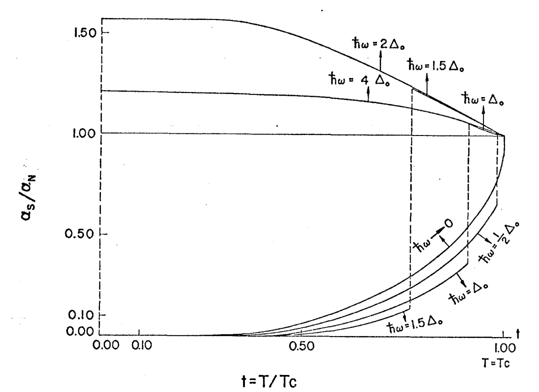

FIG. 2. Ratio of attenuations in superconducting and normal states as a function of temperature for different frequencies.

IV. RESULTS OF THE CALCULATION

1. r as a Function of a and c

Making use of the numerical values of the integrals I_1 and I_2, we find r as a function of a for given c; the numerical values are listed in Table I.

Figure 1 is a plot of r as a function of the frequency for fixed temperatures. Note the large discontinuities that occur when the frequency corresponds to the gap, $\hbar\omega=2\Delta(T)$; when $\hbar\omega>2\Delta$ it is possible to create a pair of excitations. At $T=0°\text{K}$, there is no absorption for $\hbar\omega<2\Delta$. In Fig. 2, plots are given of the absorption ratio as a function of temperature for fixed frequencies. The discontinuities are again evident. It is not possible to compare fully these results with experiment since as yet there are not available experimental data in the frequency range $\hbar\omega>2\Delta$, except for temperatures very near T_c. It is hoped that recent advances in technology for generating ultrasonic waves at microwave frequencies will make such measurements possible.

ACKNOWLEDGMENTS

The author wishes to express his deepest gratitude to Professor John Bardeen for helpful comments on the manuscript, without which these calculations could not possibly have been accomplished. The author is also greatly indebted to the Physics Department of the University of Illinois for the hospitality extended him during his stay.

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Momentum-Transfer Cross Sections for Slow Electrons in He, Ar, Kr, and Xe from Transport Coefficients*

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Momentum-transfer cross sections for electrons in He, Ar, Kr, and Xe are obtained from a comparison of theoretical and experimental values of the drift velocities and of the ratio of the diffusion coefficient to the mobility coefficient for electrons in these gases. The theoretical transport coefficients are obtained by calculating accurate electron-energy distribution functions for energies below excitation using an assumed energy-dependent momentum-transfer cross section. The resulting theoretical values are compared with the available experimental data and adjustments made in the assumed cross sections until good agreement is obtained. The final momentum cross section for helium is $5.0\pm0.1\times10^{-16}$ cm^2 for an electron energy of 5×10^{-3} eV and rises to $6.6\pm0.3\times10^{-16}$ cm^2 for energies near 1 eV. The cross sections obtained for Ar, Kr, and Xe decrease from 6×10^{-16}, 2.6×10^{-15}, and 10^{-14} cm^2, respectively, at 0.01 eV to minimum values of 1.5×10^{-17} cm^2 at 0.3 eV for Ar, 5×10^{-17} cm^2 at 0.65 eV for Kr, and 1.2×10^{-16} cm^2 at 0.6 eV for Xe. The agreement of the very-low-energy results with the effective-range theory of electron scattering is good.

I. INTRODUCTION

THE total scattering cross sections for electrons in the rare gases have been studied extensively at energies above about one electron volt using electron-beam techniques.[1] However, these methods are difficult to apply at lower energies. In this low-energy range one generally obtains the elastic-scattering cross sections, actually the cross sections for momentum transfer, from analyses of electron-transport coefficient data.[2] Until recently, such analyses avoided the complexities of solving the Boltzmann transport equation for each gas

* This work was supported in part by the Advanced Research Projects Agency through the Office of U. S. Naval Research.

[1] These experiments have been reviewed by R. B. Brode, Rev. Mod. Phys. 5, 257 (1933).

[2] For reviews of the earlier analyses see R. H. Healey and J. W. Reed, *The Behavior of Slow Electrons in Gases* (Amalgamated Wireless Ltd., Sydney, Australia, 1941); H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Clarendon Press, Oxford, 1952); L. B. Loeb, *Basic Processes in Gaseous Electronics* (University of California Press, Berkeley, California, 1955); and L. G. H. Huxley and R. W. Crumpton, in *Atomic and Molecula, Processes*, edited by D. R. Bates (Academic Press Inc., New York, 1962), Chap. 10.

by assuming that the cross sections were independent of electron energy and that the distribution of electron energies was either Maxwellian or Druyvesteyn.[2] Unfortunately, none of these assumptions is generally correct. Simplifications are possible for the cases of very low electric fields where the electrons are in thermal equilibrium with the gas and of very high-frequency electric fields at low gas densities,[3,4] for which the electron-energy distribution function is Maxwellian. Analyses based on the Maxwellian energy distribution and on an assumed power-law dependence of the cross section have been applied to a number of gases[4–7] over the energy range from 0.003 to 0.06 eV and to helium[8] and neon[9] for energies from 0.01 to 2 eV. In another recent analysis[10] for relatively high-electron energies, the cross section was assumed to vary as a power of the electron energy and the transport coefficients were calculated using the electron-energy distribution obtained by analytical solution of the Boltzmann transport equation appropriate to the assumed cross section. In the present paper we extend these results by allowing the energy dependence of the momentum-transfer cross section to be arbitrary and by covering the energy range from 0.003 eV to energies such that excitation becomes important.

The procedure used in this paper for the determination of momentum-transfer cross sections from transport coefficients is a simplification of that reported previously[11,12] for H_2, D_2, and N_2. Although the techniques used in this paper were largely developed in Ref. 11, the notation which we will use is that of Ref. 12, which will be referred to as I. According to this procedure, one makes an assumption as to the energy-dependent momentum-transfer cross section based on the best available data and then uses this to calculate the electron-energy distribution and the desired transport coefficients. Up to this point our calculations for the rare gases are an extension of those of Allen[13] and of Barbiere[14] to lower electron energies. The calculated transport coefficients are then compared with the measured drift velocities[5,6,10] and characteristic energies[12] and any differences are used to adjust the assumed cross section. The final cross section obtained by repeated application of this procedure is not unique in that rapid changes in the true momentum-transfer cross section with energy will be averaged out because of the relatively large fractional energy spread present in the equilibrium electron-energy distribution.

Some new aspects of the theory basic to our present analysis are discussed in Sec. II and the results of the analyses for various gases are given in Sec. III.

II. THEORY

In this section we will derive relations between the desired momentum-transfer cross section and the experimentally measurable quantities which will allow us to make estimates of sensitivity of our analysis. It should be kept in mind that the actual calculations do not depend upon these relations but are carried out numerically using cross sections which have an arbitrary dependence on electron energy.

Since inelastic collisions are neglected in the present analysis, the electron energy distribution $f(\epsilon)$ can be found by integrating the Boltzmann equation, e.g., Eq. (2) of I, to give

$$f(\epsilon)=A\exp\left[-\int_0^\epsilon\left(\frac{ME^2}{6mN^2Q_m{}^2(\epsilon)\epsilon}+\frac{kT}{e}\right)^{-1}d\epsilon\right]. \quad (1)$$

Here $\epsilon=mv^2/2$ is the energy of electrons of mass m and speed v, E is the electric field, N is the gas density, $Q_m(\epsilon)$ is the momentum-transfer cross section for electrons of energy ϵ, e is the charge of the electron, M is the atomic mass, k is Boltzmann's constant, and T is the gas temperature. The constant A is chosen such that $\int_0^\infty\epsilon^{1/2}f(\epsilon)d\epsilon=1$. If we define $Q_m(\epsilon_0)$ as the momentum-transfer cross section at some reference energy ϵ_0, then Eq. (1) shows that $f(\epsilon)$ for a given gas is a function only of $E[NQ_m(\epsilon_0)]^{-1}$ and T. Similarly, Eqs. (1), (6), and (7) of I show that for this $f(\epsilon)$ and for very low frequencies and zero magnetic field the electron drift velocity w and characteristic energy ϵ_K are functions only of $E[NQ_m(\epsilon_0)]^{-1}$ and T. This argument allows us to estimate the uncertainties in the final cross section due to uncertainties in the measured values of w or ϵ_K. Thus, if experiment at a fixed temperature shows that w or ϵ_K is proportional to $(E/N)^n$, then $Q_m(\epsilon_0)$ is proportional to the nth root of the measured quantity. This means that the greatest accuracy in the determination of the cross section occurs when n is large, e.g., in the thermal region where $n=1$ for drift-velocity measurements.[5]

We can obtain more specific relations between the measured quantities and the cross section by assuming

[3] H. Margenau, Phys. Rev. **69**, 508 (1946). See also W. P. Allis, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 21.

[4] A. V. Phelps, O. T. Fundingsland, and S. C. Brown, Phys. Rev. **84**, 559 (1951).

[5] J. L. Pack and A. V. Phelps, Phys. Rev. **121**, 798 (1961).

[6] J. L. Pack, R. E. Voshall, and A. V. Phelps, Phys. Rev. **127**, 2084 (1962).

[7] C. L. Chen, Phys. Rev. **131**, 2550 (1963).

[8] L. Gould and S. C. Brown, Phys. Rev. **95**, 897 (1954).

[9] A. L. Gilardini and S. C. Brown, Phys. Rev. **105**, 25 and 31 (1957).

[10] J. C. Bowe, Phys. Rev. **117**, 1411 and 1416 (1960). The analysis used by Bowe was first applied to H_2 by G. Bekefi and S. C. Brown, Phys. Rev. **112**, 159 (1958) and has been applied to other molecular gases by I. P. Shkarofsky, T. W. Johnston, and M. P. Bachynski, Planetary Space Sci. **6**, 24 (1961). The apparently low drift-velocity values obtained by Bowe for the rare gases are consistent with the low values he obtained in N_2. See the results of Lowke discussed in Ref. 33.

[11] L. S. Frost and A. V. Phelps, Phys. Rev. **127**, 1621 (1962).

[12] A. G. Engelhardt and A. V. Phelps, Phys. Rev. **131**, 2115 (1963). Note that the notation used in the present paper is the same as that of this reference.

[13] H. W. Allen, Phys. Rev. **52**, 707 (1937).

[14] D. Barbiere, Phys. Rev. **84**, 653 (1951).

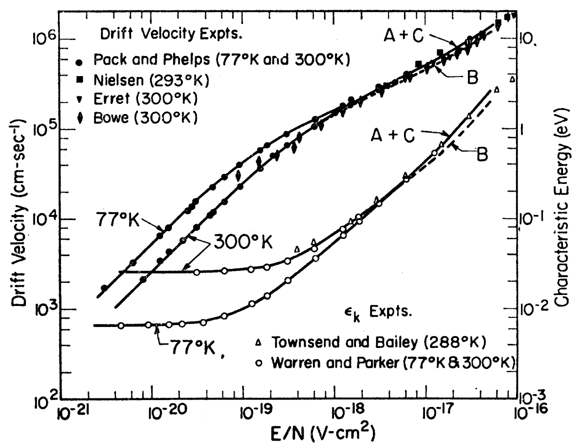

FIG. 1. Drift velocity and characteristic energy values for electrons in helium as a function of E/N at 77 and 300°K. The points show the results of various measurements. The smooth curves marked A, B, and C show the values calculated using the corresponding cross sections shown in Fig. 2. No calculations are shown for $E/N > 5\times10^{-17}$ V-cm² since the effects of inelastic collisions are neglected in this paper.

that the frequency of momentum-transfer collisions, $\nu(\epsilon) = vQ_m(\epsilon)$, is given by

$$\nu(\epsilon) = \nu_0(\epsilon/\epsilon_0)^{j/2}, \tag{2}$$

where $\nu_0 = (2\epsilon_0/m)^{1/2}Q_m(\epsilon_0)$. When this energy dependence is substituted into Eq. (1) and Eqs. (1), (6), and (7) of I, we find that for $j > -1$ and large values of E/N, $w \propto [E/NQ_m(\epsilon_0)]^{1/(1+j)}$ and that $\epsilon_K \propto [E/NQ_m(\epsilon_0)]^{2/(1+j)}$. This result suggests that when the power law approximation to the energy-dependent collision frequency is valid, e.g., for mean electron energies sufficiently removed from a sharp minimum or maximum in the cross section and for $\epsilon_K \gg kT$, the cross section can be determined with greater accuracy by using ϵ_K data rather than drift-velocity data. Unfortunately, the recent work of Warren and Parker[15] shows that the determination of ϵ_K may be subject to systematic errors, the origin of which is poorly understood. Although they apparently were able to eliminate the errors in their measurements, we are unable to estimate the errors in earlier experiments. As will be evident later in this paper, the heavier rare gases are good examples of gases in which the effective j value is so large at electron energies above the Ramsauer minimum that the accuracy of the cross-section determination from pure gas data[10,16] is low.

Finally, we note that in principle one can combine transport coefficients in such a way as to obtain a linear relation between the cross section and a suitable combination of measured transport coefficients. Thus, if we define an effective frequency for momentum-transfer collisions[17] ν_m by the relation

$$\nu_m/N = eE(mwN)^{-1}, \tag{3}$$

and consider its dependence on ϵ_K, Eqs. (1) and (2) and Eqs. (1), (6), and (7) of I can be solved to show that

$$\nu_m/N \propto \epsilon_K{}^{j/2}Q_m(\epsilon_0). \tag{4}$$

This relation is also valid at very low E/N, where $\epsilon_K \to kT/e$. Using these relations for small errors, the fractional error in $Q_m(\epsilon_0)$ is proportional to the fractional errors in the drift velocity and E/N, and to $j/2$ times the fractional error in ϵ_K.

III. DETERMINATION OF CROSS SECTION

A. Helium

Figure 1 shows the values of electron drift velocity and ϵ_K for electrons in helium as a function of E/N for gas temperatures of 77 and 300°K. The points show the experimental values of the drift velocity measured by Nielsen,[18] Pack and Phelps,[5,19] Bowe[10] and Errett[20] and the values of ϵ_K obtained by Townsend and Bailey[21] and by Warren and Parker.[15] The smooth curves show the results of substituting various assumed cross sections into the theoretical expressions given in Sec. I and in I. The smooth curves marked A, B, and C of Fig. 1 are calculated using the corresponding momentum transfer cross-section curves shown in Fig. 2. The cross-section curve marked A is chosen to fit the experimental drift-velocity data of Nielsen[18] and of Pack and Phelps[5,19] and that marked B fits the drift-velocity data of Errett[20] and of Bowe.[10] Curve B is not adjusted to fit Bowe's data for $E/N < 10^{-18}V$-cm² because of the rapid variation of $Q_m(\epsilon)$ with energy which would be required. The cross-section curve marked C in Fig. 2 is constructed so as to oscillate about curve A with approximately the amplitude and frequency found using electron-beam techniques by Ramsauer and Kollath[22] and by Normand.[23] Note that the drift velocity or

[15] R. W. Warren and J. H. Parker, Jr., Phys. Rev. **128**, 2661 (1962). See also R. W. Crompton and R. L. Jory, Australian J. Phys. **15**, 451 (1963) for a discussion of errors in this type of experiment.

[16] L. S. Frost and A. V. Phelps, Bull. Am. Phys. Soc. **5**, 371 (1960).

[17] Other combinations of transport coefficients have been used in connection with analyses which assume that the distribution function is either Maxwellian or Druyvesteyn. See, for example, Ref. 2. As indicated in Ref. 11, our choice has the advantage of simplicity, of being independent of assumptions as to the energy distribution, and of separating the effects of elastic and inelastic collisions when both are present.

[18] R. A. Nielsen, Phys. Rev. **50**, 950 (1936).

[19] A. V. Phelps, J. L. Pack, and L. S. Frost, Phys. Rev. **117**, 470 (1960). The μN values given in this reference for $E/N < 10^{-18}$ V-cm² are about 10% too small because of an overestimate of the end effect correction. The $Q_m(\epsilon)$ values are correspondingly too large for energies below about 0.2 eV. See Ref. 5.

[20] D. Errett, Ph.D. thesis, Purdue University, 1951 (unpublished).

[21] J. S. Townsend and V. A. Bailey, Phil. Mag. **46**, 675 (1923) and **44**, 1033 (1922).

[22] C. Ramsauer and R. Kollath, Ann. Physik **3**, 536 (1929). Electron beam measurements of Q_t have been reported recently by D. E. Golden and H. W. Bandel, Paper G-2, 17th Gaseous Electronics Conference, Atlantic City, 1964 (unpublished). Their results show no structure and agree well in magnitude with our values.

[23] C. E. Normand, Phys. Rev. **35**, 1217 (1930).

characteristic energy data shown in Fig. 1 for cross-section C are indistinguishable from those obtained using cross-section A. This illustrates the lack of sensitivity of the transport coefficient analyses to rapid variations in cross section with energy.

The cross-section curves of Fig. 2 for energies below 0.2 eV show that there is rather good agreement among determinations of $Q_m(\epsilon)$ from most drift velocity[5,19] and microwave conductivity data.[4,7,8,24] Since the $Q_m(\epsilon)$ curve obtained from the present analysis is required to fit the measured drift velocities in the nonthermal as well as the thermal region, the $Q_m(\epsilon)$ values differ somewhat from the results of Pack and Phelps[5] although the thermal data is the same. This change illustrates the importance of basing an analysis of this type on data obtained over the widest possible range of ϵ_K.

Figures 1 and 2 show that the comparison of calculated and experimental drift velocities for electron energies greater than about 0.2 eV yields cross-section curves which differ by as much as 20%. This is the result of uncertainties as to which electron drift-velocity data is correct.[25,26] We note that for $\epsilon_K \sim 2$ eV the fractional difference in the drift velocities predicted using curves A and B is about half the fractional difference in the assumed cross sections for $\epsilon \sim 3$ eV. This is expected since[2,3] $\epsilon_K \sim (2/3)\bar{\epsilon}$ and since $w \propto (E/N)^{1/2}$ and, according to Sec. II, $w \propto [1/Q_m(\epsilon_0)]^{1/2}$. In the present case, the use of the effective momentum-transfer collision frequency defined by Eq. (6) is not much help because the basic difficulty is the differences in the experimental data. We therefore compare the theoretical curves and experimental points for ϵ_K versus E/N in Fig. 1. Here

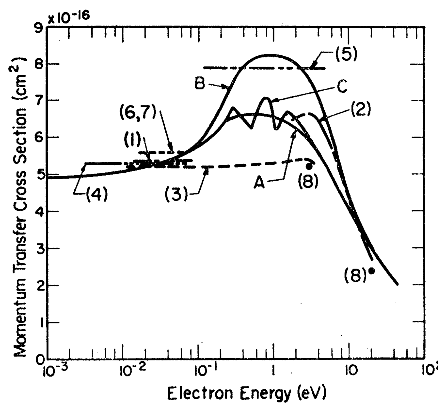

FIG. 2. Momentum-transfer scattering cross sections for electrons in helium. The solid curves show the three cross sections labeled A, B, and C used to calculate the drift velocity and characteristic energy curves of Fig. 1. The broken curves and points show the results obtained previously by the following authors: curve (1) Phelps, Fundingsland, and Brown; (2) Barbiere; (3) Gould and Brown; (4) Pack and Phelps; (5) Bowe; (6) Chen, Leiby, and Goldstein; (7) Hirschfield and Brown; and (8) McClure.

we see that the agreement between experiment and theory for $E/N > 5\times10^{-18} V$-cm^2 is significantly better for curve A than for curve B. We therefore propose that curve A is the correct momentum-transfer cross section.

The cross sections for energies above 0.2 eV which we have found to fit the measured drift velocities are compared in Fig. 2 with those derived by Barbiere[14] from the electron-beam experiments of Ramsauer and Kollath,[22] by Gould and Brown[8] from measured microwave-transport coefficients, by Bowe[10] from drift-velocity data, and by McClure[27] from a diffusion experiment. We note that Barbiere's analysis of electron-beam results is in somewhat better agreement with curve B than with curve A. However, as pointed out by Bowe,[10] the interpretation of the electron-beam data by Barbiere gives considerably smaller values of $Q_m(\epsilon)$ than does the analysis of Westin.[28] We have no explanation for the discrepancy between our results and those of Gould and Brown for energies above 0.1 eV. The disagreement with Bowe has been discussed above. The analysis presented here is not expected to yield significant data regarding $Q_m(\epsilon)$ for electron energies above 5 to 10 eV, since the largest value of ϵ_K for which inelastic collisions can be neglected is 2.5 eV.

Recent theoretical calculations of electron collision cross sections in helium have been discussed by Moiseiwitsch[29] and by Hashino and Matsuda.[30] Much of the recent work has been directed toward the calculation of elastic-scattering cross section at zero electron en-

[24] An exception to the generally good agreement in this energy range is the curve of $Q_m(\epsilon)$ derived from microwave conductivity measurements by J. M. Anderson and L. Goldstein, Phys. Rev. **102**, 933 (1956). However, more recent experiments using the same basic technique lead to cross sections in good agreement with the values shown in Fig. 2. See C. L. Chen, C. C. Leiby, and L. Goldstein, Phys. Rev. **121**, 1391 (1961) and Ref. 7. Other results which are in essential agreement with those shown are by J. L. Hirschfield and S. C. Brown, J. Appl. Phys. **29**, 1749 (1958).

[25] The question of errors in time of flight measurements of drift velocities has recently been considered by J. J. Lowke, Australian J. Phys. **15**, 39 (1962) and by M. Böhning, thesis, University of Heidelberg, 1962, (unpublished). Although Lowke shows that in an extreme case a single reading using the apparatus of Refs. 5, 6, and 19 may yield an apparent drift velocity which is high by as much as 30%, he agrees that differential measurements such as those of these references should be subject to much smaller errors. Unfortunately, the theory for finite pulse widths and partially absorbing control grids is not available. Böhning claims that the drift velocities shown[5] in Fig. 3 for Ar at $E/N < 10^{-19}$ cm^{-3} are too high by at least a factor of 2. We do not understand the source of the discrepancy between his results and those of Ref. 5. We believe that the consistency between drift velocity and characteristic energy measurements and calculations in pure argon and in argon-molecular gas mixtures (Ref. 35) rules out the possibility of such a large error in the data of Ref. 5.

[26] Some support for curve B of Fig. 2 arises from measurements of the angle through which electrons are deflected at high magnetic fields. See M. J. Bernstein, Phys. Rev. **127**, 335 (1962). However, since the relation between the measured quantities and $Q_m(\epsilon)$ at high magnetic fields is the same as at high angular frequencies,[12] one would expect the derived cross sections to agree with that of Gould and Brown (Ref. 8) rather than to differ by about 60%.

[27] B. T. McClure, Bull. Am. Phys. Soc. **9**, 90 (1964).

[28] See Ref. 9 for a discussion of this work.

[29] B. L. Moiseiwitsch, *Atomic and Molecular Processes*, edited by D. R. Bates (Academic Press Inc., New York, 1962), Chap. 9.

[30] T. Hashino and H. Matsuda, Progr. Theoret. Phys. (Kyoto) **29**, 370 (1963).

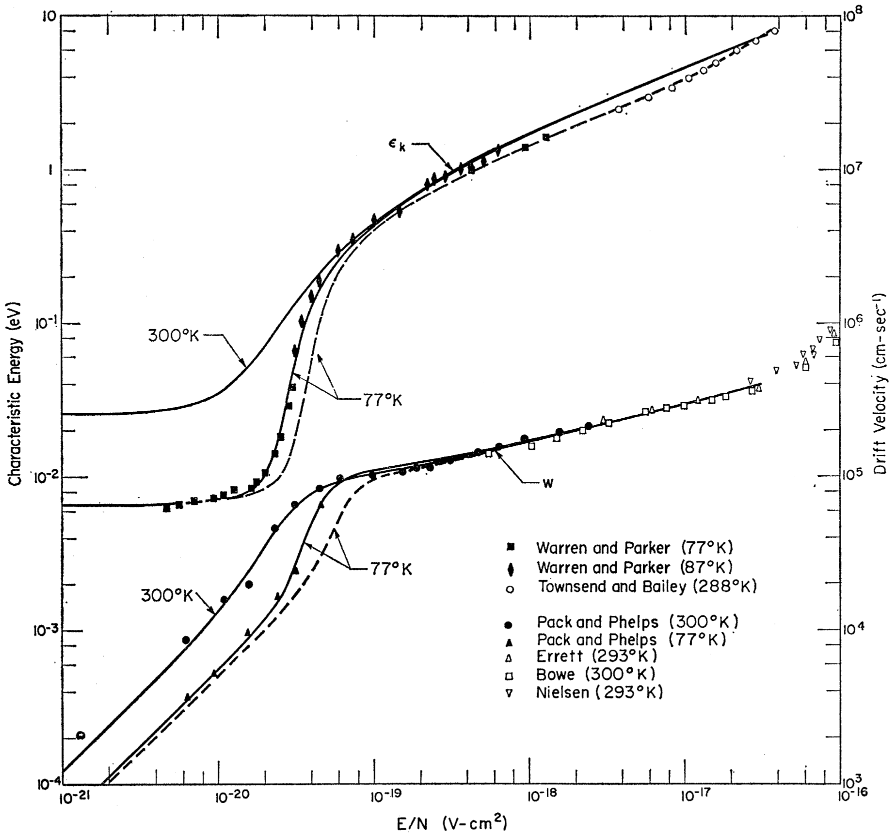

FIG. 3. Drift velocity and characteristic energy for electrons in argon at 77 and 300°K. The points show experimental data. The solid curve shows the results of calculations using our final $Q_m(\epsilon)$ curve plotted in Fig. 4. The dashed curve gives the result of calculations based on O'Malley's $Q_m(\epsilon)$ curve as plotted in Fig. 4.

ergy. While no experiments have been proposed to allow measurements of the cross section at zero energy, the analysis presented above is expected to yield useful data for electron energies down to about 0.003 eV. In addition, one can use the theoretical procedure developed by O'Malley[31] to extrapolate the experimental data to zero energy. The low-energy portions of curves A and B of Fig. 2 are chosen to have the slope calculated by O'Malley from measured polarizabilities.[31] We believe that the effect of overlapping polarization interactions proposed by Kivel[32] and refined by O'Malley[31] is less than the scatter of the low E/N data for He discussed in this paper.[33] The scattering length obtained[31] from this data is $1.18a_0$, which is essentially the same as the value given by O'Malley. Most of the theoretical calculations[29,30] give cross sections at zero energy which are about 50% larger than the value derived from this analysis. [*Note added in proof.* Good agreement with the low-energy portion of our results has been obtained from theory by R. W. LaBahn and J. Calloway, Phys. Rev. **135**, A1539 (1964).]

B. Neon

The results of our attempts to analyze the drift-velocity measurements in Ne are not given in this paper because we find an apparent 30% discrepancy in the cross sections needed to fit the 77 and 300°K data of Pack and Phelps[5] at low E/N, i.e., $E/N=3\times10^{-19}V$-cm^2. As indicated by these authors measurements with a pressurized apparatus would be necessary in order to obtain more reliable data in the near thermal region. The high gas densities which could be obtained with such an apparatus are necessary because of the very low momentum-transfer cross section for low-energy electrons in neon. A comparison of our results with recent experiments has been made by Chen.[34]

[31] T. F. O'Malley, Phys. Rev. **130**, 1020 (1963).

[32] B. Kivel, Phys. Rev. **116**, 1484 (1959).

[33] According to O'Malley (Ref. 31 and private communication) the fractional correction, F, in Q_m at zero energy is approximately $(\alpha/Aa_0)(4\pi N/3)^{1/3}$ where α is the polarizability, A is the scattering length, a_0 is the Bohr radius, N is the gas density and $4\pi A^2=Q_m(0)$. The effect should disappear for energies above about $\epsilon=13.6\times[a_0(4\pi N/3)^{1/3}]^2$ or about 0.01 eV for a density of 4×10^{19} atoms/cc. At the highest density used in the measurements in He at 77°K ($\bar{\epsilon}=0.01$ eV) of Ref. 5, $N\sim4\times10^{19}$ cm^{-3}, so that for $A=1.2a_0$ and $\alpha=1.36a_0^3$, $F\sim4\times10^{-2}$. The largest error in the experiments reported in Refs. 5 and 6 would occur in Ar at the highest density used (2.7×10^{19} at 77°K), for which $F\sim0.15$. The data of Refs. 5 and 6 show no evidence for a density dependence of the drift velocity at low temperatures and low E/N to within the scatter of the data of about ±5%, although in the case of Ar there is little very low-energy data and the scatter is larger. It is possible that the decrease in drift velocity with increasing N_2 density found in the precision measurements of Lowke and present in the limited data of Ref. 5 is evidence for this effect. See J. J. Lowke, Australian J. Phys. **16**, 115 (1963).

[34] C. L. Chen, Phys. Rev. **135**, A627 (1964).

C. Argon

The results of our analysis in argon are shown by the solid curves of w and ϵ_K as a function E/N at 77 and 300°K in Fig. 3 and the solid curve for $Q_m(\epsilon)$ in Fig. 4. Because of the pronounced Ramsauer minimum in $Q_m(\epsilon)$ at ϵ near 0.3 eV, the determination of the $Q_m(\epsilon)$ curve has been very difficult. For example, it was found that the comparison of computed and experimental drift velocities, used in a previous analysis[16] of Ar, is a significantly less accurate method for determining the rising portion of the $Q_m(\epsilon)$ for $\epsilon < 0.7$ eV than is the comparison of computed and experimental values of ϵ_K. Furthermore, it was found that because of the importance of the Ramsauer minimum and the large effective value of the exponent in the power-law approximation to the collision frequency, both w and ϵ_K were very insensitive to the choice of $Q_m(\epsilon)$ for $\epsilon > 0.7$ eV. Accordingly, our final $Q_m(\epsilon)$ curve for $\epsilon > 0.7$ eV is that obtained by Engelhardt and Phelps[35] from drift-velocity measurements by Errett[20] in a mixture of 10% H_2 and 90% Ar.

For electron energies above about 1 eV our final $Q_m(\epsilon)$ curve is about 10% below the curve obtained by Barbiere[14] from an analysis of electron-beam experiments.[36]

Our $Q_m(\epsilon)$ curve is from 20 to 40% below that obtained by Bowe[10] because of small differences between his drift-velocity data and that of Pack and Phelps.[5] The $Q_m(\epsilon)$ curve given by O'Malley[31] was obtained by fitting the effective range theory to the total cross-section measurements of Ramsauer and Kollath[36] over the energy range common to the theory and experiment. In view of the rather large differences between our result and that of O'Malley[37] for electron energies above 0.07 eV, we have shown by the dashed curve of Fig. 3 the drift velocity and ϵ_K values calculated for 77°K using O'Malley's $Q_m(\epsilon)$ curve for $\epsilon < 0.5$ and Barbiere's $Q_m(\epsilon)$ for $\epsilon > 1.5$ eV with a smoothed interpolation between these curves. We conclude that our $Q_m(\epsilon)$ curve gives a significantly better fit to the available experimental data[38] for $E/N < 5\times10^{-19}$V-cm². The agreement with O'Malley at very low energies means that we agree with his derived scattering length, and it suggests that his theory is accurate only for $\epsilon < 0.07$ eV in Ar. We note that as predicted by O'Malley[31] there is a tendency for the region of the minimum in the $Q_m(\epsilon)$ curve to occur at lower energies than that of the minimum of the curve of total cross section, $Q_t(\epsilon)$.

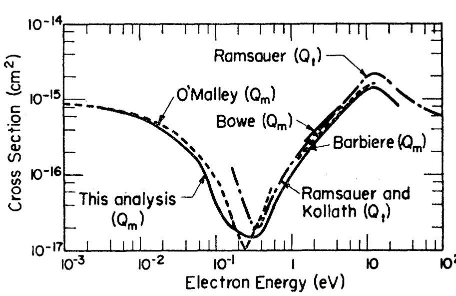

FIG. 4. Momentum-transfer, Q_m, and total, Q_t, cross sections for electrons in argon. The solid curve gives our final momentum-transfer cross section which was used to calculate the drift velocity and characteristic energy values shown by the solid curve of Fig. 3.

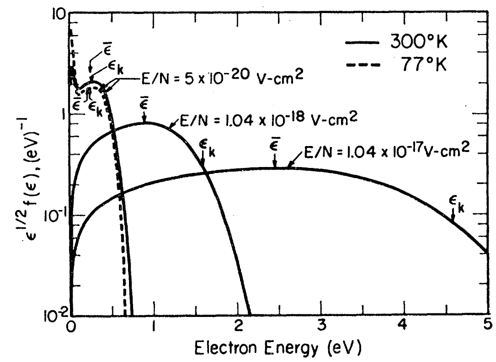

FIG. 5. Electron energy distribution function, $\epsilon^{1/2} f(\epsilon)$, for electrons in Ar at 77 and 300°K and various E/N. The distributions are essentially independent of temperature for $E/N > 5\times10^{-19}$ V-cm². The arrows indicate the values of the mean electron energy $\bar{\epsilon}$ and the characteristic energy ϵ_K. The distribution function $f(\epsilon)$ is defined such that the integral of $\epsilon^{1/2} f(\epsilon)\, d\epsilon$ over all energies is unity.

In view of the rather unusual behavior of the electron-energy distribution functions found in Ar due to the rapidly varying $Q_m(\epsilon)$, we have shown typical distribution functions in Fig. 5. At low $E/N (< 6\times10^{-20}$ V-cm²) there is a tendency for electrons to be found at thermal energies and at the energy of the Ramsauer minimum (~0.3 eV). At the higher E/N the distribution functions are characterized by a very rapid decrease with increasing electron energy as a result of the rapidly rising $Q_m(\epsilon)$ curve. As indicated by the arrows in Fig. 5 the relatively narrow electron-energy distribution functions lead to values of ϵ_K which are much larger than the mean electron energy $\bar{\epsilon}$. This is to be compared to $\epsilon_K = 2\bar{\epsilon}/3$ for a Maxwellian electron-energy distribution such as is predicted[2,3] and found for very low E/N.

A second result of the rapid variation in $Q_m(\epsilon)$ with energy is the large departures of the "magnetic deflec-

[35] A. G. Engelhardt and A. V. Phelps, Phys. Rev. **133**, A375 (1964).

[36] C. Ramsauer and R. Kollath, Ann. Physik **12**, 529 (1932).

[37] Part of the discrepancy between our results and those of O'Malley may be due to differences in the reading of the graphs given by Ramsauer and Kollath in Ref. 22.

[38] One surprising result obtained using O'Malley's $Q_m(\epsilon)$ curve is the good fit to Townsend and Bailey's ϵ_K data[21] and to the drift velocity data for $3\times10^{-18} < E/N < 3\times10^{-17}$ V-cm². However, there seems little likelihood that both the w and ϵ_K data for $2\times10^{-20} < E/N < 10^{-19}$ V-cm² could be in error by the amount required. It should be pointed out that the difference between our $Q_m(\epsilon)$ curve and that of Barbiere for energies between 1 and 10 eV is much too small to account for the change in ϵ_K calculated for the two cross sections. The apparent improvement is due to the differences in the assumed cross sections in the vicinity of the Ramsauer minimum. We note that for $3\times10^{-20} < E/N < 7\times10^{-19}$ the only ϵ_K data available is for 87°K. Calculations for this temperature show that the 87°K values of ϵ_K are about 10% higher than the 77°K values for E/N between 3 and 5×10^{-20} V-cm².

tion coefficient," w_M/w, from unity.[11,12,39] The results of our calculations are compared with the available experimental data in Fig. 6. The departure of this coefficient from unity is a measure of the range of values of the collision frequencies covered by the electron-energy distribution.[40] The data of Fig. 6 show that (a) at 77°K the collision frequency is effectively constant for thermal electrons and then the range of collision frequencies increases as the electric field causes some electrons to drop into the Ramsauer minimum, (b) at 300°K the range of collision frequencies covered by thermal electrons is rather high and decreases as most of the electrons drop into the Ramsauer minimum, and (c) at E/N greater than 3×10^{-17} inelastic collisions produce increased numbers of low-energy electrons which appear in the Ramsauer minimum and the calculated value of w_M/w is as large as 3.7. The only experimental measurements of w_M in Ar are those of Townsend and Bailey.[21] While their results[41] show the same general trend as our calculated values of w_M/w, the departure from unity is smaller and the onset of the peak is less abrupt. The experimental measurements by Anderson[42] of the Hall voltage developed in an argon positive column can in principle yield values of w_M/w. Unfortunately, his experimental data show too much variation with discharge conditions to be useful for this purpose.

Finally, it should be pointed out that in contrast to the situation for the dc electric fields considered above, the mobility integrals[12] appropriate at high magnetic fields or high ac electric fields show that the higher values of $Q_m(\epsilon)$ are more important than the lower values. This means that measurements at high magnetic fields or high frequencies in pure Ar should yield more accurate values of $Q_m(\epsilon)$ for $\epsilon>0.7$ eV than the measurements discussed above. Calculations made for high magnetic fields using the cross section of Fig. 4 show that the departures of the relative values of the mobility tensor components from the values expected for the case of constant collision frequency are about a factor of 2 for electrons with a temperature of 2000°K. These departures from the relations given by Tonks[43] for a constant collision frequency gas are often ignored in the theory of devices such as magnetohydrodynamic generators.[44]

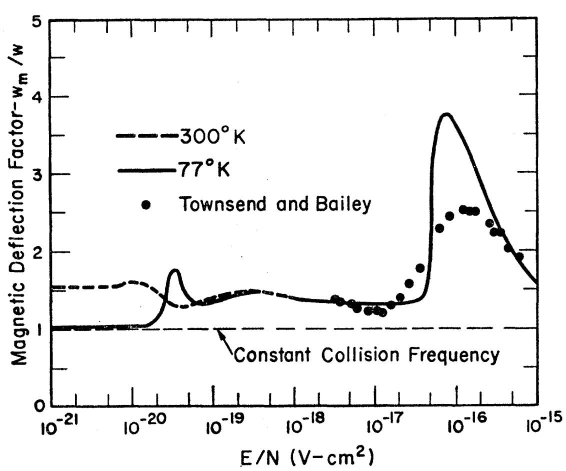

FIG. 6. Magnetic deflection factor w_M/w for electrons in argon. The smooth curves are calculated and the points are obtained from experimental measurements of the deflection of a swarm of electrons in a magnetic field. Inelastic collisions between electrons and argon atoms are important for $E/N>3\times10^{-17}$ V-cm².

D. Krypton

The $Q_m(\epsilon)$ values for krypton obtained from our analysis of electron drift-velocity data are shown in Fig. 7 along with other determinations of the momentum transfer and total scattering cross sections. At electron energies below about 0.1 eV our $Q_m(\epsilon)$ curve is about 20% below that derived by O'Malley[31] from the data of Ramsauer and Kollath[36] and about 40% below that of Chen.[7] If the scattering length parameter of O'Malley's cross-section relation were reduced by about 10% from his value, the calculated $Q_m(\epsilon)$ would be in good agreement with our result for energies below about 0.1 eV. The correction due to high gas densities[33] is expected to be less than 10%. We have no explanation for the discrepancy with Chen.[45] As pointed out in connection with the analysis of Ar drift-velocity data, the accuracy of $Q_m(\epsilon)$ determinations for ϵ significantly above the Ramsauer minimum ($\epsilon>2$ eV) is relatively low because of the rapid variation of $Q_m(\epsilon)$ with energy. A lack of validity in the power-law approximation to $Q_m(\epsilon)$ in

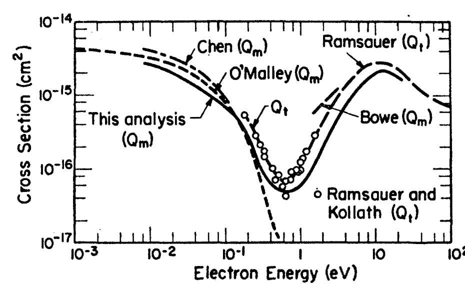

FIG. 7. Momentum-transfer and total cross sections for electrons in krypton. The solid curve shows the cross section which was used to calculate the drift velocities for Kr shown in Fig. 8. The broken curves show the results obtained by the various authors discussed in the text.

[39] A. G. Engelhardt, A. V. Phelps, and C. G. Risk, Phys. Rev. **135**, A1566 (1964).

[40] This statement is a qualitative generalization which is consistent with various cases discussed in Refs. 11, 12, and 39.

[41] We have used the calculated values of w from Ref. 35 to calculate the w_M/w values shown.

[42] J. M. Anderson, Bull. Am. Phys. Soc. **8**, 423 (1963).

[43] L. Tonks, Phys. Rev. **51**, 744 (1937). The errors resulting in the case of constant Q_m are considered by L. Tonks and W. P. Allis, Phys. Rev. **52**, 710 (1937).

[44] H. Hurwitz, R. W. Kilb, and G. W. Sutton, J. Appl. Phys. **32**, 205 (1961).

[45] As was pointed out in Ref. 6 the results of the microwave measurements of Ref. 4 are in good agreement with the results of the analysis of drift velocity data given in Figs. 7 and 9 of the present paper.

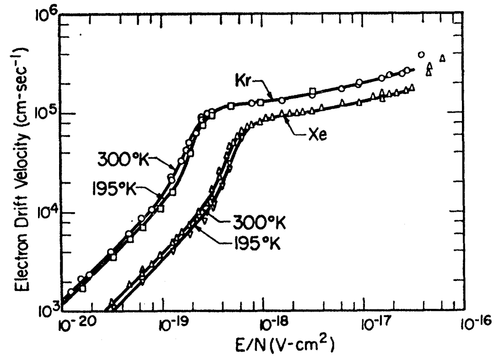

FIG. 8. Calculated and experimental drift velocities for electrons in Kr and Xe as calculated from the cross sections of Figs. 7 and 9, respectively.

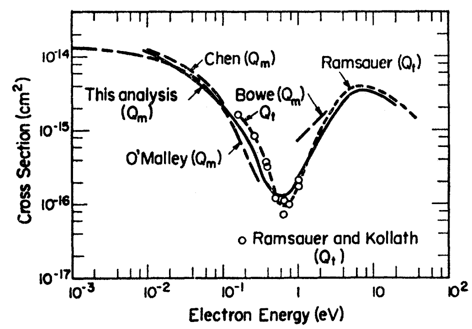

FIG. 9. Momentum-transfer and total cross sections for electrons in xenon. The solid curve shows the values of the cross section used to calculate the drift velocities for Xe in Fig. 8. The broken curves show the results obtained by the various authors discussed in the text.

this case may be the source of the large (2 to 4) ratio of the $Q_m(\epsilon)$ values found by Bowe[10] to our values for only a 10 to 20% difference in drift velocities.[6] We note that our Q_m curve is well below the measured[36] Q_t values.[46] The drift velocities calculated using the cross section given by the solid line of Fig. 7 are compared with the experimental values of Pack, Voshall, and Phelps[6] in Fig. 8. This plot shows that it has been possible to obtain very good agreement with this experimental data. However, the disagreement between various investigators at energies below the Ramsauer minimum shows the need for further experimental studies.

E. Xenon

The momentum-transfer cross sections obtained from our analysis of the drift velocity for Xe is shown in Fig. 9. The agreement with O'Malley is good for $\epsilon<0.15$ eV. However, the use of O'Malley's cross section for energies up to 0.3 eV together with essentially our result for higher energies, results in a calculated drift velocity which rises much too rapidly with E/N for $E/N>1.5\times10^{-19}$ V-cm². Our $Q_m(\epsilon)$ curve is about 20% below that of Chen[7] and about a factor of 2 below that of Bowe.[10] The differences between our derived $Q_m(\epsilon)$ curve and the measured $Q_t(\epsilon)$ curve[36] are considerably smaller than in the case of Kr. The drift velocities calculated using the cross section shown by the solid curve of Fig. 9 are compared with the experimental data of Pack, Voshall, and Phelps in Fig. 8. The agreement is within about 10%.

[46] It should be kept in mind that the total scattering cross section Q_t weights all scattering angles equally whereas the momentum-transfer cross section, Q_m, weights the scattering by $(1-\cos\theta)$, where θ is the scattering angle.

IV. SUMMARY

The results presented in the preceding section[47] show that it is possible to derive a set of momentum-transfer cross sections for electrons in the rare gases which yield electron drift velocities and characteristic energies in agreement with experimental data. In general the sensitivity of the technique is good (better than ±10%) except for electron energies well above that of any deep minimum in the cross section and below that of the maximum cross section. Because of the wide spread of electron energies present in the experiments, this technique is not capable of resolving rapid variations in cross section such as that found for helium in some electron-beam experiments. Our derived cross sections are generally in good agreement with those derived by O'Malley from electron-beam experiments using an effective-range theory. There is some indication that in Ar, Kr, and Xe the range of energies over which the effective-range theory is valid is somewhat smaller than indicated by O'Malley. The results presented in this paper are generally believed to be accurate to ±10% except possibly in the case of krypton at energies below the Ramsauer minimum.

ACKNOWLEDGMENTS

The authors wish to express their appreciation for valuable discussions of this problem with their associates in the Atomic Physics Group. The authors are especially indebted to B. Sherman, L. Morgan, and A. G. Engelhardt for assistance in programming and carrying out the numerical calculations. The authors wish to thank B. T. Barnes for the opportunity to see the results of his similar analysis prior to publication.

[47] A tabulation of the final values of the derived cross sections and calculated transport coefficients is available on request.