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Shell-Model States in Ca^{49}†

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The energy-level structure of Ca^{49} has been investigated by the deuteron-stripping reaction $Ca^{48}(d,p)Ca^{49}$. Ten excited levels have been observed below 6.1 MeV, and the energies, angular momenta, and parities have been determined for most of the levels. The analysis of the angular distributions of the reaction protons permitted the identification of single-particle configurations at the following energies: $2p_{3/2}=0$ MeV; $2p_{1/2}=$ 2.028 MeV; $1f_{5/2}=3.95$ MeV; $1g_{9/2}\geqslant 4.02$ MeV; and $2d_{5/2}>4.4$ MeV.

I. INTRODUCTION

THE energy-level structure of Ca^{49} is expected to be simple in that the Ca^{48} core is doubly magic, consisting of closed shells of both protons (20) and neutrons (28). On the basis of the shell model, it would be expected that the addition of a single neutron to the Ca^{48} core would form levels of Ca^{49} corresponding to each of the possible shell-model states of the twenty-ninth neutron. Evidence to this effect is already in the literature, with identification of the ground state and the 2.028-MeV excited level of Ca^{49} as corresponding to the $2p_{3/2}$ and $2p_{1/2}$ neutron states, respectively.[1] The present investigation confirms this and extends the range of investigation to 6.1-MeV excitation. A number of additional levels are assigned with respect to single-particle configurations corresponding to higher shell-model states of the last neutron. In addition, the angular distributions of the reaction protons provide a good test for various theoretical calculations from which spectroscopic factors and reduced widths are to be determined. The positions of the single-particle levels are expected to be unambiguous in this investigation, especially where only one level is observed corresponding to each state of the twenty-ninth neutron.

II. EXPERIMENTAL PROCEDURE

The deuteron beam of the MIT–ONR Van de Graaff electrostatic accelerator was made to impinge upon a target of $CaCO_3$ enriched to 84.1% in Ca^{48} and evaporated onto a thin gold foil. The targets had an additional layer of Formvar which was added to secure the target to the target frame. The incident deuteron energies used were 7.0 and 7.5 MeV. The reaction protons were observed as tracks in nuclear emulsions using the MIT multiple-gap, broad-range magnetic spectrograph.[2] The nuclear emulsions were covered with aluminum foil of sufficient thickness to stop the elastically scattered deuterons yet thin enough to allow passage of the reaction protons. In order to be able to observe protons leaving Ca^{49} in levels at high excitation energy, the magnetic field was set for the experiment with 7.5-MeV deuterons in such a manner that the proton group corresponding to a level at 3.595 MeV in Ca^{49} was focused close to the high-energy end of the nuclear track plates. In addition to the two experiments using the multiple-gap spectrograph, a third using the single-gap broad-range spectrograph[3] was performed at a deuteron bombarding energy of 7.5 MeV, with protons observed at a laboratory angle of 90 deg. The most accurate ground-state Q value for the $Ca^{48}(d,p)Ca^{49}$ reaction and the

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[1] W. W. Buechner, in *Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics* (Pergamon Press, Inc., London, 1954).
[2] H. A. Enge and W. W. Buechner, Rev. Sci. Instr. **34**, 155 (1963).
[3] C. P. Browne and W. W. Buechner, Rev. Sci. Instr. **27**, 899 (1956).

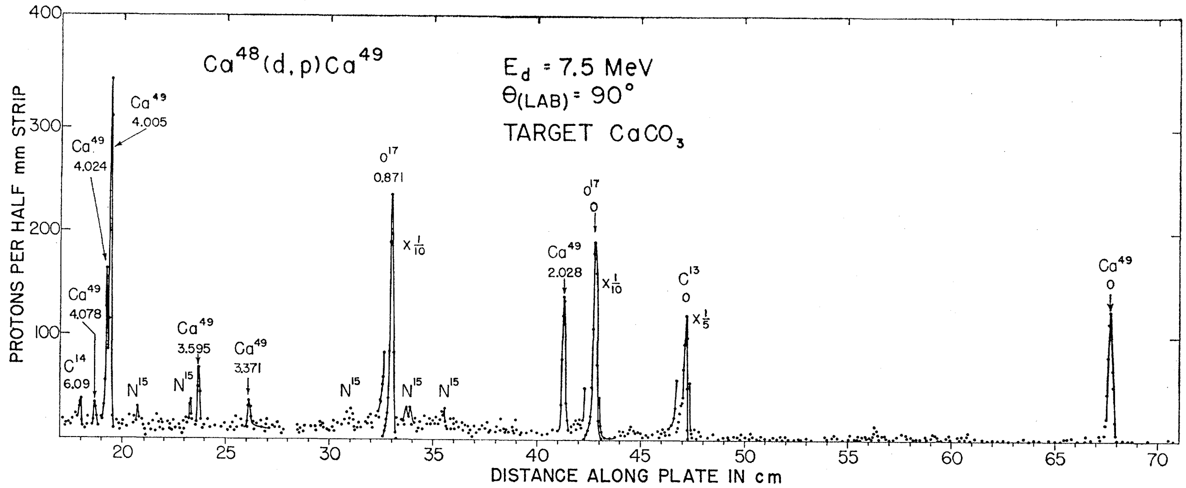

FIG. 1. Momentum spectrum of protons from the $Ca^{48}(d,p)Ca^{49}$ reaction at a laboratory angle of 90 deg.

excitation energies of the first four levels were obtained in this latter experiment.

A spectrum of the reaction protons taken at 90 deg with the single gap is shown in Fig. 1. The closely spaced 4.005–4.024-MeV doublet has been well resolved in this exposure, as can be seen from Fig. 2, where a small part of the momentum spectrum has been enlarged. Part of the spectrum of protons recorded at 30 deg with the broad-range multiple-gap spectrograph is shown in Fig. 3. The real width (60 keV) of the 6.095-MeV virtual level is apparent here. Note also the two broad proton groups that correspond to virtual levels in O^{17} at 4.55 and 5.08 MeV. These groups were the source of considerable difficulty in extracting cross sections for the 6.09-MeV level especially at small angles where the 5.08-MeV group from O^{17} overlaps the 6.09-MeV group from Ca^{49}. Also seen in Fig. 3 is a number of peaks from the $C^{12}(d,p)C^{13}$ and $C^{13}(d,p)C^{14}$ reactions. The rather large background is mainly due to the low-energy "tails" of the intense contaminant peaks and is also partially caused by proton groups from the $N^{14}(d,p)N^{15}$ and the $Ca^{40}(d,p)Ca^{41}$ reactions; the latter because Ca^{40} constituted approximately 15% of the calcium in the target.

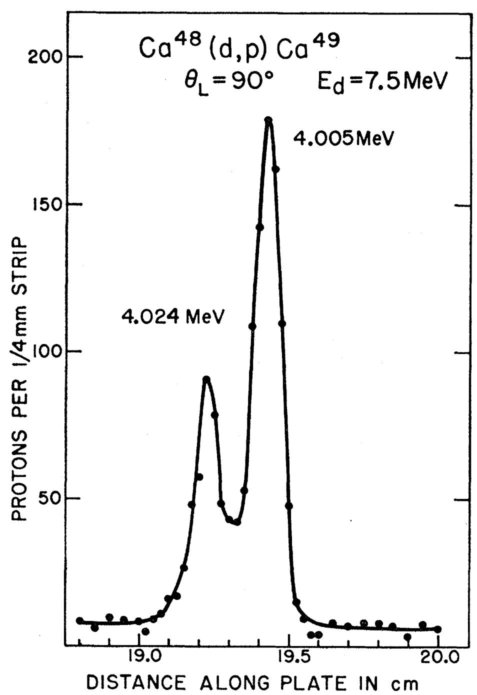

FIG. 2. Enlarged section of momentum spectrum of protons from $Ca^{48}(d,p)Ca^{49}$ reaction showing the 4.005- and 4.024-MeV levels of Ca^{49}.

In order to determine absolute cross sections in these experiments, the target was bombarded with 2.0-MeV protons, and the elastic-scattering yield was observed with the target in the same position as was used in the (d,p) exposures. The elastically scattered protons were observed at 67- and 90-deg laboratory angles, and the cross section was assumed to result from Coulomb scattering only. While it would have been preferable to do the scattering experiment at smaller angles where the Coulomb cross section is more reliable, this was not feasible because of the large yield of elastically scattered protons from the gold-foil backing of the target. Aside from this, the method employed for obtaining absolute cross sections tends to combine errors, such as those ascribed to charge integration, target thickness, and solid angle, into one uncertainty. The standard error in absolute cross section is estimated to be 25%. This does not include the errors arising from statistics of counting or from background subtraction. The latter uncertainties are indicated in the figures for the angular distributions as error flags on the data points.

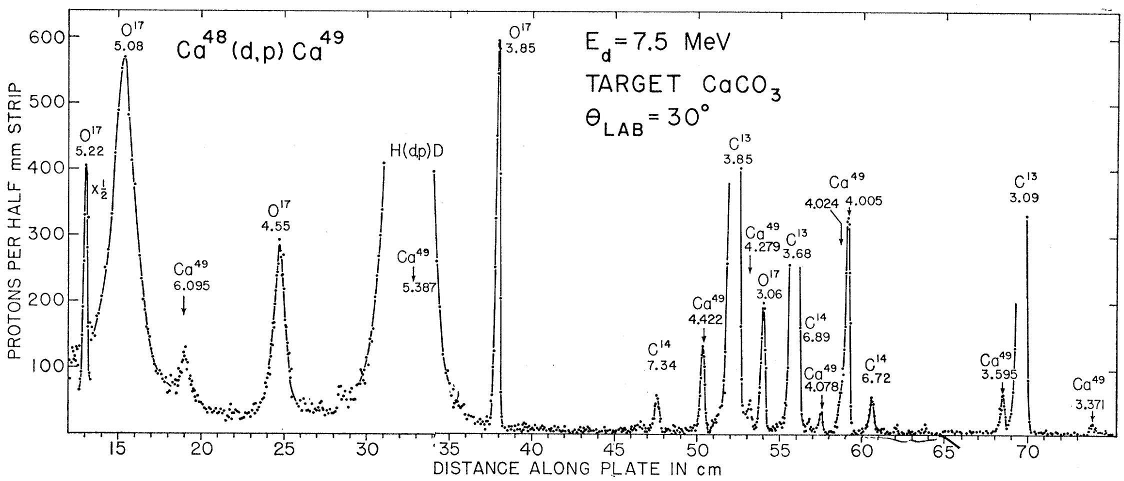

FIG. 3. Momentum spectrum of protons from the $Ca^{48}(d,p)Ca^{49}$ reaction at a laboratory angle of 30 deg.

III. RESULTS

Of the proton groups observed in the various experiments, eleven were identified as resulting from the (d,p) reaction on Ca^{48}. From the ground-state proton group, a Q value of 2.924±0.005 MeV was calculated for this reaction, in agreement with results of an earlier investigation of the $Ca^{48}(d,p)Ca^{49}$ reaction by Braams,[4] who measured a Q value of 2.919±0.006 MeV. The Q values and the energies for the excited states are listed in Table I. It is quite possible that some levels with small proton yields lying above 4.1-MeV excitation ($Q<-1.2$ MeV) may have been missed because of the large background. An upper limit of 0.3 mb/sr can be established for the (d,p) cross section from 30 to 60 deg in any level of Ca^{49} that may have been missed.

For the 6.095-MeV ($Q=-3.171$ MeV) level, the relatively large uncertainty in energy reflects the natural width of that particular level, which was determined to be $\Gamma=60\pm10$ keV. This is not surprising, since this level lies 0.946 MeV above the dissociation energy of a neutron from Ca^{49}. Although the 5.387-MeV level ($Q=-2.463$ MeV) is also a virtual level with an available disintegration energy of 238 keV, the observed width of the proton group is no larger than the experimental resolution.

There was no difficulty in obtaining angular distributions for the proton groups corresponding to the ground state and the first and third excited states. The proton angular-distribution data for these are shown in Figs. 4 and 5. For the 4-MeV doublet the separation was determined to be 19±2 keV, and some difficulty was encountered in separating the yields of the two groups. The task was complicated in that the smaller of the two groups corresponded to the higher excitation energy (4.024 MeV) and was superimposed upon the low-energy end of the 4.005-MeV group, the latter having a considerably larger yield at small angles. However, a careful analysis of the peak shares for the experiments at 7.0 and 7.5 MeV gave essentially the same angular distributions for these groups at both energies. The angular distributions taken with 7.5-MeV incident deuterons are shown in Figs. 6 and 7. The angular-distribution data for the remaining levels are shown in Figs. 8 and 9. The error flags reflect mostly the uncertainty in background subtraction and also contain the statistical uncertainty.

The analysis of the proton angular distributions was carried out on a computer using distorted-wave Born approximation (DWBA). Two computer codes were used to carry out these calculations, one being Tobocman's[5] Los Alamos DWBA code, and the other, the Oak Ridge DW code.[6] Since approximately the

TABLE I. Results for the Ca^{49} level structure.

Level	Q (MeV)	E_x (MeV)	l_n	Shell-model config.	$S=\sigma_{exp}/\sigma_{theor}$
0	2.924±0.005	···	1	$2p_{3/2}$	1.03
1	0.896±0.005	2.028±0.004	1	$2p_{1/2}$	1.33
2	−0.447±0.005	3.371±0.007	2	$(2d_{5/2})$	0.02
3	−0.671±0.005	3.595±0.007	3	$(1f_{5/2})$	0.11
4	−1.081±0.005	4.005±0.008	3	$1f_{5/2}$	0.72
5	−1.100±0.005	4.024±0.008	4	$1g_{9/2}$	0.31
6	−1.154±0.005	4.078±0.008	3	$1f_{5/2}$	0.06
7	−1.355±0.005	4.279±0.010	···	···	···
8	−1.498±0.005	4.422±0.009	2	$2d_{5/2}$	0.05
9	−2.463±0.005	5.387±0.010	···	···	···
10	−3.171±0.005	6.095±0.015	···	···	···

[4] C. M. Braams, Ph.D. thesis, University of Utrecht, Utrecht, Holland, 1956 (unpublished). The Q values and excitation energies quoted in the present paper are based on the value of 5.3042 MeV for the energy of Po^{210} alpha particles used in our spectrograph calibration since 1960.

[5] W. Tobocman and W. R. Gibbs, Phys. Rev. **126**, 1076 (1962).

[6] R. H. Bassel, R. N. Drisko, and G. R. Satchler, Oak Ridge National Laboratory, Report No. 3240 (unpublished).

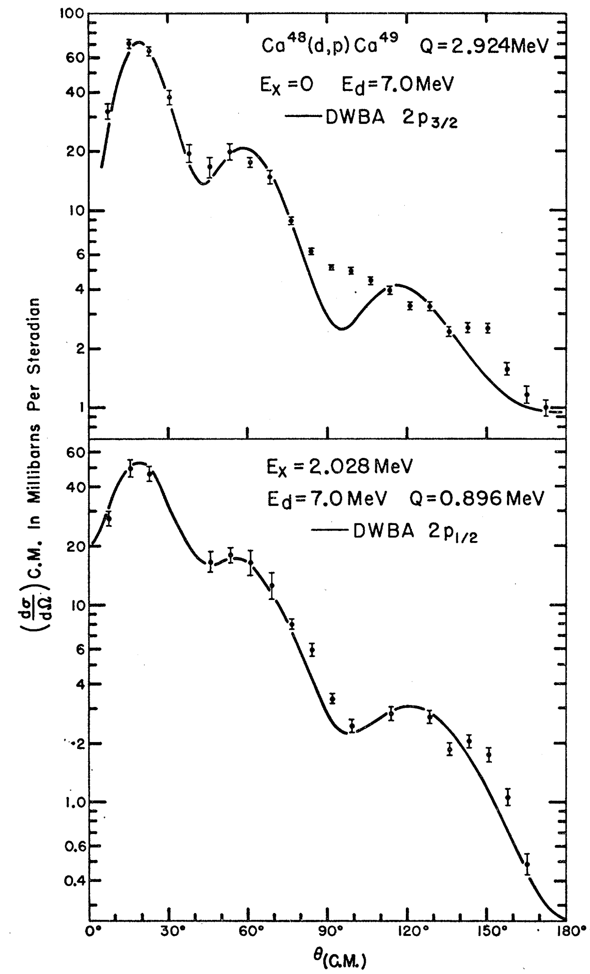

FIG. 4. Angular distributions of protons leaving Ca^{49} in its ground level and in its 2.028-MeV level.

same spectroscopic information was extracted from both sets of calculations, we have included here only the one carried out with the Oak Ridge DW code. All the calculations, at both 7.0 and 7.5 MeV, were made with a single set of optical parameters.[7] These were obtained from a recent investigation by Smith and Ivash,[8] who used our 7.0-MeV $Ca^{48}(d,p)Ca^{49}$ data for the ground state and for the 2.028-MeV level in part of their investigations. It is not clear whether one is justified in using a single set of optical parameters to extract spectroscopic factors for a series of transitions whose Q values vary from 2.9 to -1.4 MeV, as has been done here, especially since the outgoing proton energy varies from 5.5 to 10 MeV, and the optical well parameter for these protons may vary considerably. Nevertheless, a comparison of the DW calculations with the experimental cross sections does convey spectroscopic information.

A. Transitions to the Ground State and to the 2.028-MeV Level: $2p_{3/2}$ and $2p_{1/2}$

The angular distributions of protons to the ground state and to the 2.028-MeV first excited level show definite $l_n=1$ stripping patterns. The theoretical calculation of the stripping curves, together with the experimental data, is shown in Fig. 4, where it is seen that there is good agreement between theory and experiment. The spectroscopic factors for these transitions, obtained from a comparison of the experimental data with the theoretical calculations, were $S(2p_{3/2})=1.03$ and $S(2p_{1/2})=1.33$. These values of S are in good agreement with the values of unity expected for single-particle transitions. These two levels are clearly those for which the neutron states are $2p_{3/2}$ and $2p_{1/2}$.

The data shown in Fig. 4 exhibit quite obviously the effect associated with the total angular momentum for $l_n=1$ transitions. This has been recently demonstrated by Lee and Schiffer[9]; that is, for the 2.028-MeV $\frac{1}{2}^-$ level, the angular distribution has a dip at around 100 deg, while no such dip is found in the angular distribution corresponding to the $\frac{3}{2}^-$ ground-state level.

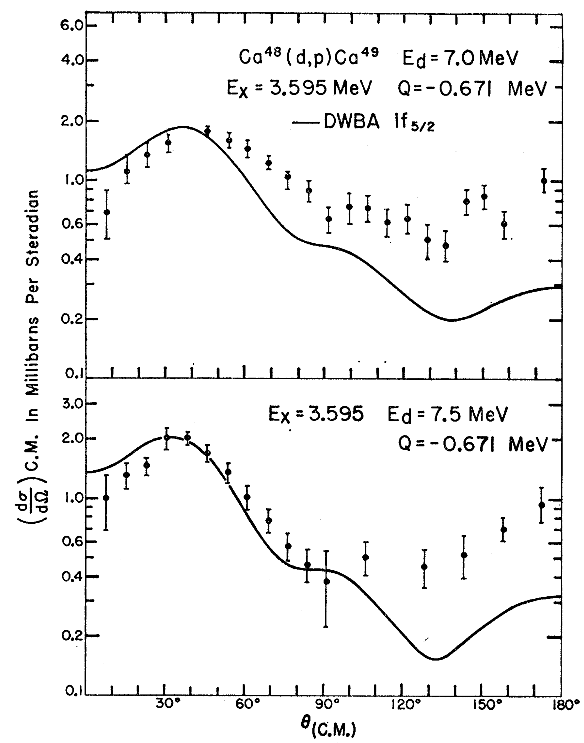

FIG. 5. Angular distributions of protons leaving Ca^{49} in its 3.595-MeV level (with deuteron bombarding energies of 7.0 and 7.5 MeV).

[7] The optical parameters used were: For the deuterons: $V=63$ MeV; $W=15$ MeV; $r_0=r_c=1.4$ F; $a=0.7$ F. For the protons: $V=55$ MeV; $W=8$ MeV; $r_0=r_c=1.25$ F; $a=0.5$ F. For the neutron: $r=1.4$ F; $a=0.7$ F.

[8] W. R. Smith and E. V. Ivash, Phys. Rev. **131**, 304 (1963).

[9] L. L. Lee and J. P. Schiffer, Phys. Rev. Letters **12**, 108 (1964).

Since the deuteron stripping to the Ca^{49} ground state and to the 2.028-MeV level is expected to yield spectroscopic factors of 1, absolute values of S could be extracted for $l_n=1$ transitions in neighboring nuclei by using optical parameters similar to those used here, and the theoretical results could be normalized to our experimental results. However, since the present absolute values of differential cross sections have a rather large uncertainty (about 25%), the accuracy of this method is limited. Because much of the previous (d,p) data in this energy region have been analyzed in terms of Butler's plane-wave Born approximation (PWBA),[10] this was also done here for the two $l_n=1$ levels. Through the use of tables for determining reduced widths,[11] one obtained $\theta^2(2p_{3/2})=0.039$ and $\theta^2(2p_{1/2})=0.035$ (indeed quite close to the same value).

It is interesting to compare the value of 2.028 MeV, representing the $2p_{3/2}$–$2p_{1/2}$ energy splitting, with the value obtained in cases where a large number of $l_n=1$ transitions are observed. A recent analysis[12] of seven

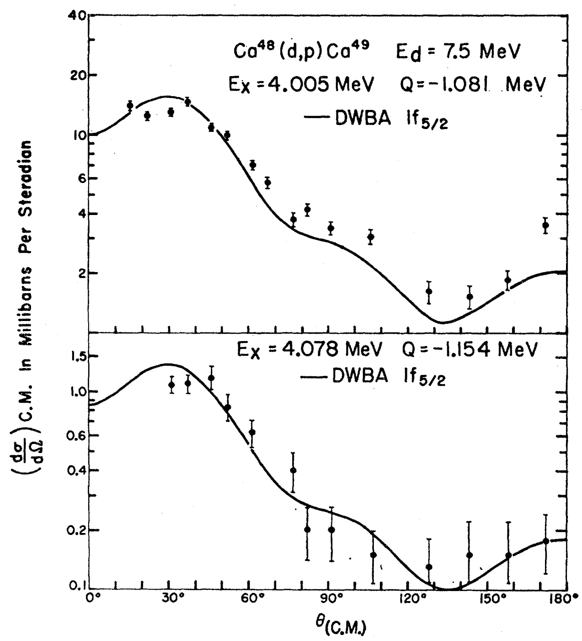

FIG. 6. Angular distributions of protons leaving Ca^{49} in its 4.005- and 4.078-MeV levels.

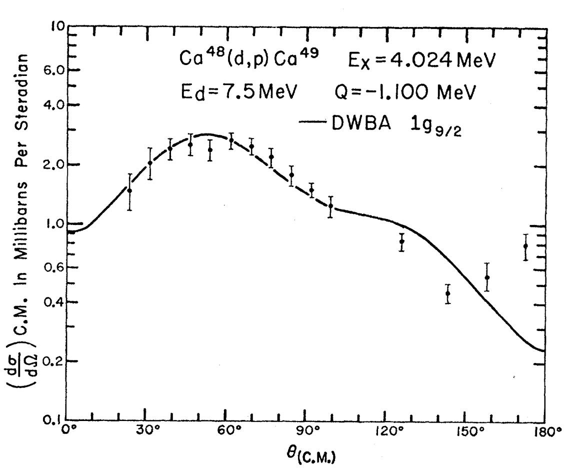

FIG. 7. Angular distribution of protons leaving Ca^{49} in its 4.024-MeV level.

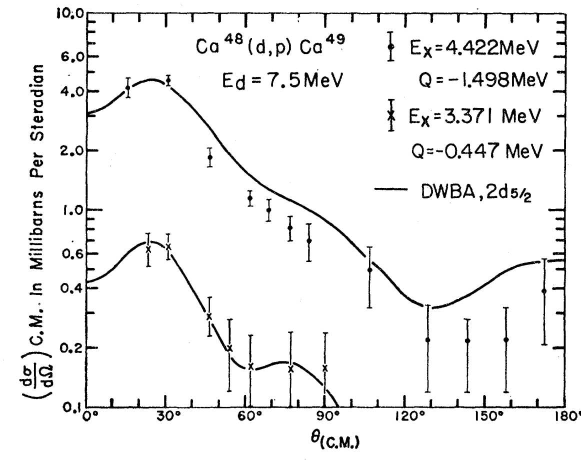

FIG. 8. Angular distributions of protons leaving Ca^{49} in its 3.371- and 4.422-MeV levels.

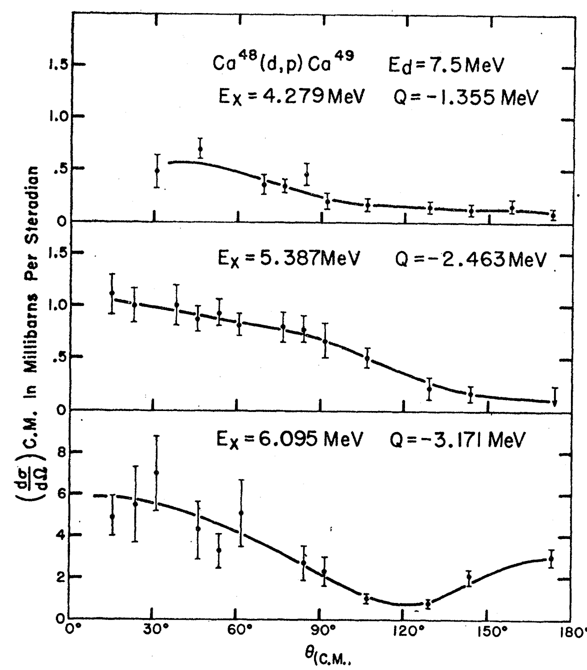

FIG. 9. Angular distributions of protons to the 4.279-, 5.387-, and 6.095-MeV levels of Ca^{49}. The solid curves are drawn through the experimental data.

[10] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).

[11] H. Y. Chen, H. A. Enge, and H. F. Lutz, MIT-LNS Report No. 74, NYO-2666 (unpublished).

[12] T. A. Belote, J. Rapaport, and W. W. Buechner, Bull. Am. Phys. Soc. **9**, 79 (1964).

$l_n=1$ transitions from the $Ca^{40}(d,p)Ca^{41}$ reaction indicates a value of 2.04 MeV for this splitting. A less favorable case is the $Ar^{40}(d,p)Ar^{41}$ reaction where thirteen $l_n=1$ transitions are observed[13] and where, in addition, some recent theoretical calculations[14] may be used to differentiate between $J^\pi=\frac{1}{2}^-$ and $J^\pi=\frac{3}{2}^-$ levels. The results show $E_0(2p_{3/2})$, where $E_0=\sum \theta_i^2 E_i/\sum \theta_i^2$, at 1.66 MeV above the $1f_{7/2}$ ground level and $E_0(2p_{1/2})$ at 3.49 MeV. This gives a $2p_{3/2}$–$2p_{1/2}$ separation of 1.83 MeV in Ar^{41}, still in good agreement with the Ca^{49} and Ca^{41} values.

B. The 3.595-, 4.005-, and 4.078-MeV Levels: $1f_{5/2}$

Angular distributions of protons leaving Ca^{49} at 3.595-MeV excitation were obtained at deuteron bombarding energies of 7.0 and 7.5 MeV (Fig. 5). Both angular distributions agree reasonably well with the DWBA calculations, assuming $l_n=3$, especially at the forward angles. The 3.595-MeV level is believed to correspond in part to a $1f_{5/2}$ neutron added to the Ca^{48} ground state. The spectroscopic factor $S(1f_{5/2})=0.11$ obtained from both the 7.0- and 7.5-MeV deuteron data, indicates that this transition would account for only a small fraction of the total single-particle strength. It should be noted that the determination of $l_n=3$ for the angular distribution in this transition limits the assignment of the 3.595-MeV level to $J^\pi=\frac{5}{2}^-$ (as opposed to $J^\pi=\frac{7}{2}^-$) only if the neutrons in excess of 20 in the Ca^{48} target nucleus are described by a pure $(1f_{7/2})^8$ configuration. If higher shell admixture is present ($2p_{3/2}$ admixture, for example), it would then be possible and, in fact, expected that some $l_n=3$ transition corresponds to leaving the neutron in a $1f_{7/2}$ orbital. This could well be the case for the 3.595-MeV level, particularly if one should speculate that the difference in the back-angle distributions between the 3.595-MeV level (Fig. 5) and those of the 4.005- and 4.078-MeV levels (Fig. 6) are significant as in the case of the $l_n=1$ distributions.[9]

The proton angular distribution for the 4.005-MeV level was initially thought to correspond to the secondary maximum of an $l_n=0$ distribution. However, the DW calculation ruled this out and, rather, indicated an $l_n=3$ distribution. The differential cross section of 13 mb/sr at 30 deg is rather large, and this level probably represents the "single-particle" $1f_{5/2}$ transition. The DW calculations' comparison with the data is shown in Fig. 6, with $S(1f_{5/2})=0.72$.

For the 4.078-MeV level, the experimental angular distribution did not extend forward past 30 deg because of interference from a proton group from $C^{12}(d,p)C^{13}$. This makes the identification of an l_n value difficult. However, a comparison of its angular distribution with those for the other levels observed limits the number of possibilities for the angular momentum involved in the transition. One can see a resemblance between this angular distribution and that for the 4.005-MeV state. The DW calculations, assuming $l_n=3$, are shown together with the data in Fig. 6, where it is seen that there is good agreement between data and theory. The spectroscopic factor obtained, assuming $1f_{5/2}$, was 0.06. From these results and assuming that the 3.595-, 4.005-, and 4.078-MeV levels comprise all the $1f_{5/2}$ transitions, we find a mean energy of the $1f_{5/2}$ configuration at 3.95 MeV and a total value of S for the three transitions of 0.89, indeed as close to unity as expected.

C. The 4.024-MeV Level: $1g_{9/2}$

This level was observed as one of the peaks in the closely spaced doublet, shown well resolved in Fig. 2. The proton angular distribution obtained with 7.5-MeV deuterons is shown in Fig. 7, where DWBA calculations are also shown, assuming $l_n=4$. The differential cross section has a broad maximum in the neighborhood of 55 deg and is well fitted by the theory at forward angles, although there is some disagreement at the very backward angles. It appears that this level corresponds to the $1g_{9/2}$ neutron state in Ca^{49}. The spectroscopic factor extracted from comparison of theory and experiment was $S(1g_{9/2})=0.33$. This value is somewhat small, and it is possible that other $l_n=4$ transitions lie at higher excitation energies.

D. The 3.371- and 4.422-MeV Levels: $2d_{5/2}$

The angular distributions corresponding to the 3.371- and 4.422-MeV levels of Ca^{49} are shown in Fig. 8. Both show strong maxima and appear to be well explained in terms of the addition of an $l_n=2$ neutron to Ca^{48}. The cross section to the 3.371-MeV level is very small, with a peak value of 0.65 mb/sr. At backward angles its cross section could not be obtained since the yield was not significantly larger than the background. Therefore, data are only shown for forward angles in Fig. 8. For the 4.422-MeV level, the (d,p) cross section is relatively large, with a value of 4.5 mb/sr at the peak. If we assume that the $1d_{3/2}$ neutron shell is filled, both transitions would be to $2d_{5/2}$ levels; that is, levels with $J^\pi=\frac{5}{2}^+$. Comparison of the present results with DW calculations yield $S(2d_{5/2})=0.02$ and 0.05 for the 3.371- and 4.422-MeV levels, respectively. It appears therefore that these are only small lower lying "fragments," having only a small fraction of the total $2d_{5/2}$ single-particle transition strength, and that the main transition lies above an excitation of 4.4 MeV. This may be compared with the situation in Ca^{41}, where at least seven $l_n=2$ transitions up to an excitation energy of 6.6 MeV contribute only about 15% of the total single-particle $2d_{5/2}$ strength.[12]

[13] E. Kashy, A. M. Hoogenboom, and W. W. Buechner, Phys. Rev. **124**, 1917 (1961).
[14] Y. Shadmi and I. Talmi, Phys. Rev. **129**, 1286 (1963).

E. The 4.279-, 5.387-, and 6.095-MeV Levels

The experimental angular distribution to the 4.279-MeV level, shown in Fig. 9, is too incomplete to enable extraction of angular-momentum information. The 5.387- and 6.095-MeV levels are both unbound and can decay by neutron emission. Their angular distributions, which are also shown in Fig. 9, did not resemble any of the theoretical predictions, although it should be noted that in the calculations, the neutron was assumed bound by about 1 MeV in order that the available DW codes could be used. The transition to the 6.095-MeV level, which has a rather large cross section, and the large width (60 keV) of the level, are interesting; this may reflect the fact that the transition has a sizable spectroscopic factor.

All results obtained for the level structure of Ca^{49} are listed in Table I, and an energy-level diagram is shown in Fig. 10. One of the interesting aspects of the present investigation is the comparison of the level structure and level density of Ca^{49} with those of Ca^{41}. In both instances, the final nucleus can be thought of as having filled proton and neutron shells plus one additional neutron, so that, except for the ground state of Ca^{41} being $1f_{7/2}$, the level structures should be very similar. While from 0- to 4.2-MeV excitation in Ca^{41}, twenty-five levels have been observed,[15] and a considerably larger number (>80) between 4.2 and 6.8 MeV have been reported[12]; in the present investigation, only ten excited levels are observed in Ca^{49} up to 6.1-MeV excitation. This is a somewhat puzzling result. Part of the explanation may be that Ca^{48} apparently is a more stable core than Ca^{40}, as evidenced by the fact that the first two excited states in Ca^{48} are at 3.825 and 4.499 MeV, as compared with 3.348 and 3.730 MeV for Ca^{40}. Another part of the explanation may be found in the fact that a higher multiplicity of states can be formed for the lowest configuration in which a proton pair is broken in Ca^{41} when compared with Ca^{49}. The lowest configuration of this kind in Ca^{41} is presumably $(d_{3/2})_\pi^{-1}(f_{7/2})_\pi^{1}(f_{7/2})_\nu^{1}$ with a multiplicity of 28. The corresponding configuration in Ca^{49} is $(d_{3/2})_\pi^{-1}(f_{7/2})_\pi^{1}(p_{3/2})_\nu^{1}$ with a multiplicity of 16. Similar ratios of multiplicities are found when other likely pairs are broken. In general, the higher numbers for Ca^{41} arise from the fact that an odd proton and an odd neutron simultaneously can occupy the $f_{7/2}$ shell, which has a relatively high multiplicity.

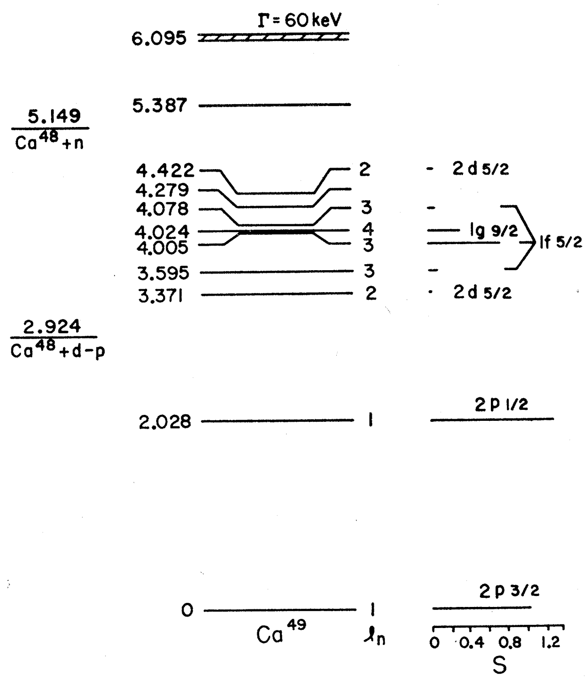

FIG. 10. Energy-level diagram of Ca^{49}, with spectroscopic factors for the $Ca^{48}(d,p)Ca^{49}$ reaction shown.

ACKNOWLEDGMENTS

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[15] C. M. Braams, Phys. Rev. **103**, 1310 (1956).