Perturbed Rotational Band Spectra of Some Two-Quasiparticle $K\pi=0^-$ States in Deformed Even-Even Nuclei*

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In deformed nuclei two states with $K_1\pi_1=\frac{1}{2}^-$ and $K_2\pi_2=\frac{1}{2}^+$ can couple to form a degenerate doublet with projections 0 and 1. We have investigated the effect of the Coriolis force and the spin-dependent residual interaction on such states. As a result of these interactions, the rotational bands are strongly mixed, and the level ordering becomes quite complicated. For certain $K=\frac{1}{2}$ states, the lowest lying members of the rotational band resemble the $K\pi=0^-$ bands observed in deformed even nuclei in that the band ordering is 1, 3, 5, and the effective moment of inertia is significantly larger than that of the ground-state band. However, such bands are usually assigned as $K\pi=0^-$ projections of $\lambda=3$ (octupole) vibrations. Thus the difference in appearance between the bands belonging to collective and two quasiparticle $K\pi=0^-$ base states may in some cases be less than previously suspected, and hence definite assignments should require more detailed experimental data than the band ordering alone.

I. INTRODUCTION

In even-even nuclei, principally in the regions $A\sim154$ and $A\geq226$, states of low excitation energy (<1 MeV) with $I\pi=1^-$ are systematically observed. The branching ratios of the $E1$ radiations which de-excite these levels to the ground and first excited 2+ states, when compared with predictions of the collective model, have led to their being assigned a $K=0$ quantum number.[1] In some cases, states with $I=3$ and $I=5$ are observed which follow the rotational $I(I+1)$ relationship relative to the 1^- states. No even-spin states are observed in these bands. The moments of inertia computed from the energy level spacing are approximately 1.7 times those computed for the ground-state bands. These facts led to the suggestion that the states are $K=0$ projections of collective octupole ($\lambda=3$) oscillations.[2] Some force is added to this assignment by the recent observation of strong electric octupole transitions to the 3− states of these bands in Coulomb excitation experiments using heavy ion beams,[3] and strong excitation of the bands in inelastic scattering.[4]

Recently an attempt has been made to understand the $\lambda=3$ oscillations from a "microscopic" viewpoint. An examination of the wave functions of the $K=0$ component of the octupole vibration shows that in some nuclei the amplitudes of the two-quasiparticle components of the states are very large.[5] These calculations have led us to consider whether it is possible to reproduce the properties of the $K\pi=0^-$ bands, assuming they are pure two-quasiparticle excitations.

II. THEORY

Rotational bands based on $K\pi=0^-$ levels of $\lambda=3$ oscillations contain no even-spin states if the nucleus is axially symmetric. A well-known result is that their effective moments of inertia should also appear to be larger than those of the ground-state ($K\pi=0^+$) band through their coupling to the $\lambda=3$ oscillation with a $K=1$ projection. In contrast, in bands based on $K\pi=0^-$ two-quasiparticle states, the even-spin states exist, and the level ordering and apparent increase of the moment of inertia must originate through residual interactions. Thus the question we wish to investigate is clear: if a $K\pi=0^-$ band base state is a two-quasiparticle state, can residual interactions reproduce the observed level ordering, effective moments of inertia, and other properties? In the present paper we discuss the results of this investigation.

Of the many possible residual interactions we shall only use those which are known to affect significantly the particle spectra in deformed nuclei. The types of interaction which are empirically important, at least with respect to the basis of the unified model plus Nilsson Scheme[6–9] are the RPC (rotational particle coupling[6,10]) and the spin-dependent interactions which are well established in deformed odd-odd nuclei.[11–13]

As a starting point in the analysis we recall that

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[1] F. S. Stephens, F. Asaro, and I. Perlman, Phys. Rev. **96**, 1568 (1954).
[2] R. F. Christy, widely quoted private communication to A. Bohr; P. O. Lipas and J. P. Davidson, Nucl. Phys. **26**, 80 (1961); D. P. Leper, *ibid.* **50**, 234 (1964).
[3] F. S. Stephens and R. Diamond (private communication, 1963).
[4] B. Elbeck (private communication, 1964).
[5] V. G. Soloviev, P. Vogel, and A. A. Korneichuk, Doklady Akad. Nauk SSSR (to be published); also, private communication, 1964.
[6] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **26**, No. 14 (1952).
[7] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).
[8] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).
[9] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter **1**, No. 8 (1959).
[10] A. K. Kerman, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **30**, No. 15 (1956).
[11] C. J. Gallagher, Jr., and S. A. Moszkowski, Phys. Rev. **111**, 1282 (1958).
[12] C. J. Gallagher, Jr., Phys. Rev. **126**, 1525 (1962).
[13] C. J. Gallagher, Jr., and V. G. Soloviev, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter **2**, No. 2 (1962).

Newby[14] has pointed out that in configurations in odd-odd nuclei in which $|\Omega_p-\Omega_n|=0$, there are, besides the usual spin-dependent interactions which lead to the removal of the $|\Omega_p\pm\Omega_n|$ degeneracy, additional matrix elements of the residual interaction which can cause a displacement of even- and odd-spin bands. Several experimental examples of such displacements are known.[15] Newby's results, although derived for odd-odd systems, are essentially directly applicable to two-quasiparticle states in even-even nuclei. In general, when $K>\frac{1}{2}$ for the two states coupling to form the $K=0$ band, the moments of inertia of the displaced bands should not significantly differ from the ground-state band, in conflict with the observed results. This rules out the possibility that pure states with $K>\frac{1}{2}$ produce the observed rotational bands. However, a special case of the $K=0$ system occurs when two $K=\frac{1}{2}$ particles couple to form $K=0$ and $K=1$ bands. In this case the RPC as well as the residual spin-dependent force plays a role, leading to the result that the even and odd bands will not in general have the same effective moments of inertia, and both will differ from that of the ground-state band. In this case, the magnitude of the difference will depend on the decoupling constants of the two $K=\frac{1}{2}$ states, and hence the effective moments of the bands will depend on the intrinsic states. Thus it appears qualitatively that the only two-quasiparticle $K=0$ states that might reproduce the effective moments of inertia are those involving the coupling of two states with $K=\frac{1}{2}$.

Now if we attempt to relate these considerations to the observed $K\pi=0^-$ bands, a reasonable assumption is that they must involve the coupling of two $K=\frac{1}{2}$ states near the Fermi surface since the experimental excitation energies are so low. As a gross check on this assumption, consider the Nilsson diagrams[9] for the region $A>208$ shown in Figs. 1(a) and 1(b) and the systematics of $I\pi K=1-0$ states energies shown in Table I. Marked on the Nilsson diagram by broadened lines are the $K=\frac{1}{2}$ states available. In the proton spectrum the two states are [530↑] and [660↑]. The [530↑] state is the well-known ground state of the Pa isotopes. The state [660↑] is never observed as a ground state but should be a low-lying excitation. Both states have large decoupling factors for finite nuclear deformation. Two neutron states, [770↑] and [640↑], are also available at considerably greater excitation energy. Thus it appears that in Ra and Th, two of the lowest lying

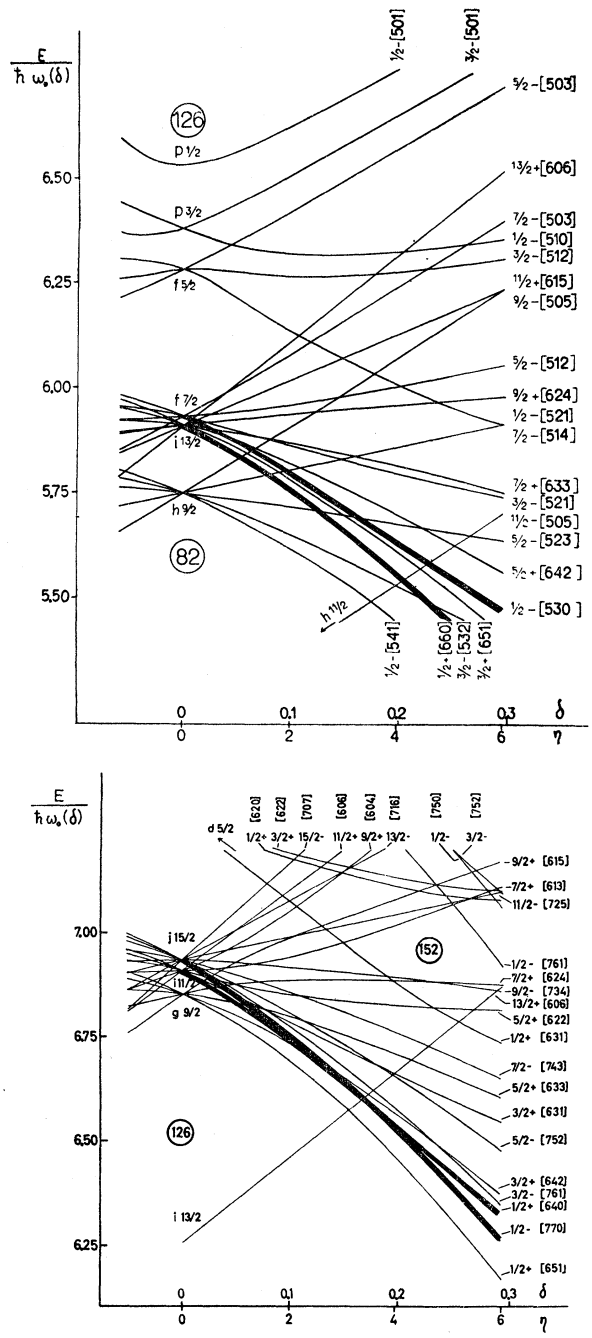

FIG. 1(a) Nilsson diagram for the proton orbitals in the region $Z>82$ ($A>208$). The two $K=\frac{1}{2}$ orbitals which can couple to form the lowest lying proton two-quasiparticle $K\pi=0, 1-$ doublet in the Ra-Th region are indicated by broadened lines. The [530↑] state is observed as the ground state of protoactinium. (b) Nilsson diagram for the neutron orbitals in the region $N>126$ ($A>208$). The two $K=\frac{1}{2}$ orbitals which can couple to form the lowest lying neutron two-quasiparticle $K\pi=0, 1-$ doublet in the Ra-Th region are indicated by broadened lines. Neither of these states has as yet been experimentally found in deformed odd-A nuclei.

two-quasiparticle states should be made up of $K=\frac{1}{2}$ states coupling to $K\pi=0^-$. A similar situation exists in the Sm region, where the states [530↑] and [660↑]

[14] N. D. Newby, Phys. Rev. **125**, 2063 (1962).

[15] F. Asaro, I. Perlman, J. O. Rasmussen, and S. G. Thompson, Phys. Rev. **120**, 934 (1960); R. G. Helmer, and S. B. Burson, *ibid.* **119**, 788 (1960); J. S. Geiger, R. L. Graham and G. Ewan, Bull. Am. Phys. Soc. **5**, 255 (1960); I. Rezanka, J. Frana, J. Adam, and L. K. Peker, Izv. Akad. Nauk SSSR Ser. Fiz. **26**, 127 (1961); C. J. Gallagher, Jr., A. Nanemson and P. C. Simms, Nucl. Phys. **49**, 443 (1963); G. Struble, N. Shelton, and R. K. Sheline, Phys. Rev. Letters **10**, 58 (1963); R. K. Sheline, Bull. Am. Phys. Soc. **8**, 80 (1963); J. Valentin, D. J. Horen, and J. M. Hollander, Nucl. Phys. **31**, 373 (1962); S. Bjornholm and O. B. Nielsen, *ibid.* **30**, 488 (1962).

occur in the neutron diagram and [550↑] and [420↑] in the proton diagram. In the remaining nuclei in both regions, however, such configurations have much higher excitation energies. These results suggest qualitatively that only in the Ra-Th and Sm regions do low-lying states exist which might have properties like those observed. In the calculations discussed below we attempt quantitatively to determine if this could be the case.

A. Energies of Two-Quasiparticle States

Soloviev has given tables of two-quasiparticle excitation energies in the heavy-element region.[16] His calculations do not include much of this mass region, however, because of the rapidly varying self-consistent-field energies in the region of deformed nuclei with lower A. Recently some corrections that should be applied to his model have been pointed out.[17] However, his estimates of the unsplit $|\Omega_1 \pm \Omega_2|$ doublet energies appear valid for the observed states[18] to within his estimated error of 15%, so we have used them for orientation purposes. His calculated energies for $K\pi = 0^-$ states are greater than 1.5 MeV throughout most of the region. A simple unsplit-doublet interpretation of the states is clearly inadequate.

TABLE I. Systematics of $I\pi K = 1-0$ states in the region $A > 208$.[a]

Nuclide	Energies of rotational states[b]			Inertial constants $\hbar^2/2\mathcal{I}$ of rotational bands (keV)[c]	
	$1-$	$3-$	$5-$	$K\pi=0-$	$K\pi=0+$
Rn^{218}	(800)				
Rn^{220}	(650)				
Rn^{222}	(610)				
Ra^{220}	(410)				
Ra^{222}	242				
Ra^{224}	217	(289)		7.2	14.1
Ra^{226}	253	320	445	6.7	11.3
Th^{224}	246				
Th^{226}	230				
Th^{228}	327	396	514	6.9	9.6
Th^{230}	503	572		6.9	8.8
Th^{232}	1045	1095		5.1	8.3
U^{232}	564	630		6.6	7.9
U^{234}	787				
U^{238}	679	724		4.5	7.5
Pu^{238}	605				

[a] All data as are reported (with references) in the compilation of B. S. Dzhelepov, L. K. Peker, and V. O. Sergeev, Academy of Sciences of the USSR, 1963 (unpublished).
[b] Parentheses indicate assignments not definitely established.
[c] Calculated from $1-$ to $3-$ spacing for the $K\pi=0-$ bands and from the 0^+ to 2^+ spacing from the $K\pi=0+$ (ground state) bands. All values have been given to nearest tenth of a keV.

B. Forces Removing the Degeneracy of the $K=0$, 1 Doublet

1. The Rotational Hamiltonian and RPC

We consider an even-even nucleus which contains two unpaired like nucleons; all other particles are paired in the appropriate Nilsson orbitals. The two unpaired particles have intrinsic wavefunctions $\chi_\kappa(1)$ and $\phi_{\kappa'}(2)$ where κ and κ' are the components of particle angular momentum along the nuclear symmetry axis, z. The parity of the rotational band based on these intrinsic states is $\pi = \pi(1)\pi(2)$. When $\kappa = \kappa'$, we can combine these two states to form both a $K=0$ and a $K=2\kappa$ band. The usual rotational wave functions are (for total spin I)

$$|I,M,K=0\rangle = ((2I+1)/16\pi^2)^{1/2} \sum_{jj'} C_j D_{j'} (-)^{i'-1/2} \mathfrak{D}_{M,0}{}^I \times [\chi_\kappa{}^j(1)\phi_{-\kappa}{}^{j'}(2) + (-)^{I-j-j'} \chi_{-\kappa}{}^j(1)\phi_\kappa{}^{j'}(2)] \quad (1)$$

and

$$|I,M,K=2\kappa\rangle = ((2I+1)/16\pi^2)^{1/2} \sum_{jj'} C_j D_{j'} [\chi_\kappa{}^j(1)\phi_\kappa{}^{j'}(2)\mathfrak{D}_{M,2\kappa}{}^I + (-)^{I-j-j'} \chi_{-\kappa}{}^j(1)\phi_{-\kappa}{}^{j'}(2)\mathfrak{D}_{M,-2\kappa}{}^I]. \quad (2)$$

The C's and D's are expansion coefficients,

$$\chi_\Omega = \sum_j C_j \chi_\Omega{}^j,$$

and can be obtained from the Nilsson wave functions. Since we are dealing with identical particles, both of the above wave functions must be antisymmetric under interchange of particles 1 and 2, although, for the sake of brevity, this will not be explicitly shown.

The rotational energy operator is given by

$$T_{\rm rot} = (1/2\mathcal{I})[(J-j_1-j_2)^2 - (J-j_1-j_2)_z{}^2] + (1/2\mathcal{I}_3)(J-j_1-j_2)_z{}^2 = T_0 + T_{p-p} + T_{\rm RPC}, \quad (3)$$

where

$$T_0 = (1/2\mathcal{I})[J^2 + j_1{}^2 + j_2{}^2 + 2j_{1z}j_{2z} - 2J_z(j_1+j_2)_z], \quad (4a)$$

$$T_{p-p} = (1/2\mathcal{I})(j_{1+}j_{2-} + j_{1-}j_{2+}), \quad (4b)$$

and

$$T_{\rm RPC} = -(1/2\mathcal{I})[J_+(j_1+j_2)_- + J_-(j_1+j_2)_+]. \quad (4c)$$

T_0 is the simple rotational energy operator; T_{p-p} contains the rotational particle-particle interaction; $T_{\rm RPC}$ is the Coriolis contribution which connects the nuclear rotation to the rotation of the individual particles. All three operators are diagonal in I. Only $T_{\rm RPC}$ is capable of connecting states of different K under certain conditions.

We calculate the matrix elements of (4a–4b) using the wave functions (1) and (2). The results are

$$\langle|T_0|\rangle_{I,M,K=0} = (1/2\mathcal{I})[I(I+1) - 2\kappa^2 + b_1 + b_2], \quad (5a)$$

$$\langle|T_0|\rangle_{I,M,K=2\kappa} = (1/2\mathcal{I})[I(I+1) - 6\kappa^2 + b_1 + b_2], \quad (5b)$$

$$\langle|T_{p-p}|\rangle_{I,M,K=0} = (1/2\mathcal{I})\delta_{\kappa,1/2}(-)^{I+1} a_1 a_2, \quad (5c)$$

[16] T. Voros, V. G. Soloviev, and I. Siklos, Joint Institute for Nuclear Research Report E-932, Dubna, 1962 (unpublished).
[17] S. G. Nilsson, Nucl. Phys. (to be published).
[18] The data on two-quasiparticle states in the heavy element region are extremely sparse. This statement is based on the more detailed comparison carried out in the rare-earth region.

and

$$\langle|T_{p-p}|\rangle_{I,M,K=2\kappa}=0. \tag{5d}$$

In the above, we have used

$$b_1=\sum_j|C_j|^2j(j+1)$$

and

$$a_1=-\sum_j(-)^{j+1/2}(j+\tfrac{1}{2})|C_j|^2,$$

with similar expressions for b_2 and a_2. The a's are the usual decoupling parameters. The terms involving b_1 and b_2 arise from single particle operators which may be absorbed in the self-consistent field and will not be considered further. The matrix element of T_{p-p} is nonzero only when $K=0$ and $\kappa=\frac{1}{2}$.

The only nonzero matrix element of T_{RPC} is

$$\langle I, M, K=0|T_{\mathrm{RPC}}|I, M, K=2\kappa\rangle = -\frac{1}{2\mathcal{I}}\delta_{\kappa,1/2}(I(I+1))^{1/2}[a_2+(-)^{I+1}a_1]. \tag{6}$$

T_{RPC} is off-diagonal in K; it contributes only when $\kappa=\frac{1}{2}$ and mixes the resulting $K=0$ and $K=1$ bands. If the total Hamiltonian were given by T_{rot} alone, then the energy levels could be obtained by diagonalizing the symmetric 2×2 matrix,

$$2\mathcal{I}E=\begin{pmatrix}[I(I+1)-2\kappa^2-\delta_{\kappa,1/2}(-)^Ia_1a_2] & -\delta_{\kappa,1/2}(I(I+1))^{1/2}[a_2-(-)^Ia_1]\\ \cdots & [I(I+1)-6\kappa^2]\end{pmatrix}. \tag{7}$$

One must also include the effect of the residual particle-particle interaction which would be different for the two bands. Only the contribution due to the two unpaired particles must be included. Before considering this additional complication, a few remarks regarding Eq. (7) are in order.

Since the spectrum of any band is limited by the condition $I\geq K$, there will be only one level for each integral I, $0\leq I<2\kappa$. Each of these will correspond to $K=0$ and is given by Eqs. (5a) and (5c) (E_{11} of our matrix). Even for $I\geq2\kappa$, the matrix of Eq. (7) will be diagonal unless $\kappa=\frac{1}{2}$ ($2\kappa=1$). Band mixing via RPC arises only in the latter case when we have $K=0$ and $K=1$ bands. This is precisely the situation which we wish to investigate since the strong repulsion of energy levels is capable of appreciably altering the effective moment of inertia.

2. *The Residual Interaction*

As a residual interaction, we choose

$$V=\alpha_{\mathrm{se}}V_{\mathrm{se}}+\alpha_{\mathrm{to}}V_{\mathrm{to}}, \tag{8}$$

where

$$V_{\mathrm{se}}=\tfrac{1}{8}(1+P_M)(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)V_{\mathrm{se}}(r_{12}), \tag{9a}$$

and

$$V_{\mathrm{to}}=\tfrac{1}{8}(1-P_M)(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)V_{\mathrm{to}}(r_{12}). \tag{9b}$$

P_M is the two-particle space-exchange operator. The subscript se (to) refers to the singlet-even (triplet-odd) projection operator which is explicitly shown in Eq. (9). Singlet-odd and triplet-even operators do not enter since our wave functions are totally antisymmetric. Matrix elements of V taken between rotational states conserve both I and K. Such an interaction contributes only to the diagonal terms in Eq. (7).

The matrix elements of V may be written as

$$\langle|V|\rangle_{I,K=0}=\alpha_{\mathrm{se}}(\bar{A}_0\pm\bar{B}_0)+\alpha_{\mathrm{to}}(\bar{A}_3\pm\bar{B}_3) \tag{10a}$$

and

$$\langle|V|\rangle_{I,K=2\kappa}=\alpha_{\mathrm{se}}\bar{M}_0+\alpha_{\mathrm{to}}\bar{M}_3. \tag{10b}$$

The $+(-)$ sign is used in Eq. (10a) when I is even (odd). Expressions for the $\bar{A}$'s, $\bar{B}$'s, and $\bar{M}$'s are given in the Appendix.

Since the available experimental data are so sparse, there is little point in varying parameters so as to obtain a best fit. We choose instead to fix our parameters at reasonable values and see where our model leads us. For simplicity, we take V to be Gaussian [$V\sim\exp(-r^2/\lambda^2)$]. Consequently, we have a model with five parameters: α_{se} and α_{to}, interaction strengths; λ_{se} and λ_{to}, characteristic distances of interaction; and ν, the harmonic-oscillator length parameter used in obtaining radial shell-model wave functions.

For the interaction, we employ the parameters of the Gaussian n-n potential used by True and Ford, $\alpha_{\mathrm{se}}=-32.5$ MeV, $\alpha_{\mathrm{to}}=0$, and $\lambda_{\mathrm{se}}=1.85$F.[19] These same authors also obtain an estimate for ν based on a classical turning-point argument,

$$\nu^2=R^2/(2l_{\max}+3), \tag{11}$$

where $R\sim1.2A^{1/3}$(F), and $l_{\max}$ is the largest orbital angular-momentum state which is occupied.

III. RESULTS

A. Heavy Element Region

1. *Proton Orbitals*

The proton orbitals (660↑) and (530↑) form the only low-lying $K\pi=0^-$ state of interest in the Ra-Th region. Both particles have $\kappa=\frac{1}{2}$ and can form bands with $K=0$ and $K=1$. All bands based on these states will have odd parity. Using $A=226$ and $l_{\max}=6$, we obtain $\nu=1.88$ F. The parameters ν and λ enter only in the combination $\zeta=(\nu/\lambda)^2$, which in our example has the

[19] W. True and K. Ford, Phys. Rev. **109**, 1675 (1958).

value $\zeta=1.04$. These parameters have been used to calculate the level spectrum for various values of nuclear distortion, $\eta=2$, 4, and 6. For the case $\eta=4$, we obtain

$$\bar{A}_0=0.0075,\quad \bar{B}_0=0.0071,\quad \bar{M}_0=0.0059,$$
$$\bar{A}_3=0.0134,\quad \bar{B}_3=0.0026,\quad \bar{M}_3=0.0137.$$

The matrix elements of the residual interaction are

$$\langle|V|\rangle_{I\text{even},K=0}=-\ 14\text{ keV},$$
$$\langle|V|\rangle_{I\text{odd},K=0}=-474\text{ keV},$$

and

$$\langle|V|\rangle_{I,K=1}=-191\text{ keV}.$$

Thus the main effect of the residual interaction is to depress the odd I states of the $K=0$ band relative to all other levels. Using the level spacing of the ground state $K=0$ band of Ra^{226}, we obtain a value for the inertial constant,

$$(1/2\mathcal{I})=11.27\text{ keV}.$$

If this same constant is used for the two-particle excited state, we are now in a position to diagonalize the total energy matrix (rotational plus interaction). The lowest levels have $I=1$, 3, and 5 in that order. From a simple rotational model we would expect $E_I\sim I\ (I+1)$ or

$$(E_5-E_1)/(E_3-E_1)=(30-2)/(12-2)=2.8.$$

This should be compared with a value of 2.89 obtained from our model.

If our energy levels are interpreted as a simple rotational band, the effective inertial constant is

$$(1/2\mathcal{I})=8.19\text{ keV},$$

about $\frac{2}{3}$ of the corresponding quantity for the ground-state band.[20] Also one should notice that the low lying $I=1$, 3, and 5 levels are predominantly composed of $K=0$ components (98, 94, and 87%, respectively) which might lead to their identification as a $K=0$ band by virtue of their gamma branching ratios. The next state in the spectrum is not the usually expected $I=7$ level, but rather $I=4$, followed by $I=2$ and 6. These states are predominantly $K=1$ (70, 81, and 64%).

Similar results have been obtained for other values of η. The corresponding spectra are plotted as a function of η in Fig. 2. (A similar plot for the case of no residual interaction is given in Fig. 3.) The low-lying states ($I=1$, 3, 5) maintain the same ordering and roughly the same spacing throughout the range of distortions considered ($2<\eta<6$). The ratio $(E_5-E_1)/(E_3-E_1)$ varies from 2.84 at $\eta=2$ to 2.96 at $\eta=6$, which again should be compared with the simple rotational value 2.80. The corresponding inertial constants are 9.1 and 7.3 keV, approximately 0.81 and 0.64 times the corresponding quantity for the ground-state band (11.27 keV).

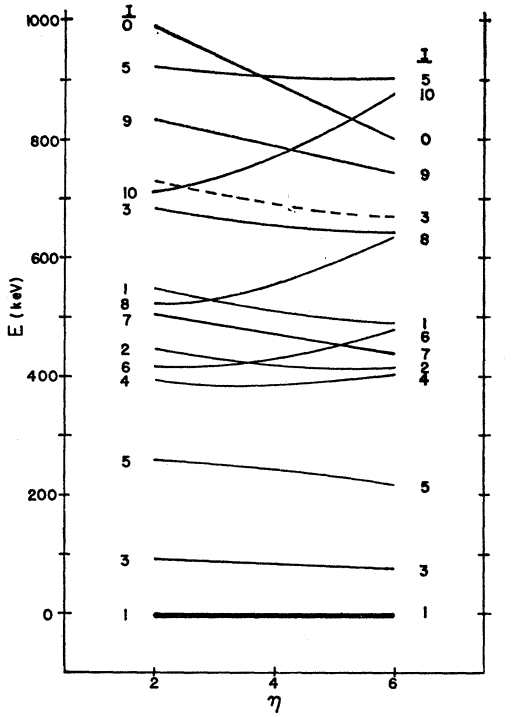

FIG. 2. Energy spectrum versus deformation η for proton states [660↑] and [530↑]. The parameters characterizing the residual interaction are $\alpha_{se}=-32.5$ MeV and $\zeta=1.04$. Energies are shown relative to the lowest $I\pi=1-$ state. The position of the unsplit doublet is indicated by the dashed line.

Contrary to the remarkable stability of the low-lying states with respect to variation of η is the sensitivity of ordering of higher states. The important point here is not the definite order for any particular value of η, but rather that even-spin states and one additional $I=1$ state make their appearance at relatively low excitations ($E_I-E_1\lesssim 0.5$ MeV). Whereas the three lowest states would be experimentally determined as $K\pi=0^-$, these higher states (except for $I=7$) would be largely $K\pi=1^-$.

The dependence on ζ has also been investigated. Calculations were performed for the range $0.8<\zeta<1.3$ ($\zeta=1.04$ was used for the previous part of the calculation). The predominantly $K=0$ odd spin states are relatively unaffected by small changes in ζ; their energies relative to the lowest $I=1$ state fluctuate by about 10% over the range in question. The positions

[20] In order to obtain a better estimate for the moment of inertia of the two-quasiparticle state, we should probably have used the empirical fact that for two-quasiparticle states in deformed even-even nuclei the moments of inertia, $\mathcal{I}_{ee}(2)$, seem to be best fitted by the relationship $\mathcal{I}_{ee}(0)<\mathcal{I}_{ee}(2)\lesssim\mathcal{I}_{ee}(0)+\Delta\mathcal{I}_1+\Delta\mathcal{I}_2$. In this expression $\mathcal{I}_{ee}(0)$ is the moment of inertia of the ground-state rotational band and $\Delta\mathcal{I}_1$ and $\Delta\mathcal{I}_2$ are the incremental moments resulting from the odd-particle motion (see Ref. 9). Since roughly $\Delta\mathcal{I}\sim n_z$ (n_z is the number of nodes along the axis of symmetry of the odd-particle state), and n_z is large for all the $K=\frac{1}{2}$ states of interest here, we can quite reasonably expect that the effective moments of inertia of these bands should be considerably larger than we have calculated assuming $\mathcal{I}_{ee}(2)=\mathcal{I}_{ee}(0)$.

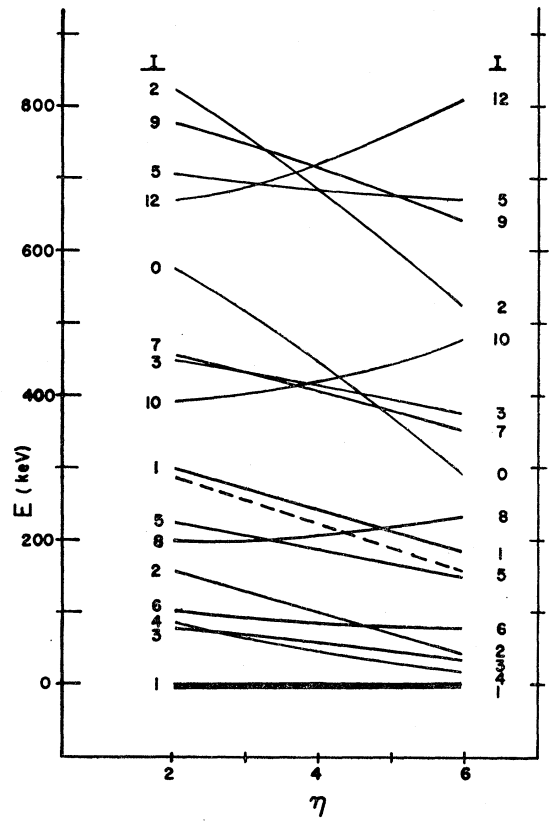

FIG. 3. Energy spectrum versus deformation η for proton states [660↑] and [530↑]. No residual interaction has been included. Energies are shown relative to the lowest $I\pi=1-$ state. The position of the unsplit doublet is indicated by the dashed line.

of the other levels depend much more sensitively on ζ since their location is determined largely by the residual interaction.

By increasing the strength (α_{se}) of the residual interaction, we increase proportionately the $K=0, K=1$ splitting which significantly affects band mixing. If, however, one correspondingly reduces the range of the potential (e.g., keeping the volume integral of the potential constant), then the spectra are not significantly affected. Small admixtures of a triplet-odd residual interaction do not change the qualitative results of this section.

2. Neutron Orbitals

The spectrum which is obtained when using the neutron states [640↑] and [770↑] is illustrated in Fig. 4. The same interaction parameters have been employed as for the previous discussion and $\zeta=0.914$. Since the decoupling parameters of these two states both have the same sign, they tend to depress the even I states below those of odd I. (Also the Coriolis interaction is strongest for even I.) This works against V which tries to depress I odd relative to I even.[21] The bands are so thoroughly mixed that for $\eta=4$ the lowest state has $I=5$. These intrinsic states seem entirely incapable of producing the desired spectrum.

B. Rare-Earth Region

1. Proton Orbitals

We have investigated the proton states [550↑] and [420↑]. Using $\zeta=0.91$, we have calculated the energy levels as a function of deformation (Fig. 5). The result is similar to that obtained for the two-proton state of the Ra region. The lowest lying states have $I=1$, 3, and 5, respectively, all predominantly with $K=0$. At excitations of about 600 keV relative to the $I\pi=1^-$ level, even-spin states begin to make their appearance. The spacing of the low-lying odd-spin states is again compressed yielding an inertial constant which is about one-half of that found for the ground-state band ($1/2\mathcal{I}\sim 20$ keV).

2. Neutron Orbitals

The neutron states in this region are the states [660↑] and [530↑], which are the proton orbitals in the heavy element region. Although the parameters of the Nilsson scheme change between the two regions,

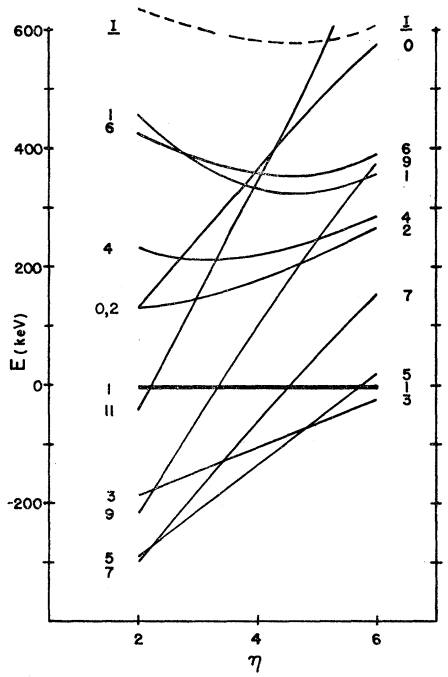

FIG. 4. Energy spectrum versus deformation η for neutron states [640↑] and [770↑]. The parameters characterizing the residual interaction are $\alpha_{se}=-32.5$ MeV and $\zeta=0.914$. Energies are shown relative to the lowest $I\pi=1-$ state. The position of the unsplit doublet is indicated by the dashed line.

[21] In the previous case of two unpaired proton states, the states had decoupling parameters of opposite sign so that both the Coriolis and residual interactions tend to lower the states with odd I.

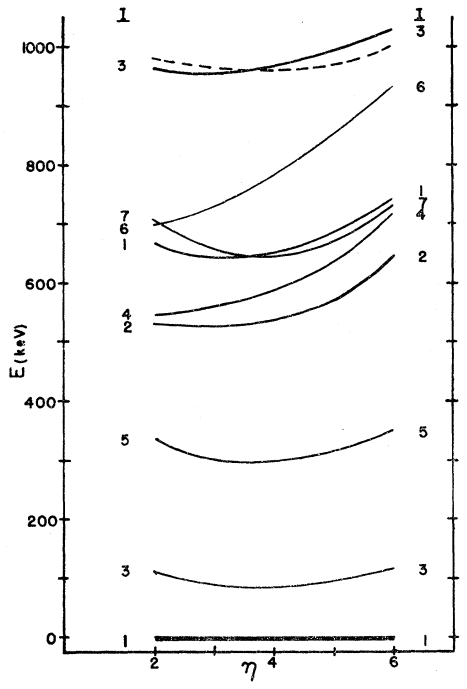

FIG. 5. Energy spectrum versus deformation η for proton states [550↑] and [420↑]. The parameters characterizing the residual interaction are $\alpha_{se}=-32.5$ MeV and $\zeta=0.91$. Energies are shown relative to the lowest $I\pi=1-$ state. The position of the unsplit doublet is indicated by the dashed line.

the qualitative results are clearly the same as shown in Fig. 2. It should be noted, however, that none of the $K=\frac{1}{2}$ states in the rare-earth region have been experimentally identified in the low-energy spectra of deformed nuclei, in contrast to the well-established appearance of the (530↑) state in the heavy region. This fact implies that the two-quasiparticle states should only appear at relatively high excitation energies.

IV. CONCLUSION

Our results indicate that over the mass region of deformed nuclei it is not in general possible, assuming reasonable residual interactions, to obtain low-lying two-quasiparticle states that have the properties of the $K\pi=0^-$ bands observed for example in Ra^{226}.[22] However, in special cases, possibly in Ra^{226}, it is possible to reproduce both the effective moment of inertia of the observed band and the ordering of levels [but not an enhanced $B(E3)$ should it be found] with a specific two-quasiparticle state. In this case, our calculations indicate that a complicated group of odd-parity levels should be observed at somewhat higher energy than the $I\pi K=5-0$ state. These states should be observable experimentally, although the even spin states would not have been observed in the $\alpha-$ decay experiments (where the odd-spin states were found) because of the parity selection rule in α decay.

The recent calculations of Soloviev, Vogel, and Korneichuk[5] indicate that it is possible to reproduce the energies of the observed $I\pi K=1-0$ states in the deformed region assuming they are octupole excitations. Enhanced $B(E3)$'s to the 3^- states are of course expected in this interpretation, although actual numbers have not as yet been calculated. These authors have also not attempted to calculate moments of inertia for the observed bands, so there is also no check other than the collective model estimate on this point.

Our results support the general view that the $I\pi K=1-0$ states are collective excitations, in that we find it generally difficult to get low-lying configurations which reproduce the observed moments of inertia. We think these results emphasize the need for clear experimental definition of the properties of the states of this type before they are assigned as collective vibrations. In addition, because the states whose properties we investigated are in the spectrum, our calculations indicate that the interpretation of odd-parity states, particularly at the beginning of the deformed regions, may be more difficult than previously suspected. This is especially true since the levels of the bands in question apparently need not follow any simple angular momentum ordering.

We would like to thank the Columbia University Computer Center for making their facilities available to us.

APPENDIX

The single-particle Nilsson wave functions may be expanded in terms of eigenfunctions of total angular momentum,

$$\chi_\Omega=\sum_j C_{j\Omega}\chi_{j\Omega},$$

or in terms of eigenfunctions of orbital angular momentum,

$$\chi_\Omega=\sum_{l\Lambda} a_{l\Lambda\Omega}\chi_{l\Lambda\Omega}.$$

(Λ is the component of orbital angular momentum along the symmetry axis.) In dealing with the residual interaction, the second expression is the more useful.

If the residual interaction is taken to be

$$V=V(r_{12})[U_0+U_1P_M+U_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+U_3P_M\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2], \quad \text{(A1)}$$

then the matrix elements with respect to the basis states given by Eqs. (1) and (2) are

$$\langle|V|\rangle_{K=0}=A_0\tilde{U}_0+A_1\tilde{U}_1+A_2\tilde{U}_2+A_3\tilde{U}_3 +(-)^I[B_1\tilde{U}_1+B_2\tilde{U}_2+B_3\tilde{U}_3] \quad \text{(A2)}$$

and

$$\langle|V|\rangle_{K=2\kappa}=M_0\tilde{U}_0+M_1\tilde{U}_1+M_2\tilde{U}_2+M_3\tilde{U}_3, \quad \text{(A2')}$$

[22] It should be noted that no systematic information about the properties of the *bands* exists, at least as far as we are aware.

where

$$\begin{aligned}\tilde{U}_0&=U_0-\tfrac{1}{2}(U_1+3U_3),\\ \tilde{U}_1&=U_1-\tfrac{1}{2}(U_0+3U_2),\\ \tilde{U}_2&=U_2-\tfrac{1}{2}(U_1-U_3),\end{aligned}\qquad\text{(A3)}$$

and

$$\tilde{U}_3=U_3-\tfrac{1}{2}(U_0-U_2).$$

The A's, B's, and M's are themselves matrix elements which may be written as

$$\begin{aligned}A_0=&\sum_{l_1l_1'l_2l_2'\mu\nu k}(V_k/(2k+1)^2)(-)^{\mu+\nu}a_{l_1,\kappa-\mu}a_{l_2,\kappa+\nu}a_{l_1',\kappa-\mu}a_{l_2',\kappa+\nu}\\&\times C(l_1l_1'k;00)C(l_2l_2'k;00)C(l_1l_1'k;\kappa-\mu,\mu-\kappa)C(l_2l_2'k;-\kappa-\nu,\kappa+\nu),\\ A_1=&\sum_{l_1l_1'l_2l_2'\mu\nu k}(\tilde{V}_k/(2k+1)^2)a_{l_1,\kappa-\mu}a_{l_2,\kappa+\nu}a_{l_1',\kappa-\mu}a_{l_2',\kappa+\nu}\\&\times C(l_1l_2'k;00)C(l_2l_1'k;00)C(l_1l_2'k;\kappa-\mu,\kappa+\nu)C(l_2l_1'k;-\kappa-\nu,\mu-\kappa),\\ A_2=&-2\sum_{l_1l_1'l_2l_2'k\mu\nu\rho}(V_k/(2k+1)^2)a_{l_1,\kappa-\mu}a_{l_2,\kappa+\nu}a_{l_1',\kappa+\rho}a_{l_2',\kappa+\mu+\nu+\rho}C(\tfrac{1}{2},\tfrac{1}{2},1;\mu,\rho)C(\tfrac{1}{2},\tfrac{1}{2},1;\nu,-\mu-\rho+\nu)\\&\times C(l_1l_1'k;00)C(l_2l_2'k;00)C(l_1l_1'k;\kappa-\mu,-\kappa-\rho)C(l_2l_2'k;-\kappa-\nu,\rho+\nu+\mu+\kappa),\\ A_3=&-2\sum_{l_1l_1'l_2l_2'\mu\nu\rho\kappa}(\tilde{V}_k/(2k+1)^2)(-)^{\mu+\nu}a_{l_1,\kappa-\mu}a_{l_2,\kappa+\nu}a_{l_1',\kappa+\rho}a_{l_2',\kappa+\mu+\nu+\rho}\\&\times C(\tfrac{1}{2},\tfrac{1}{2},1;\mu,\rho)C(\tfrac{1}{2},\tfrac{1}{2},1;\nu,-\mu-\nu-\rho)C(l_1l_2'k;00)\\&\times C(l_2l_1'k;00)C(l_1l_2'k;\kappa-\mu,\kappa+\mu+\nu+\rho)C(l_2l_1'k;-\kappa-\nu,-\kappa-\rho),\qquad\text{(A4)}\\ B_1=&\pi\sum_{l_1l_1'l_2l_2'\mu\nu k}(\tilde{V}_k/(2k+1)^2)a_{l_1,\kappa-\mu}a_{l_2,\kappa+\nu}a_{l_1',\kappa+\mu}a_{l_2',\kappa-\nu}\\&\times C(l_1l_2'k;00)C(l_2l_1'k;00)C(l_1l_2'k;\kappa-\mu,\nu-\kappa)C(l_2l_1'k;-\kappa-\nu,\mu+\kappa),\\ B_2=&-2\pi\sum_{l_1l_1'l_2l_2'\mu\nu\rho k}(V_k/(2k+1)^2)a_{l_1,\kappa-\mu}a_{l_2,\kappa+\nu}a_{l_1',\kappa-\rho}a_{l_2',\kappa-\mu-\nu-\rho}\\&\times C(\tfrac{1}{2},\tfrac{1}{2},1;\mu,\rho)C(\tfrac{1}{2},\tfrac{1}{2},1;\nu,-\mu-\rho-\nu)C(l_1l_1'k;00)\\&\times C(l_2l_2'k;00)C(l_1l_1'k;\kappa-\mu,\kappa-\rho)C(l_2l_2'k;-\kappa-\nu,\rho+\nu+\mu-\kappa),\\ B_3=&-2\pi\sum_{l_1l_1'l_2l_2'\mu\nu\rho k}(\tilde{V}_k/(2k+1)^2)(-)^{\mu+\nu}a_{l_1,\kappa-\mu}a_{l_2,\kappa+\nu}a_{l_1',\kappa-\rho}a_{l_2',\kappa-\mu-\nu-\rho}\\&\times C(\tfrac{1}{2},\tfrac{1}{2},1;\mu,\rho)C(\tfrac{1}{2},\tfrac{1}{2},1;\nu,-\mu-\nu-\rho)C(l_1l_2'k;00)\\&\times C(l_2l_1'k;00)C(l_1l_2'k;\kappa-\mu,\mu+\nu+\rho-\kappa)C(l_2l_1'k;-\kappa-\nu,\kappa-\rho),\\ M_0\equiv&A_0,\\ M_1=&\sum_{l_1l_1'l_2l_2'\mu\nu k}(\tilde{V}_k/(2k+1)^2)a_{l_1,\kappa-\mu}a_{l_2,\kappa-\nu}a_{l_1',\kappa-\mu}a_{l_2',\kappa-\nu}\\&\times C(l_1l_2'k;00)C(l_2l_1'k;00)C(l_1l_2'k;\kappa-\mu,\nu-\kappa)C(l_2l_1'k;\kappa-\nu,\mu-\kappa),\\ M_2\equiv&-A_2,\\ M_3=&-2\sum_{l_1l_1'l_2l_2'\mu\nu\rho k}(\tilde{V}_k/(2k+1)^2)(-)^{\mu+\nu}a_{l_1,\kappa-\mu}a_{l_2,\kappa-\nu}a_{l_1',\kappa+\rho}a_{l_2',\kappa-\mu-\nu-\rho}\\&\times C(\tfrac{1}{2},\tfrac{1}{2},1;\mu,\rho)C(\tfrac{1}{2},\tfrac{1}{2};1,\nu,-\mu-\nu-\rho)\\&\times C(l_1l_2'k;00)C(l_2l_1'k;00)C(l_1l_2'k;\kappa-\mu,\mu+\nu+\rho-\kappa)C(l_2l_1'k;\kappa-\nu,-\kappa-\rho).\end{aligned}$$

The V_k and $\tilde{V}_k$ are matrix elements,

$$V_k=\tfrac{1}{2}[(2l_1+1)(2l_1'+1)(2l_2+1)(2l_2'+1)]^{1/2}(2k+1)\times\int r_1^2dr_1r_2^2dr_2R_{n_1l_1}(1)R_{n_1'l_1'}(1)R_{n_2l_2}(2)R_{n_2'l_2'}(2)\int_{-1}^{1}d\mu P_k(\mu)V(|\mathbf{r}_1-\mathbf{r}_2|)\qquad\text{(A5)}$$

and

$$\tilde{V}_k=\tfrac{1}{2}[(2l_1+1)(2l_1'+1)(2l_2+1)(2l_2'+1)]^{1/2}(2k+1)\times\int r_1^2dr_1r_2^2dr_2R_{n_1l_1}(1)R_{n_1'l_1'}(2)R_{n_2l_2}(2)R_{n_2'l_2'}(1)\int_{-1}^{1}d\mu P_k(\mu)V(|\mathbf{r}_1-\mathbf{r}_2|).$$

As an alternative to Eq. (A1), the interaction may be expressed in terms of singlet-even and triplet-odd potentials [Eqs. (8) and (9)]. The matrix elements of the interaction in this form are given in Eqs. (10a) and (10b). The $\bar{A}$'s, $\bar{B}$'s, and $\bar{M}$'s can be simply expressed as linear combinations of the quantities calculated in Eq. (A4),

$$\bar{A}_0=\tfrac{1}{4}(A_0+A_1-A_2-A_3)$$

and (A6)

$$\bar{A}_3=\tfrac{1}{4}(3A_0-3A_1+A_2-A_3),$$

with identical expressions for $\bar{B}_0$, $\bar{B}_3$, $\bar{M}_0$, and $\bar{M}_3$.

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Effect of Quadrupole Collective Motions on the Giant Dipole Resonance*

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The photonuclear model which includes both scalar and tensor polarizabilities is refined by considering small vibrations of the nuclear shape deformation. The effects of these zero-point vibrations on the structure of the giant dipole resonance for elastic and inelastic scattering have been investigated in an adiabatic approximation. Illustrations are given.

I. INTRODUCTION

IT has been established experimentally that there is a definite correlation between the giant dipole resonance of the photonuclear effect and nuclear deformation. These resonances are appreciably narrower in the closed-shell nuclei than those found in nuclei situated between closed shells. For deformed nuclei, the giant resonances broaden and even split into two peaks; this is especially apparent in the rare-earth region where the deformation is particularly large. For an ellipsoidal nucleus having a positive intrinsic quadrupole moment, the higher energy resonance is observed to contain about twice as much area as the lower energy resonance, the latter being always sharper than the narrowest resonances found in spherical nuclei.

These results are all in accord with the predictions of Okamoto[1] and Danos[2] that for a deformed nucleus the dipole oscillations would take place with two characteristic frequencies associated with the nuclear axes. The order of magnitude of the ratio of these frequencies follows from dimensional considerations, namely, $\omega_1/\omega_2 = 0.91R_2/R_1$ as given by a detailed hydrodynamic analysis, where R_1 and R_2 are the largest and smallest radii of the nucleus. Recently Fano,[3] and Fuller and Hayward,[4] using tensorial techniques, derive a more general theory which takes into account the dependence of the photon scattering upon the spin orientation of the nucleus with respect to the wave vector of the photon, and thus includes three possible polarizability contributions: scalar, vector, and tensor.

Many experimental absorption data of strongly deformed nuclei seem to agree as well with the three-resonance theory first proposed by Inopin.[5] On the basis of the hydrodynamic model within the framework of the theory of axially asymmetric nuclei proposed by Davidov and Filippov, Inopin showed that the resonance energies E_i corresponding to the density oscillations along the three different axes are proportional to $1/R_i$. Any experimental results for deformed nuclei can thus be interpreted by a three-line fit, for which two resonance energies E_1 and E_2 may be allowed to approach each other at a common value E_{12}. However, we know that a nonaxial deformation is generated by shape vibrations away from equilibrium axial symmetry, so called γ vibrations, and it seems sensible to assume that these vibrations might make more or less important contributions to the photoeffect resonance.[6]

Those considerations motivate us to include, if only for the sake of self-consistency of the collective model, zero-point vibrations of the nuclear shape. Conversely a successful interpretation of some aspects of the photonuclear effect on this basis might be useful in the study of nuclear structure, in revealing properties of collective levels and estimates of the zero-point vibra-

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[1] K. Okamoto, Phys. Rev. **110**, 143 (1958).
[2] M. Danos, Nucl. Phys. **5**, 23 (1958).
[3] U. Fano, Natl. Bur. Std. (U. S.) Tech. Note **83**, (1960).
[4] E. Fuller and E. Hayward, Nucl. Phys. **30**, 613 (1962).
[5] E. Inopin, Zh. Eksperim i Teor. Fiz. **38**, 992 (1960) [English transl.: Soviet Phys.—JETP **11**, 714 (1960)].
[6] P. A. Tipler, P. Axel, N. Stein, and D. C. Sutton, Phys. Rev. **129**, 2096 (1963).