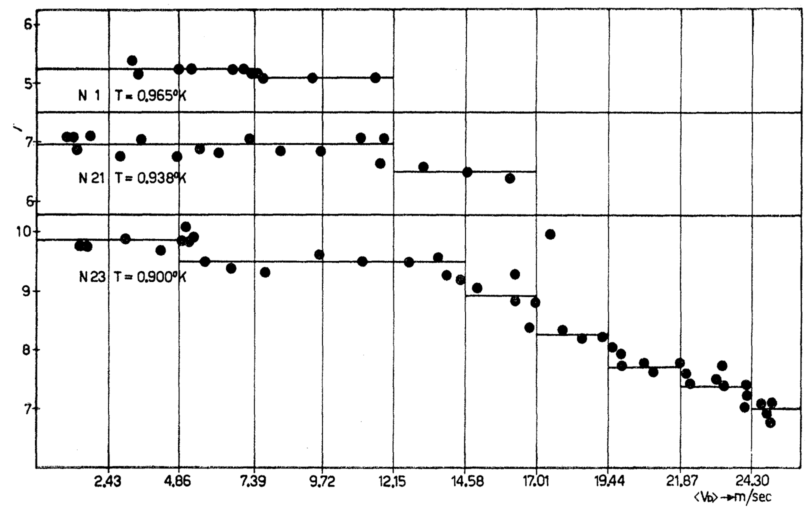

FIG. 7. Plot of the mobility μ versus drift velocity $\langle V\rangle$ of negative ions in three different runs. Note the rather large scatter of the experimental data and the absence of the first discontinuity at 2.43 m/sec, as well as other discontinuities.

apparent reason. In this way we explain the anomalous behavior of the runs of Fig. 5, where the second discontinuity is omitted.

(d) After this digression on metastability, we come back to the question of the mobility gap $\Delta\mu$ between two adjacent mobility levels. As one can notice in Figs. 1 and 3, this quantity seems to vary somewhat from one discontinuity to another in the same run and also among the different runs, but its relative value $\Delta\mu/\mu$ is always of the same order of magnitude and close to 6%. We believe the observed values of $\Delta\mu/\mu$ to be affected by some partial hindrance which in the case of the metastability discussed above, entirely prevented the onset of the phenomenon.

The anomalously large value of the fourth mobility gap displayed in the run P19 of Fig. 5, can also be explained by saying that the process of lowering the mobility was incomplete during the first three discontinuities. Then the group of ions in the metastable upper level reached the drift velocity corresponding to the fourth discontinuity at a field lower than the one adequate for those of the true level, and the jump in the mobility was therefore larger than in the previous discontinuities.

(e) Experiments quite similar to the ones reported above have been performed using negative ions as well. After many runs we arrived at the conclusion that critical velocities which are integer multiples of the quantity 2.43±0.07 m/sec can be observed at different temperatures, but that the process is now very much affected by metastability. As examples we report in Fig. 7 the results of three runs, N1, N21, and N23. Run N23 shows clearly many discontinuities which are equally spaced, while run N21 shows only one discontinuity which is identified as the fifth one, and N1 only one which is identified as the third one. Under no experimental conditions have we been able to observe the first discontinuity at 2.43 m/sec.

4. GENERAL DISCUSSION

In this section we aim to discuss the above reported results together with the results obtained by the heat-flush effect[5] in order to derive some general conclusions about the physical features of the process without reference to any particular model. We first list the partial conclusions which can be inferred from separate aspects of the experiments, and then we will summarize the relevant conslusions.

(a) The transition at the value of the critical velocity could not be sharp, unless the thermal Maxwellian distribution of the ions either is of comparable narrowness to that of the transition region or does not play any role. In the first case the ions would have to have an effective mass M larger than 10^4 helium masses, in order that their average thermal velocity be of the order of a few cm/sec; this is hardly believable, in view of the current ideas about the structure of the ions in the fluid. One could also propose that many different ions cooperate as a single unit of larger mass, but this is also difficult to understand if the independence of beam density is recalled. Moreover, a particle of such high mass should exibit a continuous mobility decrease at fields larger than $\frac{1}{10}E_c$.[8] The second and more realistic possibility is that the process leading to the abrupt transition requires a relaxation time much longer than the thermal relaxation time of the ions, so that only their drift velocity is important and not the effective velocity, which is affected by the Maxwellian distribution and which would thus cause the transition to be broadened. We believe this to be the explanation.

(b) The lack of dependence of the critical velocity on temperature suggests that the threshold for this process is not connected with the density of excitations, but only with the background fluid (the superfluid). Moreover, the lack of dependence of the critical velocity

[8] G. H. Wannier, Bell System Tech. J. 32, 170 (1953).

on the geometry of the apparatus suggests that this phenomenon takes place in the bulk liquid, during the flight of the ion between the electrodes.

(c) The different values of the critical velocity for positive and negative ions suggest that we are not dealing with an excitation characteristic only of the bulk liquid, but that these excitations (if they are produced at all) must be a characteristic of the ion-fluid system.

(d) The metastability of the transition and its periodic character suggest that this process is not the analog of the classical transition from laminar to turbulent flow. As a matter of fact, while in the classical case any external disturbance helps the transition to take place, here on the contrary, external disturbances induce metastability, i.e., they prevent the process from taking place. Furthermore, the periodic character of the transition has no known classical equivalent in the behavior of an object which moves in a fluid.

(e) The lack of reproducibility in the mobility changes observed in different runs taken at the same temperature suggests that a variable number of ions of the beam can take part in this process; some of these are prevented from participating in the process in which each ion acts independently. This conclusion is supported also by the continuous variation of the mobility after the discontinuity, shown for example in Fig. 5 by run P1.

(f) To all the above we must add an important result from the heat-flush effect,[5,8] namely that the *new* dissipation which sets in after the discontinuity is due mainly to the interaction of the ion with the normal fluid.

On the above grounds we believe that the general features of this phenomenon can be conveniently summarized as follows: Single ions, at particular values of their drift velocity relative to the superfluid, fall into a new flow regime characterized by a different interaction with the normal fluid. The transitions from one regime to another one are periodic; the period depends on the nature of the ion and is such that a relaxation time long in comparison with the thermal relaxation time is required. In analogy with the conclusions reached by Meservey[10] on the flow instabilities in liquid helium, we can also think of the dissipation of the ion in the roton sea is always of viscous character, but that the flow pattern changes in the different regions of the drift velocity of the ion relative to the background fluid.

5. A TENTATIVE MODEL

We will now put forward a tentative explanation which uses concepts already familiar in the description of liquid helium. We propose that the essential feature of the strange behavior of the single ions is due to the formation of quantized vortex rings,[11,12] and that a vortex ring remains closely bound to the ion in motion while interacting with the roton sea, thus giving rise to a new source of dissipation. This picture will be qualitatively discussed in the following:

It has long been realized[4] that the creation of one quantized vortex ring of nearly atomic size as the first quantum of circulation would provide an explanation for the observed magnitude of the critical value of $\langle V_D \rangle$, if one makes use of the Feynman[12] expressions for the vortex rings. Moreover, these expressions can also account for the observed multiplicity in terms of different quantum numbers of circulation. A difficulty was noticed, however, in the creation process of the vortex rings. In order to satisfy the Landau criterium of creation of an excitation of momentum p and energy ϵ, the drift velocity V_D must be[2]

$$V_D = (\epsilon/p) + (p/2M). \quad (1)$$

This condition requires for the mass, M, a value greater than 10^2 times the expected one.

However, if we allow the creation process to require a long relaxation time,[13] or else if the bath fluctuations are responsible for the creation, Eq. (1) does not need to be satisfied any more. We then imagine the vortex ring, once created, to remain closely bound around the ion and this entity to interact with the roton sea, giving rise to a new source of dissipation. To calculate this new interaction, we must first evaluate the size of the vortex ring. The Lamb expression [14] for the vortex-ring drift velocity requires a radius of about 100 Å for the vortex ring with the single quantum of circulation drifting at 5 m/sec. This value is reasonable, if we think of the positive ion as being enveloped by a cloud of high-density fluid; the latter also prevents the radius from being reduced at higher drift velocities.

Using the Lifshitz-Pitaevskii[15] expression for the Gorter-Mellink force due to roton scattering by a vortex filament of 600-Å length, one obtains an interaction between the ion and the rotons of the order of magnitude observed in the mobility changes at the discontinuity—if one assumes that only a fraction of the ions is subjected to this process, as the metastability suggests. Details of this simple calculation are omitted here, and are given in the Appendix. It is also seen that at increasing quantum numbers of vorticity a corresponding higher

[9] G. Careri, S. Cunsolo, and M. Vicentini-Missoni (to be published).

[10] R. Meservey, Phys. Rev. **127**, 995 (1962).

[11] L. Onsager, Nuovo Cimento **6**, Suppl. 2, 249 (1949).

[12] R. P. Feynman, in *Progress in Low-Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1955), Vol. 1, pp. 58.

[13] G. Careri, S. Cunsolo, P. Mazzoldi, and M. Vicentini-Missoni, in *Proceedings of the NATO 1963* (W. A. Benjamin Company, Inc., New York, to be published).

[14] H. Lamb, *Hydrodynamics* (Dover Publications, Inc., New York, 1945), 6th ed. p. 241.

[15] E. M. Lifshitz and L. P. Pitaevskii, Zh. Eksperim. i Teor. Fiz. **33**, 535 (1959) [English transl.: Soviet Phys.—JETP **6**, 418 (1958)].

dissipation occurs, as actually observed. Moreover, this new interaction is of decreasing importance at increasing temperature in the temperature range so far investigated, as also experimentally found.

We notice also that all the foregoing calculations give the same result if instead of one single vortex ring in the nth quantum of circulation, one allows n different rings of the same radius and of unit quantum of circulation to follow the ion in motion as a sort of wake. This alternative model requires a rather sophisticated flow pattern to explain the appearance of new vortex rings at the right multiples of the critical velocity. Moreover, from the experimental point of view it seems difficult to account for the two runs P1 and P10 with this model, because one would have difficulties in understanding why the metastability should disappear right at the third value of $\langle V_c \rangle$ rather than at some arbitrary place. A more natural explanation is offered by the formation of one single vortex of the requisite quantum number, as we said above.

It seems to us that the above interpretation can offer a simple explanation of the observed phenomena in the framework of the present picture of liquid helium, apart from the creation process. This interpretation is quite similar to the one generally accepted for the existence of critical velocities in capillaries and for the related dissipative effects, and is given in a recent paper by Meservey.[10] The main difference is that in capillaries the vortex lines are of larger dimensions and therefore they easily break up into turbulence, while in our case the positive ion is capable of maintaining the vortex ring at a constant radius around it, most probably because of its small dimension and the peculiar pressure field which exists around the ion.

However, in our opinion the above-outlined explanations suffer from the following uncertainties which prevent us from claiming the definite existence of the multiply quantized vortex rings:

(a) the validity of the Lamb[14] expressions for boundring vortices is assumed;

(b) the stability conditions for the ion-vortex ring system in motion are not investigated;

(d) the creation process of the vortex ring is obscure. The possibility that a collective effect among the excitations can give rise to a vortex line has already been suggested by Vinen.[16] Another possibility[17] is that in apparently undisturbed helium vortex lines of small length nevertheless are always present, which act as nuclei for the growth of longer lines when the superfluid velocity exceeds a certain value. Actually, the creation process of the vortex line is not yet understood by theory, either in the high- or in the low-temperature region.

We believe it to be important to emphasize the above uncertainties, because recently Rayfield and Reif[18] have described experiments in which they claim to have obtained good evidence for the creation of quantized vortex rings by ions in liquid helium. The experiments of Rayfield and Reif were also conducted by a time-of-flight method and employed both positive and negative ions but no discontinuities were observed in the drift velocities. Their conditions, however, were different from ours, because they worked with more energetic ions and in a lower temperature range, 0.3–0.6°K. The interpretation Rayfield and Reif give of their own experiments is based on the possibility of the creation by the ion of one roton directly from the superfluid. This roton would then convert into a vortex ring and follow the ion closely, while keeping the same, single quantum of circulation. It is clear that neither the experimental data nor the interpretations are in contradiction, since they refer to different temperature regions. However, it is also clear that the above-quoted uncertainties (a) and (b) are still applicable to their interpretation, and moreover, in the creation process proposed by Rayfield and Reif the strong backflow velocity field around the ion in motion may appreciably change the value of the Landau critical velocity for creating the initial roton.

6. CONCLUSION

We believe the above-reported behavior of the drift velocity of ions in liquid helium to offer a unique example of hydrodynamical motion which displays the periodic behavior typical of quantization. The microscopic size of the ion and the possibility of an external control of this probe make it possible to observe the discontinuities which in macroscopic experiments are probably smoothed out. The explanation of this phenomenon given in Sec. 4 in term of vortex rings of different quanta of circulation seems to fit into the present picture of liquid helium, but should be put on a firmer ground by a proper theoretical treatment of the vortex–ring–ion system in motion.

Note added in proof. Since this paper was written, we have extended these experiments to higher applied fields with results similar to the one indicated in the Fig. 4 of this paper. These new results will be published in the Proceedings of the Ninth International Conference on Low Temperature Physics, Columbus, Ohio.

According to our present interpretation, during the periodic discontinuities reported in this paper, an atomic-scale quantized vorticity is built close to the ion, a process accounted for by a strictly quantum hydrodynamical theory and not by a semiclassical treatment, which can be used only as a very crude approximation.

[16] W. F. Vinen, *Progress in Low-Temperature Physics* (Interscience Publishers, Inc., New York, 1961), Vol. 3, pp. 1. See particularly p. 40.

[17] W. F. Vinen, *Proceedings of the International School of Physics, Varenna 1961* (Academic Press Inc., New York, 1963), pp. 336.

[18] G. W. Rayfield and F. Reif, Phys. Rev. Letters **11**, 305 (1963).

Instead, during the hydrodynamical regime which sets in at the higher applied fields and is characterized by the negative resistance, we believe one quantum of vorticity becomes detached from the ion and increases its size by a mechanism similar to that described by Rayfield and Reif. In this latter regime the semiclassical treatment is quite successful because of the large size of the vortex rings.

APPENDIX: THE MOBILITY OF THE ION-VORTEX-RING SYSTEM

We want to evaluate the fractional forces acting on a vortex ring which moves with a drift velocity V_L in a roton gas which has a local average velocity V_R; both velocities are relative to the walls. According to Hall and Vinen,[19] the force acting on a unit length of a vortex filament by the symmetrical scattering of the rotons is

$$f=D(V_R-V_L).$$

According to Lifshitz and Pitaevskii[15]

$$D=1.2\chi\rho_n(M_0/KT)^{1/2}p_0^{-1},$$

where $\chi=h/m$ is the quantum of vorticity. M_0 and p_0 are the effective mass and momentum of the roton. The nonsymmetrical scattering is not considered here because after bending the vortex line into a circle to form the vortex ring we are interested only in the friction along the ring axis.

We first notice that the influence of the vortex filament on the roton distribution at the distance of the roton free path λ is negligible, because the roton energy in the velocity field generated by the vortex line is too small compared to the thermal energy, that is $p_0\chi\lambda^{-1} \ll KT$ in our temperature range. Therefore we put $V_R=0$ and the quantity $V_L=\mu^{(n)}E=V_D$ is simply the drift velocity of the vortex lines relative to the walls. Then for a length l of a vortex line with n quanta of circulation at equilibrium we calculate

$$\mu^{(n)}=eV_D/f=(nDl)^{-1}e$$
$$=2.2\times10^{-8}(\ln\rho_n)^{-1}T^{-1/2}\text{cm}^2/\text{V sec},$$

where e is the electric charge.

We evaluate the magnitude of l from the condition $l=2\pi r$, where r is the radius of the vortex ring. Assuming the validity of the Lamb[14] expression

$$V=(\chi/4\pi r)[\ln(8r/a)-\tfrac{1}{4}],$$

and taking[12] for the core radius of the vortex ring $a=1.2$ Å, our observed critical velocities for the positive and negative ions yield values of $r=95$ Å and 200 Å, respectively. These figures are not unrealistic, because a cloud of high-density fluid must surround the electric charge; furthermore, the vortex line must keep away from this high-density region around the core, otherwise it would increase its zero-point energy.

The friction of the moving ion arises partly from its interaction with the roton sea as a bare ion (we suppose this interaction to remain the same as the one responsible for its subcritical mobility μ_0), and partly from its interaction with the roton sea via the velocity field generated by the vortex ring with n circulation quanta, as we just calculated. Then the mobility of the ion–vortex–ring system is

$$\mu_n=1/(C_0/\mu_0+C_n/\mu^{(n)}), \quad (2)$$

where C_0 and C_n are constants which should be close to 1 in case of straightforward superposition.

Our experimental data given in Figs. 1–3, can be fitted to the above equations by choosing $C_0=1$ and $C_n=0.04\pm0.01$. This low value of C_n must be due to the effect of the bath disturbances which prevent the vortex rings from remaining permanently attached to the ion during its motion. Moreover, the contribution of $1/\mu^{(n)}$ is seen to be of decreasing importance at increasing temperatures in this temperature range.

[19] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. A238, 215 (1956).