Electromagnetic Structure of the 3-Nucleon System*

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The cross sections for elastic scattering of high-energy electrons on He^3 and H^3 are calculated. The nuclear ground-state wave functions are assumed to consist of a dominant fully symmetric S state with either Gaussian or exponential space dependence and a small component of D state. We include also estimates of effects of relativisitic terms in the nuclear current, and effects of the exchange currents. The results are compared with the experimental data of Hofstadter, Collard, and Yearian. We can fit these data to within their experimental limits for both He^3 and H^3. The best fits are not obtained by choosing the same wave parameters for both nuclei.

RECENT measurements[1] of elastic electron scattering from He^3 and H^3 have yielded information about their electric and magnetic form factors at momentum transfers up to $|q^2|=5$ F^{-2}. In particular, they show in He^3 a magnetic moment form factor appreciably larger than its charge form factor; i.e., the distribution of magnetic moment is more compact than that of charge, with rms radii 1.69 and 1.97 F, respectively. The two form factors of H^3, on the other hand, appear to be approximately equal, and furthermore equal to the He^3 moment factor.

One would expect, from the simplest model of H^3 and He^3, to find the electric and magnetic form factors approximately equal. If all particles are in S states, the magnetic moment resides entirely in the spin of the odd particle. Neglecting Coulomb effects, the neutron and proton wave functions are equal and, aside from nucleon finite size corrections, this leads to equality of the charge and moment density distributions. However, these finite size effects are not negligible and lead to $F_{\text{chg}}{}^{H^3} > F_{\text{chg}}{}^{He^3}$.[2] It is known also that the nucleon-nucleon tensor force admixes into the ground-state wave function an appreciable D-state component. Now F_{chg} has only S^2 and D^2 terms; F_{mag} has a SD interference term which may lead to $F_{\text{mag}} > F_{\text{chg}}$, in qualitative agreement with the experimental results. One would hope that these two effects would explain the differences in F_{chg} and F_{mag} for He^3 while remaining consistent with the H^3 measurements. We should note that both the He^3 and H^3 magnetic moments differ by about 0.2 nucleon magneton from the Schmidt values which correspond to total orbital angular momentum $L=0$, and that the introduction of a D state increases rather than decreases these discrepancies.[3] Presumably the meson exchange currents would account for much of the difference in the moment although this has not yet been shown explicitly.

It is of interest, therefore, to evaluate the form factors for both He^3 and H^3 using a wave function composed of both S and D states. Schiff[4] has obtained some measure of agreement with these experimental quantities, using only an admixture of two different S states but has included all D-state contributions in an experimentally determined quantity. Here we calculate the contributions from the D-state component expected to be dominant in the wave function. We also include the exchange currents, via Sachs' phenomenological model.[5] Although this model has not resolved the question of the magnetic moments, it might still give some indication of the contributions to the electron-scattering cross section.

It has been proposed that these cross sections may provide a feasible method of obtaining the neutron charge form factor. It will become apparent that, in agreement with Schiff, the present data are far too insensitive to a neutron charge density to determine it with greater accuracy than already realized in other experiments.

In first order, the matrix element describing elastic scattering is

$$\mathfrak{M} = ie^2 j^\mu (1/q^2) J_\mu(q^2).$$

Here q^2 is the invariant of the four-momentum transfer $q=k_i-k_f$, $q^2=-4E_iE_f\sin^2(\theta/2)$, j^μ is the electron's Dirac current, $j^\mu=\bar{u}_f\gamma^\mu u_i$, while $J_\mu(q^2)$ is the nuclear current for absorption of a photon of momentum q and polarization μ.

Anticipating the energy-momentum conserving delta functions in the matrix element $\mathfrak{M}$ and performing the final- and initial-state sums and averages, we obtain the cross section following the Rosenbluth formula[6]

$$d\sigma/d\Omega = \sigma_M(\theta)\{a(q^2)-(q^2/2M_T{}^2)b(q^2)\tan^2(\theta/2)\},$$

where $\sigma_M(\theta)$ is the Mott cross section

$$\sigma_M(\theta) = \frac{z^2\alpha^2}{4E_i{}^2}\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\frac{1}{1+(2E_i/M_T)\sin^2(\theta/2)}.$$

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[1] H. Collard and R. Hofstadter, Phys. Rev. **131**, 416 (1963). H. Collard, R. Hofstadter, A. Johansson, R. Parks, M. Ryneveld, A. Walker, M. Yearian, R. Day, and R. Wagner, Phys. Rev. Letters **11**, 132 (1963).

[2] For entirely symmetric S states we have (Ref. 4) $F_{\text{chg}}{}^{H^3}/F_{\text{chg}}{}^{He^3} = 2(F_{\text{chg}}{}^p+2F_{\text{chg}}{}^n)/(F_{\text{chg}}{}^n+2F_{\text{chg}}{}^p)=1$, 1.09, and 1.15 for $|q^2|=0$, 3, and 5 F^{-2}, respectively.

[3] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953), Chap. 8, p. 180.

[4] L. I. Schiff, Phys. Rev. **133**, B802 (1964).

[5] R. G. Sachs, Phys. Rev. **74**, 433 (1948).

[6] M. N. Rosenbluth, Phys. Rev. **79**, 615 (1950).

M_T is the mass of the target (H^3 or He^3). In terms of the components of J_μ (q^2), $a(q^2)$, and $b(q^2)$ are given by

$$z^2 a(q^2) = \langle J_0^+ J_0\rangle + \tfrac{1}{2}\langle \mathbf{J}^+ \cdot \mathbf{J}\rangle ,$$

$$z^2 b(q^2) = \frac{2M_T^2}{-q^2}\left\{\langle \mathbf{J}^+\cdot\mathbf{J}\rangle + \frac{q^2}{4M_T^2}\langle J_0^+ J_0\rangle\right\} .$$

The symbol $\langle\ \rangle$ denotes the sums over the target ground-state spins. Corrections to $a(q^2)$ and $b(q^2)$ of order q^2/M_T^2 compared to unity have been neglected since in the region of interest $q^2/M_T^2 \lesssim 0.02$. $a(q^2)$ and $b(q^2)$ are related to the electric and magnetic form factors of H^3 and He^3 by

$$a(q^2) = \{G_E^2 - (q^2/4M_T^2)G_M^2\}/[1-(q^2/4M_T^2)] ,$$

$$b(q^2) = G_M^2 .$$

These form factors are normalized so $a(0)=1.0$, $b(0)=\mu^2(A^2/z^2)$, with μ the nuclear magnetic moment in nucleon magnetons.

To evaluate $a(q^2)$ and $b(q^2)$ we expand $j^\mu J_\mu(q^2)$ in a power series in q/m_p via the Foldy-Wouthuysen transformation.[7] We retain terms through order q^2/m_p^2.

$$\begin{aligned} j^\mu J_\mu(q^2) &= \sum_j \bar{v}_f \langle\psi_f| \Bigg\{ \tfrac{1}{2}(f_{1S}+\tau_j^z f_{1V}) e^{i\mathbf{q}\cdot\mathbf{r}_j} - \frac{1}{4m_p}(f_{1S}+\tau_j^z f_{1V}) \\ &\times [\mathbf{p}_j\cdot\boldsymbol{\alpha} e^{i\mathbf{q}\cdot\mathbf{r}_j} + e^{i\mathbf{q}\cdot\mathbf{r}_j}\mathbf{p}_j\cdot\boldsymbol{\alpha}] \\ &- \frac{i}{4m_p}(f_{1S}+\tau_j^z f_{1V}+f_{2S}+\tau_j^z f_{2V})\boldsymbol{\sigma}_j\cdot(\mathbf{q}\times\boldsymbol{\alpha})e^{i\mathbf{q}\cdot\mathbf{r}_j} \\ &+ \frac{q^2}{8m_p^2}(f_{1S}+\tau_j^z f_{1V}+2f_{2S}+2\tau_j^z f_{2V})e^{i\mathbf{q}\cdot\mathbf{r}_j} \\ &+ \frac{i}{16m_p^2}(f_{1S}+\tau_j^z f_{1V}+2f_{2S}+2\tau_j^z f_{2V}) \\ &\times \boldsymbol{\sigma}_j\cdot[\mathbf{p}_j\times(q^0\boldsymbol{\alpha}-\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{r}_j} - e^{i\mathbf{q}\cdot\mathbf{r}_j}(q^0\boldsymbol{\alpha}-\mathbf{q})\times\mathbf{p}]\Bigg\} |\psi_i\rangle v_i . \end{aligned}$$

The charge and moment structures of the nucleons are included in the nucleon form factors f_{1S}, f_{1V}, f_{2S}, and f_{2V} which are normalized so that

$$F_{1(p,n)} = \tfrac{1}{2}(f_{1S}\pm f_{1V}) \quad F_{2(p,n)} = \tfrac{1}{2}(f_{2S}\pm f_{2V}) ,$$

($+$ for p, $-$ for n)

$$q^2=0 , \quad F_{1p}=1 , \quad F_{1n}=0 , \quad F_{2p}=K_p , \quad F_{2n}=K_n ,$$

$$G_E = F_1 - (q^2/4m_p)F_2 , \quad G_M = F_1 + F_2 .$$

The last term in $j^\mu J_\mu$, the spin-orbit term, is dropped because after doing the spin sums it does not contribute to $a(q^2)$ to order q^2/m_p^2 and its correction to $b(q^2)$ is estimated to be at most 5% at $|q^2|=5$ and less for smaller q^2 values. Due to the uncertainty in the experimental values of $b(q^2)$, this seems justified.

One expects a major correction to the above interaction from the exchange effect. This addition to the nucleon current is necessary if for no other reason than to preserve gauge invariance. A standard method of including this is that of Sachs.[5] If the nucleon-nucleon potential includes a charge exchange operator $V_\tau(r_{ij})\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$, Sachs shows that an addition to the current $\mathbf{J}$ of the form

$$\langle\psi_f|\sum_{i\neq j}\sum_j\left\{f_{1V}-\frac{q^2}{4m_p^2}f_{2V}\right\}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)_z \times V_\tau(r_{ij})\int_{\mathbf{r}_i}^{\mathbf{r}_j} d\mathbf{s}\, e^{i\mathbf{q}\cdot\mathbf{s}}|\psi_j\rangle$$

will maintain current conservation. The path over which the line integral is taken is arbitrary since only the divergence of the operator is fixed. Accordingly, we have made the simplest assumption, that the path is a straight line connecting $\mathbf{r}_i$ and $\mathbf{r}_j$. We have not investigated how strongly this choice of path affects the numerical results. Sachs' method is, in fact, highly questionable. Calculations using this method badly underestimate the exchange magnetic moments of He^3 and H^3, which are quite large. In the absence of other tractable exchange-current models, however, we have used this in calculating the cross sections to see its effect as a function of q^2.

There remains finally the choice of a suitable ground-state wave function with total spin $I=\frac{1}{2}$, isotopic spin $T=\frac{1}{2}$, and even parity. Construction of three-nucleon wave functions with these quantum numbers and satisfying the Pauli principle is discussed exhaustively by Sachs.[3] We have taken the wave function to be of the form

$$\psi = c_1\psi_S + c_7\psi_D .$$

ψ_S is an S state which is symmetric under interchange of the three particles' spatial coordinates. Using Sachs' notation

$$\psi_S = \psi_1^{m,t} h_1(\mathbf{r},\boldsymbol{\rho}) N_1 ,$$

with $\psi_1^{m,t}$ a product of spin and isospin functions giving $S=\frac{1}{2}$, $T=\frac{1}{2}$ with projections m and t ($=\pm\frac{1}{2}$ for He^3 and H^3) respectively. N_1 is the normalization constant such that $\langle\psi_S|\psi_S\rangle=1$. The coordinates $\mathbf{r}$ and $\boldsymbol{\rho}$ are, in terms of position vectors of the three particles

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2 , \qquad \boldsymbol{\rho}=\mathbf{r}_1-\mathbf{r}_3+\mathbf{r}_2-\mathbf{r}_3 ,$$

$$\mathbf{R}=\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3) ,$$

and h_1 is then a scalar function symmetric in $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$. ψ_D is that component of the D state which is expected to be dominant,[8] i.e.,

$$\psi_D = \psi_7^{m,t} h_7(\mathbf{r},\boldsymbol{\rho}) N_7 ,$$

[7] K. W. McVoy and L. Van Hove, Phys. Rev. **125**, 1034 (1962).

[8] ψ_7 is expected to be dominant since it is the only D state with a component of two single-particle s orbitals, all others having two p or two d single-particle orbitals.

where $\psi_7^{m,t}$ is an antisymmetric product of spin, isospin, and orbital functions of total orbital angular momentum $L=2$, with $\langle\psi_D|\psi_D\rangle=1$ and $c_1^2+c_7^2=1$. Time-reversal invariance requires c_1/c_7 to be real but its sign is undetermined.

The functions $h_1(\mathbf{r},\boldsymbol{\rho})$ and $h_7(\mathbf{r},\boldsymbol{\rho})$ determine the main features of $a(q^2)$ and $b(q^2)$. In the next section we shall give the results of the calculation for several types of functions.

RESULTS

Elaborate three-nucleon wave functions have been constructed for variational calculations of H^3 and He^3 binding energies.[9] These are probably more elaborate than is necessary for our purpose. Since we wish to calculate only the over-all q^2 dependence of the form factors, we shall limit ourselves to more tractable forms for the radial functions.

Gaussian:

$$h_i=e^{-a_i(u^2/2)}\,,\quad i=1,7\,.$$

Irving functions[10]:

$$h_1{}^{(n)}=\frac{e^{-(a_1/2)u}}{u^{n/2}}\,,\quad n=0,\,1,\,2$$

with

$$u^2=2\sum_{i<j}r_{ij}{}^2=\rho^2+3r^2\,.$$

The Irving functions have the virtue of the correct behavior at large distance. They also are in a sense favored by results in the photodisintegration of He^3. Berman[11] has measured the photon cross section as a function of energy. With a pure S-wave ground state they find that the Irving function ($n=2$, $a_1=0.545$ F^{-1}) fits their data quite well. Gaussian functions, on the other hand, do not. Their calculations, however, neglect corrections due to final-state rescattering, which can be quite considerable. There is thus no totally compelling reason to choose one type of function over the other. We use both in evaluating the S-state contribution to the electron scattering cross sections.

There remains the choice of parameters to be used, both those for the radial functions and the depth and range parameters for the exchange potential. We have calculated $a(q^2)$ and $b(q^2)$ using no exchange current, $V_\tau=0$, and using the values suggested by low-energy n-p scattering[12]

$$V_\tau=\sum_{i,j}{}'\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j(3\boldsymbol{\sigma}_i\cdot\hat{r}_{ij}\boldsymbol{\sigma}_j\cdot\hat{r}_{ij}-\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)v_\tau e^{-d_8r^2}\,,$$

with $v_\tau=4$ MeV and $d_8=0.21$ F^{-2}. This choice fits low-energy n-p scattering, and moreover gives good results for the He^3 electric dipole ordinary and bremsstrahlung weighted sum rules. The exchange current matrix elements vanish between S states; the leading term being the S-D interference. Since this term is quite small, we neglect the even smaller D^2 term. Finally, for the nucleon form factors we have used the equations given by Hand, Miller, and Wilson.[13]

The results of these calculations are given below, along with the experimental points. In comparing them one should remember two points. Firstly, none of these models gives the correct static magnetic moment. These moments are $\mu_{He^3}=-2.127$, $\mu_{H^3}=2.979$, in nucleon magnetons. With pure S states the calculated moments are the neutron and proton moments. If we add a D state, the additional moment has the wrong sign to account for this difference.

$$\mu(He^3)=\mu(n)\{1-\tfrac{2}{3}c_7{}^2[2+\mu(p)/\mu(n)]\}+\tfrac{2}{3}c_7{}^2\,,$$
$$\mu(H^3)=\mu(p)\{1-\tfrac{2}{3}c_7{}^2[2+\mu(n)/\mu(p)]\}+\tfrac{1}{3}c_7{}^2\,.$$

The exchange moment has the right sign, but is much too small, $\mu_{ex}\approx0.04\mu_0$. Since $b(q^2=0)=\mu^2(A^2/z^2)$, we then cannot obtain the correct absolute value of b as a function of q^2, and therefore one should compare the general shape of this curve with the data, rather than the absolute coincidence of the two. The second point is that the extrapolated value of the experimental points for b to $q^2=0$ appears much larger than $\mu^2(A^2/z^2)$. This probably indicates less reliability of the data at the smaller momentum transfers.

Figures 1 and 2 show the results for $a(q^2)$ and $b(q^2)$ assuming a pure S state, for various values of a_1 (here $c_7=0$). As a quantitative comparison the mean-square deviations are formed and shown in Table I where we have scaled $b(q^2)$.

$$\alpha_{He^3}=\sum_{q^2}\left\{\left[\frac{a_{calc}(q^2)-a_{exp}(q^2)}{a_{exp}(q^2)}\right]^2+\left[\frac{b_{calc}(q^2)(A^2\mu^2/z^2b_{calc}(0))-b_{exp}(q^2)}{b_{exp}(q^2)}\right]^2\right\}\,,$$

$$\alpha=\alpha_{He^3}+\alpha_{H^3}\,.$$

The values of α though useful as a guide can be misleading if one is not careful. This is noted in curve A for H^3 where the Irving gives quite a good fit to $a(q^2)$ but the value of α_{H^3} is due largely to $b(q^2)$ which has greater uncertainty than $a(q^2)$, and, of course, is not accurately obtained with only an S state. The three types of Irving wave functions give very similar plots differing mainly in the amount of curvature, with curvature increasing as n goes from 0 to 2. This is demonstrated in curves A and F. The values of α for $n=0$ are consistently lower

[9] J. M. Blatt, G. H. Derrick, and J. N. Lyness, Phys. Letters **8**, 323 (1962).

[10] J. Irving, Phil. Mag. 42, 338 (1951).

[11] B. L. Berman, L. J. Koester, Jr. and J. H. Smith, Phys. Rev. **133**, B117 (1964). See also R. Bösch, J. Lang, R. Müller, and W. Wölfli, Phys. Letters 8, 120 (1964) for results on H^3.

[12] P. O. Davey and H. S. Valk, Phys. Letters **7**, 155 (1963).

[13] L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963), Eqs. (47), (48), (51), and (52).

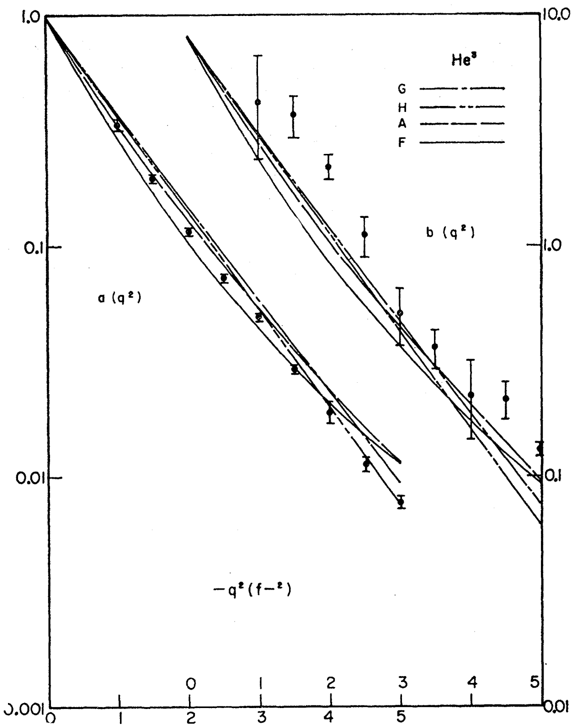

FIG. 1. $a(q^2)$ and $b(q^2)$ versus $-q^2$ for He^3 with pure S states. See Table I for description of A, F, G, and H. The experimental points are those of Collard *et al.* (Ref. 1).

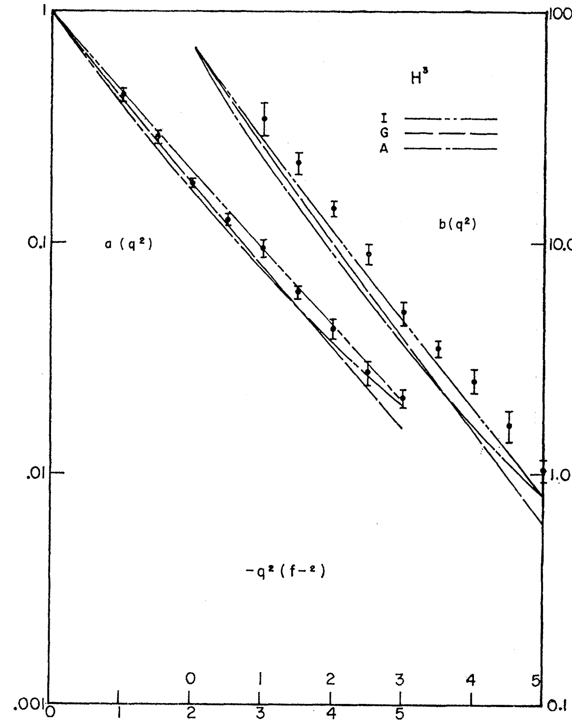

FIG. 2. $a(q^2)$ and $b(q^2)$ versus $-q^2$ for H^3 with pure S states. See Table I for description of A, G, and I. The experimental points are those of Collard *et al.* (Ref. 1).

than for $n=1$ which in turn are lower than for $n=2$. However, for $n=2$ the best fit for α_{He^3} is obtained with the parameter which also fits the energy dependence of the photo cross section,[11] $a_1=0.545$ F^{-1}. Curves G, H, and I show the results for Gaussians. These curves indicate that reasonable results, differing from each other only slightly, can be obtained with either Gaussian or Irving functions. Since the Gaussian functions involve simpler computations, we have used only these in dealing with the $S+D$ wave functions.

Here we have three parameters to choose, a_1, a_7, and c_7^2 and, in addition, the sign of c_7. A lower variational binding energy is obtained with c_7 as positive.[14] Figures 3 and 4 show the results for different values of these

TABLE I. Square deviations of calculated $a(q^2)$ and $b(q^2)$ from experimental values for various wave functions. Values in parenthesis are the lowest possible for the particular wave function considered. Case F also fits the photodisintegration of He^3 and H^3. Only case M has $c_7<0$.

Wave function	a_1	a_7 (F^{-2})	$c_7^2\times10^2$	α	α_{He^3}	α_{H^3}	Coulomb energy (point nucleons) (MeV)	
Irving$_0$	0.916 F^{-1}	...	0	(1.83)	1.28	0.55	0.776	*A*
Irving$_0$	0.882 F^{-1}	...	0	2.35	(0.923)	1.42	0.746	*B*
Irving$_0$	0.95 F^{-1}	...	0	3.07	2.85	(0.22)	0.805	*C*
Irving$_1$	0.74 F^{-1}	...	0	(2.14)	1.46	0.679	0.785	*D*
Irving$_2$	0.558 F^{-1}	...	0	(2.71)	1.59	1.12	0.789	*E*
Irving$_2$	0.545 F^{-1}	...	0	3.03	(1.41)	1.62	0.770	*F*
Gaussian	0.073 F^{-2}	...	0	(1.60)	0.98	0.626	0.759	*G*
Gaussian	0.070 F^{-2}	...	0	2.01	(0.813)	1.19	0.734	*H*
Gaussian	0.079 F^{-2}	...	0	2.88	2.70	(0.174)	0.790	*I*
Gaussian	0.0705 F^{-2}	0.10	1.1	1.28	0.621	0.660	0.743	*J*
Gaussian	0.0669 F^{-2}	0.0785	2.0	1.82	0.452	1.37	0.723	*K*
Gaussian	0.0764 F^{-2}	0.0927	1.45	2.46	2.37	0.090	0.775	*L*
Gaussian	0.0705 F^{-2}	0.10	1.1				0.743	*M*
Gaussian	0.071 F^{-2}	0.05	6	1.32	0.60	0.724	0.729	*N*
Gaussian	0.073 F^{-2}	0.035	6	1.18	0.54	0.64	0.734	*O*

[14] J. Schwinger and E. Gerjuoy, Phys. Rev. **61**, 138 (1942).

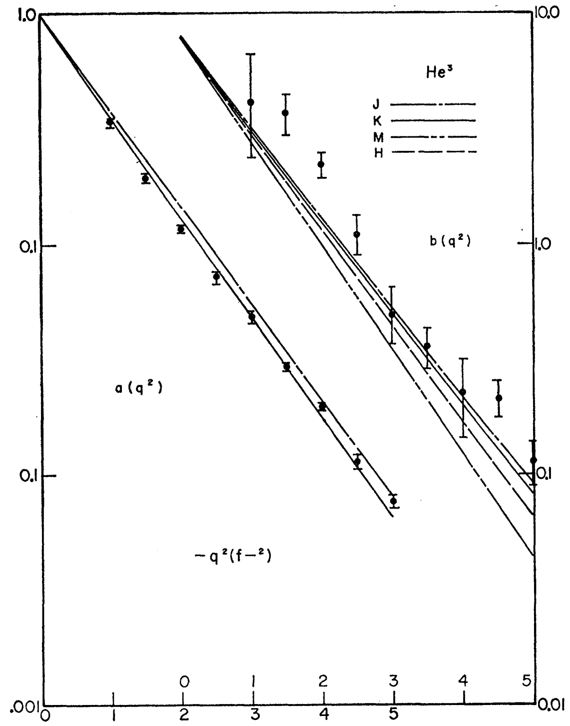

FIG. 3. $a(q^2)$ and $b(q^2)$ versus $-q^2$ for He^3 with $S+D$ states. See Table I for description of J, K, and M. The experimental points are those of Collard *et al.* (Ref. 1).

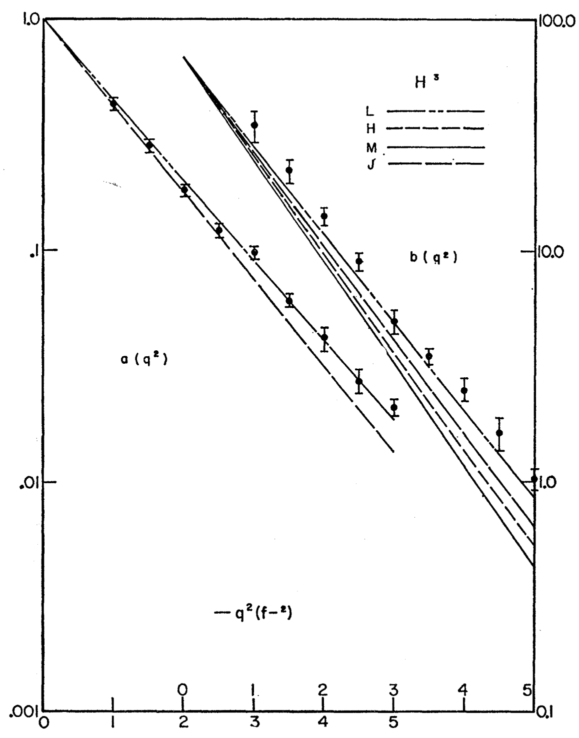

FIG. 4. $a(q^2)$ and $b(q^2)$ versus $-q^2$ for H^3 with $S+D$ states. See Table I for description of J, K, and M. The experimental points are those of Collard *et al.* (Ref. 1).

quantities. The values of a_1 and $c_7{}^2$ were chosen so one of the α's were close to a local minimum for fixed a_7. No true minimum was found since α continued to decrease for $a_7<3.5\times10^{-2}\,\text{F}^{-2}$ which makes $\langle r^2\rangle_D/\langle r^2\rangle_S=(5/3)(a_1/a_7)\gtrsim3.5$. $a(q^2)$ depends only weakly on a_7 and $c_7{}^2$ because its main contribution comes from $G_{\text{ch}}{}^2$ and G_{ch} has no SD term. $b(q^2)$ is more sensitive to the D state, but its greater experimental uncertainty precludes a final determination of these parameters. Finally, the curve M shows the effect of reversing the S-D phase. Herein lies the major difference between this work and Schiff's. We obtain an interference between the dominant S state and other components only in the magnetic currents, not in the charge. Assuming a mixture of two different S states, Schiff obtains this interference in *both* charge and magnetic currents.

TABLE II. S-state range parameters for various wave functions as given by other authors.

Reference		Wave function	a_1	
E. M. Henley and D. U. L. Yu[a]	(compilation of values of other workers)	h_G	0.065	
B. L. Berman, *et al.*[b]	photodisintegration of He^3	$h_I{}^{(2)}$	0.545	
R. Bösch *et al.*[b]	photodisintegration of H^3	$h_I{}^{(2)}$	0.545	
A. N. Gorbunov and A. T. Varfolomeev[c]	photodisintegration of He^3	$h_I{}^{(2)}$	0.71	fits peak position
			0.475	fits peak height
C. Rossetti[d]		$h_I{}^{(0)}$	0.966	
L. I. Schiff[e]	analysis of electron scattering from He^3 and H^3 uses $S+S'$ wave function	h_G	0.0738	
		$h_I{}^{(0)}$	0.898	
This work		h_G	0.073	
		$h_I{}^{(0)}$	0.916	
		$h_I{}^{(I)}$	0.74	
		$h_I{}^{(2)}$	0.558	

[a] E. M. Henley and D. U. L. Yu, Phys. Rev. **133**, B1444 (1964).
[b] See Ref. 9.
[c] A. N. Gorbunov and A. T. Varfolomeev, Phys. Letters **5**, 149 (1963).
[d] C. Rossetti, Nuovo Cimento **14**, 1171 (1959).
[e] See Ref. 3.

The net effect of the exchange current here is quite small. With the well depth and range given above, the fractional increase in $b(q^2)$ over that with $v=0$ is at most 4%. For He^3 in case J its contribution to G_M varies as $e^{-0.196q^2}$ versus $e^{-0.444q^2}$ for the other terms, indicating the exchange moment is spatially more compact than that due to the spins.

Finally, we have also computed the cross sections with the neutron charge form factor equal to zero for all q^2. $a(q^2)$ and $b(q^2)$ are affected at most by 1%.

CONCLUSIONS

The first rather obvious conclusion is that these measurements cannot give information about the neutron charge form factor, unless their accuracy is considerably improved by perhaps an order of magnitude. Even then, the theoretical uncertainties are much too great at present. It would be necessary to develop some reliable method of generating an isotopic vector current which accounts correctly for the magnetic moment. The method used thus far, that of Sachs', gives a result for this in the right direction but an order of magnitude too small. This is the most unreliable facet of any calculation of these nuclear form factors. With the provision that Sach's method of including this exchange current is reasonable, some conclusions may be drawn about the nuclear wave functions. From the curves above, it is possible to fit well the shape of the cross sections for He^3 and H^3 separately with a mixture of S and D states. These fits, however, are not good if the same parameters are chosen for each nucleus, the falloff distance in H^3 being considerably smaller than in He^3. It is doubtful whether the Coulomb repulsion between protons giving a Coulomb energy on 0.764 MeV could account for this; using the wave function from curve J the Coulomb energy for point nucleons is 0.743 MeV. The value for extended nucleons is expected to be 0.669 to 0.600 MeV or 10–20% less on the basis of calculations done by Ohmura and Ohmura.[15] Thus, while a purely symmetric S state is ruled out by the differences in F_{chg} and F_{mag} for He^3, neither can a mixture of S and D states give these form factors correctly for both nuclei, with the same parameters. The differences in finite sizes effects between proton and neutron accounts partially for this difference, but only for about half of it.

Finally, Table II summarizes the best values of the parameters from these curves, and contrasts them with the corresponding quantities found by Schiff and with those found in other types of experiments.

[15] H. Ohmura and T. Ohmura, Phys Rev. 128, 729 (1962).

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Low-Lying Collective States of Sm^{152} and Sm^{154}†

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The excited states up to 2 MeV in Sm^{152} and Sm^{154} have been studied by magnetic analysis of the inelastic protons from thin targets bombarded by a 12-MeV proton beam from the Florida State University tandem Van de Graaff accelerator. The results are compared with previous studies and with the predictions of collective nuclear models. The ground-state band levels up to spin 6 are excited in these experiments. The 2+, 3+, and 4+ states in the gamma vibrational band, the 0+ and 2+ states in the beta vibrational band, and the 1−, 3−, and 5− states in the octupole band have been observed in Sm^{152}. In Sm^{154} the gamma band head is observed at 1443 keV and the octupole band head is observed at 927 keV. Additional levels in these bands and the beta band are suggested. Several other levels are observed above 1100 keV in these nuclei.

INTRODUCTION

THE excited states of deformed nuclei near the boundaries of the deformed regions have been the subject of intensive experimental investigation in recent years. This has come about as a natural extension of the success of the Bohr-Mottelson description[1] of highly deformed nuclei, in order to understand more clearly the relationship between the collective states of spherical and deformed nuclei. The stable isotopes of samarium ($Z=62$) are very appropriate for such a study since they extend from the $N=82$ closed neutron shell to $N=92$ which is well into the region of deformation beyond $N=89$. Here we are concerned with the two deformed even isotopes Sm^{152} and Sm^{154}. Further results concerning the odd isotopes of samarium will be forthcoming in a later paper.

While the low-lying levels of Sm^{152} are relatively well studied[2–8] through the decay of Eu^{152} and of Eu^{152m}, only Coulomb excitation data[9–12] have been previously available for states in Sm^{154} above the 2+

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[1] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 27, No. 16 (1953).

[2] J. M. Cork, M. K. Brice, R. G. Helmer, and D. E. Sarason, Phys. Rev. 107, 1621 (1957).

[3] O. Nathan and M. A. Waggoner, Nucl. Phys. 2, 548 (1957).

[4] B. V. Bobykin and K. M. Novik, Izv. Akad. Nauk SSSR, Ser. Fiz. 21, 1556 (1957).

[5] O. Nathan and S. Hultberg, Nucl. Phys. 10, 118 (1959).

[6] L. Grodzins and H. Kendall, Bull. Am. Phys. Soc. 1, 163 (1956).

[7] S. B. Burson and L. C. Schmid, Atomic Energy Commission Report ANL-5911 1958, p. 8 (unpublished).

[8] I. Marklund, Nucl. Phys. 9, 83 (1958).

[9] J. de Boer, G. Goldring and H. Winkler, Bull. Am. Phys. Soc. 8, 387 (1963).

[10] Y. Yoshizawa, B. Elbek, B. Herskind, and M. C. Oleson, Bull. Am. Phys. Soc. 9, 107 (1964).

[11] G. G. Seaman, J. S. Greenberg, D. A. Bromley, and F. K. McGowan, Bull. Am. Phys. Soc. 9, 108 (1964).

[12] O. Hansen and O. Nathan, Nucl. Phys. 42, 197 (1963).