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Field Theory of Matter

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A speculative field theory of matter is developed. Simple computational methods are used in a preliminary survey of its consequences. The theory exploits the known properties of leptons by means of a principle of symmetry between electrical and nucleonic charge. There are fundamental fields with spins 0, $\frac{1}{2}$, 1. The spinless field is neutral. Spin $\frac{1}{2}$ and 1 fields can carry both electrical and nucleonic charge. The multiplicity of any nonzero charge is 3. Explicit dynamical mechanisms for the breakdown of unitary symmetry and for the muon-electron mass difference are given. A more general view of lepton properties is proposed. Mass relations for baryon and meson multiplets are derived, together with approximate couplings among the multiplets. The weakness of ϕ production in $\pi-N$ collisions and the suppression of the $\phi\rightarrow\rho+\pi$ decay is explained.

THE THEORY

Two exact conservation laws of nature, those of electrical and nucleonic charge, have received similar dynamical interpretations through the structure of vector gauge fields.[1] An obvious physical difference between these charges is also accounted for dynamically by the observation that a massless physical particle will exist only for sufficiently weak interactions.[2] It is worth asking how far this qualified analogy can be pursued. The significance of the attempt lies in the basic concept of field theory[3] that a dynamical link must be established between the fundamental field variables and the observed particles. That relationship may be quite immediate for particles having predominantly electromagnetic interactions, with corresponding implications about the primitive fields, while it may be rather remote for the strongly interacting entities. The postulated analogy thus becomes a tool for analyzing the structure of the otherwise inaccessible substratum of fundamental fields and interactions.

The analogy between nucleonic charge N and electrical charge Q will be used first to explore the structure of the fundamental spin $\frac{1}{2}$ Fermi field ψ. Based upon the known leptons, $\mu^{\pm}$, $e^{\pm}$, ν, $\bar{\nu}$, it can be assumed that such a field carries $Q=1, 0, -1$, and therefore also carries $N=1, 0, -1$. Furthermore, each value of Q occurs twice among the leptons, which have $N=0$. A table of multiplicities based upon this information and the postulated symmetry between N and Q would then appear as

$N \backslash Q$	1	0	−1
1	n_+	2	n_-
0	2	2	2
−1	n_-	2	n_+

which also incorporates a requirement of symmetry for the reflection of both charges. The total number of charged fields, of either type, is $4+2n$, $n=n_+ + n_-$. The simplest assumption is that $n=1$. Then twelve fields can be divided, for each sense of charge, into six charged fields and six neutral fields. The choice between the two possibilities $n_+=1$, $n_-=0$ and $n_-=1$, $n_+=0$ is a physical one since it must imply the correlation between N and Q that is embodied in the distinction between the proton $(N=Q)$ and the charged cascade particle $(N=-Q)$. We shall make the provisional assumption that $n_-=1$, $n_+=0$. Then the $N=\pm1$ parts of the fundamental Fermi field have three components each, one with $Q=\mp1$ and two with $Q=0$.

It has been proposed that the vector electromagnetic field A_μ, the dynamical instrument of electrical charge, is the neutral partner of a unit charged vector field Z_μ, which is coupled universally to unit changes of charge.[4] That idea, which also anticipated the existence of two neutrinos, implies the phenomenological current—current couplings of the weak interactions if a sufficiently massive particle is associated with the Z field. Searches for more direct manifestations of this particle are in progress.

The qualitative analogy between electric and nucleonic charge suggests that B_μ, the vector gauge field that is strongly coupled to N, is the neutral partner of a multicomponent vector field V_μ, which is coupled to changes of nucleonic charge. The Bose field V_μ must carry compensating amounts of nucleonic charge. Now, there appears to be an approximate meaning to a world without leptons and the electromagnetic field, in the sense of a restricted time scale, which implies that rapid exchanges of nucleonic charge are limited to the

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **98**, 1501 (1955).

[2] J. Schwinger, Phys. Rev. **125**, 397 (1962); **128**, 2425 (1962).

[3] The term field theory must be qualified, since there are diverse opinions on the subject. I contend that the fundamental dynamical variables are field operators, while particles are identified as the stable or quasistable excitations of the coupled field system. There is no *a priori* relation between the primary dynamical fields and the secondary phenomenological fields that can be associated with the observed particles. For some further discussion see the *Miramare, Trieste, Lectures* (International Atomic Energy Agency, Vienna, 1963).

[4] J. Schwinger, Ann. Phys. **2**, 407 (1957).

nonleptonic world, and therefore only connect the $N=\pm 1$ components of the fundamental Fermi field. It is for this reason that we deviate from the strict analogy and postulate that V_μ carries *two* units of nucleonic charge. One might speculate on the role that a unit nucleonic charged boson field could play for weak interactions, but we shall assume that it is not relevant to strong interactions.

What values of electrical charge can we reasonably suppose the V_μ field to carry? Here we shall only indicate the most plausible hypothesis, in the light of its physical consequences. It is that $V_\mu(N=\pm 2)$ has the same charge structure as $\psi(N=\pm 1)$; there are three components, one with $Q=\mp 1$ and two with $Q=0$. This is a threefold way.[5]

A simplified picture of the nucleonic world is obtained by considering the Fermi field $\psi(N=\pm 1; Q=\mp 1, 0, 0)$ and the Bose field $V_\mu(N=\pm 2; Q=\mp 1, 0, 0)$, each coupled to the neutral gauge field B_μ, but without the direct interaction between ψ and V_μ that exchanges nucleonic charge. The nucleonic charge interaction term in the Lagrange function is

$$f B_\mu[\tfrac{1}{2}\psi\alpha^\mu\nu_3\psi + iV^{\mu\nu}2\nu_3 V_\nu] = f B_\mu[\bar{\psi}\gamma^\mu\psi + i(\bar{V}^{\mu\nu}2V_\nu - \bar{V}_\nu 2V^{\mu\nu})], \quad (1)$$

where the two equivalent forms correspond to the use of Hermitian fields that are eigenvectors of charge reflection matrices, or non-Hermitian fields that are charge eigenvectors. The matrix ν_3 is the antisymmetrical nucleonic charge matrix with eigenvalues ± 1. Symmetrization or antisymmetrization of operator multiplication in accordance with statistics is understood. Also understood is a summation over the three components contained in each non-Hermitian field, as illustrated by

$$\bar{\psi}\gamma^\mu\psi = \sum_{a=1}^{3} \bar{\psi}_a\gamma^\mu\psi_a .$$

This structure is invariant under three-dimensional unitary transformations of the ψ_a.

With appropriate restrictions on the kinematical part of the Lagrange function, the group of three-dimensional unitary transformations of the ψ_a with coordinate-independent coefficients, the group U_3, is an exact invariance group in the simplified description. That property is expressed by the local conservation laws

$$\partial_\mu\, {}^{(\psi)}j_{ab}{}^\mu(x) = 0 ,$$

where

$${}^{(\psi)}j_{ab}{}^\mu = {}^{(\psi)}j_{ba}{}^{\mu\dagger} = -\bar{\psi}_b\gamma^\mu\psi_a = \psi_a(\bar{\psi}_b\gamma^\mu) .$$

The associated integral quantities

$${}^{(\psi)}T_{ab} = {}^{(\psi)}T_{ba}{}^\dagger = \int (d\mathbf{x})(-)\bar{\psi}_b\gamma^0\psi_a$$

obey the unitary group-generator commutation relations

$$[T_{ab}, T_{cd}] = \delta_{bc}T_{ad} - \delta_{ad}T_{cb} .$$

According to our hypothesis about the structure of the Bose field V_μ, a similar but independent U_3 invariance group can be assumed to exist for that field. The corresponding current operators are given by

$${}^{(V)}j_{ab}{}^\mu = -i(\bar{V}_b{}^{\mu\nu}V_{\nu a} - \bar{V}_{\nu b}V_a{}^{\mu\nu}) .$$

The nucleonic charge currents for the two fields are

$${}^{(\psi)}j_N{}^\mu = -\sum_{a=1}^{3} {}^{(\psi)}j_{aa}{}^\mu ,$$

$${}^{(V)}j_N{}^\mu = -2\sum_{a=1}^{3} {}^{(V)}j_{aa}{}^\mu .$$

They are conserved independently in this idealization. The additional invariance transformations that do not refer to nucleonic charge belong to the subgroup of U_3, the unimodular or special unitary group, SU_3. The infinitesimal generators of this group, $'T_{ab}$, obey the commutation relations of the T_{ab}, together with the trace condition

$$\sum_{a=1}^{3} {}'T_{aa} = 0 .$$

A few properties of these operators will be noted here. Eight independent Hermitian operators are introduced by the notation[6]

$$'T_{12} = T_1 + iT_2 ,\quad 'T_{21} = T_1 - iT_2 ,\quad 'T_{11} - 'T_{22} = 2T_3 ,$$
$$'T_{23} = U_1 + iU_2 ,\quad 'T_{32} = U_1 - iU_2 ,\quad 'T_{22} - 'T_{33} = 2U_3 ,$$
$$'T_{31} = V_1 + iV_2 ,\quad 'T_{13} = V_1 - iV_2 ,\quad 'T_{33} - 'T_{11} = 2V_3 ,$$

where

$$T_3 + U_3 + V_3 = 0 .$$

We also define

$$'T_{11} = Q ,\quad 'T_{11} + 'T_{22} = -'T_{33} = Y$$

so that

$$'T_{22} = Y - Q$$

and

$$T_3 = Q - \tfrac{1}{2}Y ,\quad U_3 = Y - \tfrac{1}{2}Q ,\quad V_3 = -\tfrac{1}{2}(Q + Y) .$$

[5] That a threefold internal multiplicity could be basic was suggested quite long ago [S. Sakata, Progr. Theoret. Phys. (Kyoto) **16**, 686 (1956)], as was the symmetry represented by the group SU_3 [S. Ogawa, Progr. Theoret. Phys. (Kyoto) **21**, 209 (1959)]. An inability to describe the known baryons transferred attention to the eight-dimensional representation of this group [Y. Ne'eman, Nucl. Phys. **26**, 222 (1961); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962)]. The threefold multiplicity is introduced here at a deeper dynamical level than the observed particles. An independent attempt in this direction has been made by M. Gell-Mann, Phys. Letters (to be published). He introduces particles of fractional charge which can be detected, presumably, only by their "palpitant piping, chirrup, croak, and quark." [Hartley Burr Alexander, quoted in *Webster's New International Dictionary of the English Language* (G. & C. Merriam Company, Publishers, Springfield, Massachusetts, 1944), 2nd. ed., p. 2033].

[6] Compare S. P. Rosen, Phys. Rev. Letters **11**, 100 (1963) and references therein to contributions of C. A. Levinson, H. J. Lipkin and S. Meshkov.

The various ordered sets, $T_{1,2,3}$; $U_{1,2,3}$; $V_{1,2,3}$; and $2T_2$, $2U_2$, $2V_2$, each obey the commutation relations of three-dimensional angular momentum. The T operators commute with Y, the U operators commute with Q, and the V operators commute with $Y-Q$. An operator that commutes with all eight generators is

$$\frac{1}{2}\sum_{a,b=1}^{3}{}'T_{ab}\,{}'T_{ba}$$

$$=\sum_{a=1}^{3}(T_a^2+U_a^2+V_a^2)-\tfrac{1}{9}[(T_3-U_3)^2+(U_3-V_3)^2+(V_3-T_3)^2].$$

The relation between gauge invariance and a zero-mass, unit-spin particle is such that the latter ceases to exist when the coupling becomes sufficiently strong that an excitation produced by the field descends to zero mass. Under these conditions of strong binding between oppositely charged fields, we can anticipate the existence of low-lying boson excitations with spin and parity $0^\pm$, $1^\pm$, $\cdots$. Odd-parity, spin-zero, and spin-one excitations, for example, will be produced by the effect on the vacuum state of operators such as

$$\bar{\psi}_a\gamma_5\psi_b-\tfrac{1}{3}\delta_{ab}\bar{\psi}\gamma_5\psi$$

and

$$\bar{\psi}_a\gamma^\mu\psi_b-\tfrac{1}{3}\delta_{ab}\bar{\psi}\gamma^\mu\psi,$$

or

$$\bar{V}_a{}^{\mu\nu}\epsilon_{\mu\nu\lambda\kappa}V_b{}^{\lambda\kappa}-\tfrac{1}{3}\delta_{ab}\bar{V}^{\mu\nu}\epsilon_{\mu\nu\lambda\kappa}V^{\lambda\kappa},$$

and

$$\bar{V}_a{}^{\mu\nu}V_{\nu b}-\bar{V}_{\nu a}V_b{}^{\mu\nu}-\tfrac{1}{3}\delta_{ab}(\bar{V}^{\mu\nu}V_\nu-\bar{V}_\nu V^{\mu\nu}),$$

which form independent unitary octuplets. It is not possible to decide in any general way which of these octuplets is least massive. The spin-one, odd-parity unitary singlets generated by the operators $\bar{\psi}\gamma^\mu\psi$ and $\bar{V}^{\mu\nu}V_\nu-\bar{V}_\nu V^{\mu\nu}$ are physically connected through the gauge field B^μ. Thus, a particle of the corresponding quantum numbers is not associated with a specific field, but is a joint physical attribute of all three fields.

Strong nucleonic binding forces also operate between the sets of oppositely charged fields $V(N=\pm2)$ and $\psi(N=\mp1)$. We therefore anticipate the existence of low-lying fermion excitations of nucleonic charge ±1, with spins and parities $\frac{1}{2}^\pm$, $\frac{3}{2}^\pm$, $\cdots$. Spin-$\frac{1}{2}$ excitations, for example, are produced by the effect on the vacuum state of the non-Hermitian operators

$$\bar{\psi}_a\gamma_\mu V_b{}^\mu,\quad \bar{V}_a{}^\mu\gamma_\mu\psi_b.$$

With an additional γ_5 factor, the spin-$\frac{1}{2}$ states of opposite parity are produced. It is important to recognize that these excitations form nonuplets. The multiplicity is $3\times3=9$ since the unitary transformations of the ψ and V fields are still independent. A different kind of fermion multiplet structure is generated by operator products of the form (omitting spinor and vector indices)

$$[(\bar{\psi}_a\bar{\psi}_b+\bar{\psi}_b\bar{\psi}_a)\psi_c-\tfrac{1}{2}(\delta_{bc}\bar{\psi}_a+\delta_{ac}\bar{\psi}_b)\bar{\psi}\psi]V_d,$$

and their adjoints. The multiplicity of this set is $3\times15=45$. The interchange of ψ, $\bar{\psi}$ with V, $\bar{V}$ gives another set. One should also note the boson states generated by

$$(\bar{\psi}_a\bar{\psi}_b\pm\bar{\psi}_b\bar{\psi}_a)V_c$$

and their adjoints. The multiplicity is $3\times6=18$ or $3\times3=9$ according to the choice of sign.

The interaction between the Fermi field ψ and the Bose field V_μ must now be introduced. Nucleonic charge conservation dictates the non-Hermitian combinations $\psi_a\psi_b\bar{V}_c$ and $\bar{\psi}_a\bar{\psi}_bV_c$. But no linear combination of these forms can be invariant under common unitary transformations of the ψ and V fields. The mechanism for the breakdown of unitary symmetry is thereby identified.

The violation of unitary symmetry is minimized if scalar products are formed between $\bar{V}$ and a ψ, V and a $\bar{\psi}$, which leaves objects that have the transformation properties of a component of ψ and $\bar{\psi}$. We shall designate the preferred direction in the three-dimensional unitary space as the third axis. It must be associated with an electrically neutral component of ψ. The resulting theory is invariant under U_2, a single two-dimensional unitary group of transformations on the ψ and V fields. The generators of the subgroup SU_2 are the total isotopic spin operators

$$T_a={}^{(\psi)}T_a+{}^{(V)}T_a,\quad a=1,2,3$$

and the multiplicative phase group is generated by the total hypercharge

$$Y={}^{(\psi)}Y+{}^{(V)}Y.$$

These operators are constants of the motion and the related currents are conserved.

The three components of $\bar{\psi}_a$ and $\bar{V}_a$ can now be given standard isotopic spin and hypercharge labels. Thus $\bar{\psi}_1$ and $\bar{\psi}_2$ are designated $Y=+1$, $T=\frac{1}{2}$, $T_3=+\frac{1}{2}$ and $-\frac{1}{2}$, respectively while $\bar{\psi}_3$ is assigned $Y=T=0$. The precise significance of these statements, together with the electrical charge labels, is given by commutation relations of the type

$$[\bar{\psi},T_a]=t_a\bar{\psi},\quad [\bar{\psi},Y]=y\bar{\psi},\quad [\bar{\psi},Q]=q\bar{\psi},$$

where

$$t_3=\tfrac{1}{2}\begin{pmatrix}+1&&\\&-1&\\&&0\end{pmatrix},\quad y=\begin{pmatrix}+1&&\\&+1&\\&&0\end{pmatrix},\quad q=\begin{pmatrix}+1&&\\&0&\\&&0\end{pmatrix}$$

and

$$t_1+it_2=\begin{pmatrix}\cdot&1&\cdot\\\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot\end{pmatrix},\quad t_1-it_2=\begin{pmatrix}\cdot&\cdot&\cdot\\1&\cdot&\cdot\\\cdot&\cdot&\cdot\end{pmatrix}.$$

For completeness we record the matrices u_a and v_a that are similarly associated with the operators that are not constants of the motion,

$$U_a={}^{(\psi)}U_a+{}^{(V)}U_a, \qquad a=1,2.$$
$$V_a={}^{(\psi)}V_a+{}^{(V)}V_a,$$

They are given by

$$u_1+iu_2=\begin{bmatrix}\cdot&\cdot&\cdot\\ \cdot&\cdot&1\\ \cdot&\cdot&\cdot\end{bmatrix},\quad u_1-iu_2=\begin{bmatrix}\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot\\ \cdot&1&\cdot\end{bmatrix},$$

$$v_1+iv_2=\begin{bmatrix}\cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot\\ 1&\cdot&\cdot\end{bmatrix},\quad v_1-iv_2=\begin{bmatrix}\cdot&\cdot&1\\ \cdot&\cdot&\cdot\\ \cdot&\cdot&\cdot\end{bmatrix}.$$

All these matrices comprise a three-dimensional representation of the SU_3 generator commutation relations. Note that the commutation relations for the field operators are of the form illustrated by

$$[V_\mu,T_a]=-V_\mu t_a=-t_a{}^{\mathrm{tr}}V_\mu$$

so that the negative transposed matrices appear. The latter constitute an inequivalent three-dimensional representation of the SU_3 commutation relations.

The structure of the $\psi-V$ coupling term is not yet completely specified. There is the choice of using a symmetrical or an antisymmetrical combination of ψ_a and ψ_b, before one forms the scalar product with $\bar{V}_b$ and sets $a=3$. If the antisymmetrical combination is chosen, we get, effectively,

$$\psi_3\psi_1\bar{V}_1+\psi_3\psi_2\bar{V}_2,$$

and its adjoint, so that $\bar{V}_3$ and V_3 do not appear in this coupling. Such a theory is invariant under independent phase transformations of the V_3 field, which would imply the existence of another conservation law on the same dynamical level as isotopic spin and hypercharge conservation. Since there is no evidence for additional selection rules, we reject that possibility and choose the symmetrical combination. The space-time character of the vector coupling is then also fixed. If we use Hermitian fermion fields, the possible double nucleonic charged vector structures are

$$\psi_a\alpha^\mu\nu_{1,2}\psi_b,\quad \psi_a\gamma_5\alpha^\mu\nu_{1,2}\psi_b,$$

where ν_1 and ν_2 are the real, symmetrical nucleonic charge reflection matrices. In view of the symmetry of the real matrices α^μ, the antisymmetry of the real matrix γ_5, and Fermi-Dirac statistics, the two structures are, respectively, antisymmetrical and symmetrical in a and b. This selects the pseudovector combination. The result of these considerations is the Hermitian coupling term

$$f'\tfrac{1}{2}[\psi_3 i\gamma_5\alpha^\mu\nu_1\psi V_\mu{}^{(1)}+\psi_3 i\gamma_5\alpha^\mu\nu_2\psi V_\mu{}^{(2)}]$$
$$=f'(1/2^{1/2})[\bar{\psi}_3\gamma_5\gamma^\mu\beta\bar{\psi}V_\mu+\psi_3\gamma_5\beta\gamma^\mu\psi\bar{V}_\mu], \quad (2)$$

where β is the real, antisymmetrical matrix $i\gamma^0$, and

$$V_\mu{}^{(1)}=2^{-1/2}(V_\mu+\bar{V}_\mu),\quad V_\mu{}^{(2)}=2^{-1/2}i(V_\mu-\bar{V}_\mu).$$

The latter is the explicit diagonalization transformation for the nucleonic charge matrix

$$\nu_3=\begin{pmatrix}0&-i\\ i&0\end{pmatrix}.$$

The equivalence of the two forms also depends upon the specific identifications

$$\nu_1=\begin{pmatrix}1&\\ &-1\end{pmatrix},\quad \nu_2=\begin{pmatrix}&1\\ 1&\end{pmatrix}.$$

The description of V_μ as a pseudovector field is to some extent conventional but it is a very natural one in the framework of Hermitian fields, where space reflection is represented by the purely geometrical transformation (spatial coordinates are omitted)

$$P:\quad \psi\to\beta\psi,\quad (V_0,V_k)\to(-V_0,V_k),$$
$$(B_0,B_k)\to(B_0,-B_k).$$

The repetition of this operation reverses the sign of ψ, while leaving the Bose fields unchanged. This can be expressed by the following property of the parity operator in relation to nucleonic charge, or angular momentum,

$$P^2=(-1)^N=(-1)^{2J}.$$

One can always combine P with a nucleonic phase transformation such as $\exp[-\frac{1}{2}\pi iN]$, which obeys

$$(Pe^{-\frac{1}{2}\pi iN})^2=1.$$

The latter is represented in its effect on the non-Hermitian fields by

$$Pe^{-\frac{1}{2}\pi iN}:\quad \psi\to\gamma^0\psi,\quad \bar{\psi}\to\bar{\psi}\gamma^0,\quad (V_0,V_k)\to(V_0,-V_k),$$
$$(B_0,B_k)\to(B_0,-B_k).$$

The theory possesses another discrete invariance operation in the combined reflection of nucleonic and hypercharge. It is represented by

$$\psi\to\bar{\psi}\gamma^0e^{-\pi it_2},\quad \bar{\psi}\gamma^0\to e^{+\pi it_2}\psi,$$
$$G:\quad V_\mu\to\bar{V}_\mu e^{-\pi it_2},\quad \bar{V}_\mu\to e^{+\pi it_2}V_\mu,$$
$$B_\mu\to -B_\mu,$$

where the isotopic spin rotation is introduced to maintain the structure of the isotopic spin currents. The repetition of this operation, which we shall designate as hyperparity, reverses the sign of those field components that carry hypercharge and leaves the others unchanged. This is expressed by

$$G^2=(-1)^Y=(-1)^{2T}.$$

To complete the theory we must specify the dynamical properties of the A_μ and Z_μ fields and introduce the leptonic component of the Fermi field,

$$\chi=\psi(N=0).$$

The six elements of this field can be classified by a leptonic charge $L=\pm1$ in such a way that all values of Q are represented for a given L,

$$\chi=\chi(L=\pm1;Q=0,\pm1,\mp1).$$

One may arbitrarily identify the physical realization of $L=Q$ with the muon, and of $L=-Q$ with the electron. We shall label these components of χ as χ_2 and χ_3, respectively. In accordance with the various electric charge assignments, the coupling with the electromagnetic field is given by

$$eA_\mu[-\bar{\psi}_1\gamma^\mu\psi_1-i(\bar{V}_1^{\mu\nu}V_{\nu1}-\bar{V}_{\nu1}V_1^{\mu\nu})+\bar{\chi}_2\gamma^\mu\chi_2-\bar{\chi}_3\gamma^\mu\chi_3+i(\bar{Z}^{\mu\nu}Z_\nu-\bar{Z}_\nu Z^{\mu\nu})]. \quad (3)$$

At this dynamical level the internal symmetry groups are restricted to the phase transformations that define the charges N, Y, and Q. There is also the discrete operation of charge reflection,

$$C:\quad \begin{array}{ll}\psi\leftrightarrow\bar{\psi}\gamma^0, & \chi\leftrightarrow\bar{\chi}\gamma^0,\\ V_\mu\leftrightarrow\bar{V}_\mu, & Z_\mu\leftrightarrow(\pm)\bar{Z}_\mu,\\ A_\mu\to-A_\mu, & B_\mu\to-B_\mu.\end{array}$$

The charge parity operator obeys

$$C^2=1.$$

The structure of the interaction term between the Z field and the leptonic field is relatively well established in the form[7]

$$e'\bar{Z}_\mu[\bar{\chi}_1\gamma^\mu(1+i\gamma_5)\chi_2+\bar{\chi}_3\gamma^\mu(1-i\gamma_5)\chi_1]+e'Z_\mu[\bar{\chi}_2\gamma^\mu(1+i\gamma_5)\chi_1+\bar{\chi}_1\gamma^\mu(1-i\gamma_5)\chi_3]. \quad (4a)$$

This coupling term incorporates the connection between neutrino polarization and the electrical charge of the accompanying lepton that is expressed by the requirement of invariance under the substitution

$$\chi_1\to e^{i\lambda(i\gamma_5)}\chi_1,\quad \chi_2\to e^{i\lambda}\chi_2,\quad \chi_3\to e^{-i\lambda}\chi_3.$$

It also contains an assertion of dynamical equivalence between μ and e in the sense of the charge reflection transformation

$$(1+i\gamma_5)\chi_2\leftrightarrow(1-i\gamma_5)\chi_3,\quad Z\leftrightarrow\bar{Z}.$$

Finally, the nonequivalence of the left-polarized neutrinos that accompany μ^+ and e^+ is represented by the distinction between the field operators $\bar{\chi}_1$ and χ_1. The only parity operation admitted by this coupling term is

$$CP:\quad \begin{array}{l}\chi\to i\bar{\chi},\quad \bar{\chi}\to i\chi,\\ (Z_0,Z_k)\leftrightarrow(-\bar{Z}_0,\bar{Z}_k),\\ (A_0,A_k)\leftrightarrow(-A_0,A_k).\end{array}$$

We shall introduce an interaction between the Z field and nucleonic charge-bearing fields on the basis of an analogy between χ_a and ψ_a, $a=1,2,3$, one that has been foreshadowed in the labelling of field components. The single electrically charged and the two uncharged components of ψ are to be set into correspondence with the single uncharged and the two charged components of χ. The substitution $\bar{\chi}_1\chi_2\to\bar{\psi}_1\psi_2$ preserves the charge property of the operator while $\bar{\chi}_1\chi_3\to\bar{\psi}_1\psi_3$ reflects it. Hence the latter must be accompanied by a Z-charge reflection. The result is the $\psi-Z$ coupling term

$$e'\bar{Z}_\mu[\bar{\psi}_1\gamma^\mu(1+i\gamma_5)\psi_2+\bar{\psi}_1\gamma^\mu(1-i\gamma_5)\psi_3]+e'Z_\mu[\bar{\psi}_2\gamma^\mu(1+i\gamma_5)\psi_1+\bar{\psi}_3\gamma^\mu(1-i\gamma_5)\psi_1]. \quad (4b)$$

The specific identifications of ψ_2 and ψ_3 relative to γ_5 is in tentative anticipation of the empirical requirements.

An interaction between the V and Z fields is suggested by the recognition that $-\bar{\psi}_2\gamma^\mu\psi_1$ and $-\bar{\psi}_3\gamma^\mu\psi_1$ are the fermion contributions to the current vectors associated with T_1+iT_2 and V_1-iV_2, respectively. The electromagnetic field interacts with the current of total charge $Q=T_3+\frac{1}{2}Y$. It is very natural to suppose that the charged partners of the electromagnetic field also interact with the total quantities of the indicated type. This supplies the coupling term

$$e'\bar{Z}_\mu[i(\bar{V}_1^{\mu\nu}V_{\nu2}-\bar{V}_{\nu1}V_2^{\mu\nu})+i(\bar{V}_1^{\mu\nu}V_{\nu3}-\bar{V}_{\nu1}V_3^{\mu\nu})]+e'Z_\mu[i(\bar{V}_2^{\mu\nu}V_{\nu1}-\bar{V}_{\nu2}V_1^{\mu\nu})+i(\bar{V}_3^{\mu\nu}V_{\nu1}-\bar{V}_{\nu3}V_1^{\mu\nu})]. \quad (4c)$$

We shall include no boson counterparts to the γ_5 terms in (4b), although a basis for such terms could be found in the similarity between $(1\pm i\gamma_5)\psi$ and

$$V^{\mu\nu}\pm i(\epsilon V)^{\mu\nu};$$
$$(\epsilon V)^{\mu\nu}=\tfrac{1}{2}\epsilon^{\mu\nu\lambda\kappa}V_{\lambda\kappa}.$$

This hypothesis assigns the entire dynamical origin of the nonconservation of space parity to the fundamental fermion field. It must meet such experimental tests as those provided by the observed polarization properties in the nonleptonic decays of hyperons.

The complete interaction between the Z field and the nucleonic charge bearing fields possesses the weak parity property

$$CP:\quad \begin{array}{l}\psi\to i\bar{\psi},\quad \bar{\psi}\to i\psi,\\ (V_0,V_k)\leftrightarrow(-\bar{V}_0,\bar{V}_k),\\ (B_0,B_k)\leftrightarrow(-B_0,B_k).\end{array}$$

The repetition of this operation is described, for all fields, by

$$(CP)^2=(-1)^{N+L}=(-1)^{2J}.$$

The dynamical structure of the theory is contained in the four coupling terms (1), (2), (3), and (4a)–(4c).

FURTHER SPECULATIONS

An alternative view of the Z field and of leptonic properties is suggested by an attempt to extend the pattern of multiplicities that have been established for the fundamental spin $\frac{1}{2}$ field ψ to the set of fundamental

[7] Apart from notational differences and a change in sign for γ_5, this is the coupling term (14) of Ref. 4.

unit spin fields A, B, Z, and V. We recall that ψ has six charged components and six neutral components, with either sense of charge. Of the unit spin fields, the number of nucleonic charged components is six, being the fields V and $\bar{V}$, and the number of electrically neutral fields is also six, comprising the fields A, B, and the four electrically uncharged components of V and $\bar{V}$. But when we count the $N=0$ components $[A,B,Z,\bar{Z}]$ and the electrically charged components $[Z,\bar{Z},V_1,\bar{V}_1]$ we find each number to be only four. Should the occult power of multiplicity six be sufficiently impressive, we would be led to introduce another spin one field carrying $N=0$ and $Q=\pm 1$, that is, a second Z field!

We shall try to accommodate two Z fields dynamically by using a vector form of coupling for one field and pseudovector coupling for the other. This is illustrated by the leptonic coupling term

$$2^{1/2}e'[Z_\mu \tfrac{1}{2}\chi\alpha^\mu t\chi + Z_\mu' \tfrac{1}{2}\chi\alpha^\mu i\gamma_5\{t_3,t\}\chi], \qquad (1')$$

which is written in terms of Hermitian fields. Here

$$Zt = Z_1 t_1 + Z_2 t_2$$

and $t_{1,2,3}$ are the imaginary antisymmetrical unit isotopic matrices (they are the SU_3 matrices $2u_2$, $2v_2$, $2t_2$, respectively). The structure (1′) is left invariant under the neutrino parity reflection

$$\chi \to [-it_3 + \gamma_5(1-t_3^2)]\chi,$$

combined with the interchange

$$Z_\mu \leftrightarrow Z_\mu'.$$

An alternative form employs the fields

$$W_\mu = 2^{-1/2}(Z_\mu + Z_\mu'), \quad W_\mu' = 2^{-1/2}(Z_\mu - Z_\mu'),$$

namely,

$$e'[W_\mu \tfrac{1}{2}\chi\alpha^\mu(1+i\gamma_5 t_3)t(1+i\gamma_5 t_3)\chi + W_\mu' \tfrac{1}{2}\chi\alpha^\mu(1-i\gamma_5 t_3)t(1-i\gamma_5 t_3)\chi]. \qquad (2')$$

The transformation induced by neutrino parity reflection now appears as

$$W_\mu \to W_\mu, \quad W_\mu' \to -W_\mu'.$$

If the W' term were absent (2′) would be the leptonic Z-field coupling (4a). But the same practical result appears if the particles of the W' field are sufficiently more massive than the W particles. This is analogous to the empirical mass relation between the two types of charged leptons, μ and e. One might hope that both kinds of mass splitting have a common dynamical origin, which would also identify the mechanism for the individual violation of P and C invariance, since the effect of either transformation is to interchange W and W'.

We thus seek a field that is coupled to the $N=0$, $Q=\pm 1$ members of the spin $\frac{1}{2}$ and spin 1 duodecuplets. In view of the unique role it must play, the field may be presumed to lie outside the established framework, which suggests that it is a spin 0 field. This field ϕ does not carry nucleonic charge and we infer that it is also electrically neutral. (Compare the σ field of Ref. 4.) The proposed analogy between the (non-Hermitian) fields W' and χ_2 is realized by the coupling terms

$$-h\phi\bar{\chi}_2\chi_2 - h'\tfrac{1}{2}\phi^2\bar{W}'^\mu W_\mu' \qquad (3')$$

in which h and h' are dimensionless coupling constants. The spinless field is a weak parity scalar. It also reverses sign under the charged lepton parity reflection

$$\chi \to [(1-t_3^2) + i\gamma_5 t_3]\chi,$$
$$W_\mu \to W_\mu, \quad W_\mu' \to -W_\mu'.$$

In consequence of the large mass displacements it must produce, we cannot assume that (3′) is a weak interaction. An important aspect of the field ϕ is its vacuum expectation value ϕ_0, a quantity that has the dimensions of mass and which is not required to vanish by its space-time transformation properties. If it is assumed that no mass constant is associated with ϕ, the field equations imply the vanishing vacuum expectation value

$$\langle h\bar{\chi}_2\chi_2 + h'\phi\bar{W}'^\mu W_\mu'\rangle = \phi_0 F(\phi_0^2) = 0.$$

It is conceivable that the stability of the vacuum requires that $\phi_0 \neq 0$. Its magnitude would then be given by the (presumably unique) root of

$$F(\phi_0^2) = 0.$$

Either sign of ϕ_0 can be chosen. But once a sign is adopted, one can no longer perform a charged lepton parity reflection. This implies that the muon, in particular, has acquired a mass, as the term $-h\phi_0\bar{\chi}_2\chi_2$ qualitatively suggests. The structure of the h' coupling term also indicates that particles associated with the W' and ϕ fields will receive substantial mass increments. It is interesting that the possibility of electron parity reflection ($\chi_3 \to -i\gamma_5\chi_3$) would persist if only the W-lepton coupling were considered, but fails on including the W' term. Thus, the interference of the two kinds of weak interaction imply a nonzero electron mass. It is not proposed that such higher order effects of the weak interactions produce the observed electron mass, but perhaps they supply the seed that flowers under the influence of electromagnetic interactions.

The same analogies and arguments used in constructing (4b) and (4c) give the vector and pseudovector forms that couple Z and Z' to the nucleonic charge-bearing fields. We have observed that a neutrino parity reflection, $\chi_1 \to \gamma_5\chi_1$, induces the interchange of Z and Z' or the substitution $W \to W$, $W' \to -W'$. No transformation of ψ and V can reverse the latter sign change. Thus, the presence of two kinds of weak interactions generates both an electron and a neutrino mass, but only the former could be amplified through electromagnetic action. It is to be hoped that the new possibilities offered by the idea of the W' field will stimulate more precise measurements of the neutrino mass and of lepton polarizations. The best available polarization measurements still allow a 10% leeway for the effect of the reversed polarizations characteristic of W' coupling. That is, a W' particle with only a little more than twice the mass of the W particle is compatible with present observations.

It should be remarked that the new hypothetical domain of strong interactions described by the W' and ϕ fields is quite insulated from the strongly interacting nuclear world. This solves the problem of giving the large muon mass a dynamical origin without implying strong nonelectromagnetic muon-nucleon interactions. A direct experimental confrontation of these ideas may require higher energies than are now available. Perhaps the most immediate possibility for an experimental test lies in the detection of anomalous electromagnetic production of muon pairs, proceeding through the charged W' field and the strong coupling of the latter to muons through the intermediary of the ϕ field.

SIMPLE CALCULATIONS

The intent of this section is to obtain a first orientation toward some of the physical implications of the theory by using very simple calculational methods. This includes the application of perturbation theory to interactions that are not weak. When such methods lead to gratifying numerical results, that must be regarded as a sign of physical processes at work to bring about unanticipated simplifications, which are exploited but not explained by the crude calculational techniques.

We begin with the highly symmetrical and physically degenerate situation that is created by the strong nucleonic charge binding forces, interaction (1), and try to estimate the effect of the symmetry-destroying coupling between the ψ and V fields, interaction (2).

Baryons ($\frac{1}{2}^+$)

Nine degenerate states of unit nucleonic charge are represented by the vectors

$$\langle ab| \sim \langle |\bar{\psi}_a V_b$$

together with their adjoints

$$|ab\rangle \sim \psi_a \bar{V}_b |\rangle .$$

The notation indicates that the states transform under the two independent groups of U_3 transformations in the same manner as the operators acting on the invariant vacuum state. An explicit construction of the states would include functions of the unitary invariants formed from the two sets of field operators.

To have a nonvanishing matrix with respect to these states, a perturbation must conserve the nucleonic charges associated with each field. The $\psi - V$ coupling is first effective in the second order of f'. That perturbation is symbolized by

$$M_{f'}^{(2)} = f'^2 \psi_3 \bar{\psi} \bar{V} V \psi \bar{\psi}_3 , \quad = f'^2 \sum_{cd} \psi_3 \psi_c \bar{\psi}_d \bar{\psi}_3 \bar{V}_c V_d .$$

In view of the product structure of the states and of the individual terms in this summation, the computation of the matrix with respect to the nonuplet of baryon states essentially reduces to

$$\langle V_b \bar{V}_c V_d \bar{V}_{b'} \rangle = \delta_{bb'} \delta_{cd} k_1 + \delta_{bc} \delta_{b'd} k_2 ,$$

according to the requirement of V unitary invariance, together with

$$\langle \bar{\psi}_a \psi_3 \psi_c \bar{\psi}_d \bar{\psi}_3 \psi_{a'} \rangle = \delta_{aa'} (\delta_{cd} + \delta_{c3} \delta_{d3}) l_1 + (\delta_{a3} \delta_{cd} \delta_{a'3} + \delta_{a3} \delta_{c3} \delta_{a'd} + \delta_{ac} \delta_{d3} \delta_{a'3} + \delta_{ac} \delta_{da'}) l_2 ,$$

which also invokes the symmetry between ψ_3 and ψ_c, $\bar{\psi}_3$ and $\bar{\psi}_d$, in the fundamental interaction. On combining the factors, and omitting a term that displaces the whole multiplet, we obtain the following matrix form:

$$\langle ab | M_{f'}^{(2)} | a'b' \rangle = \delta_{aa'} \delta_{b3} \delta_{b'3} m_1 + \delta_{bb'} \delta_{a3} \delta_{a'3} m_2 + (\delta_{ab} \delta_{a'b'} + \delta_{ab} \delta_{a'3} \delta_{b'3} + \delta_{a'b'} \delta_{a3} \delta_{b3}) m_3 .$$

Various submatrices are supplied by the conservation laws of isotopic spin and hypercharge. Thus, the two states with $b=3$ and $a=1, 2$ are characterized by $Y=1$, $T=\frac{1}{2}$, and $T_3=+\frac{1}{2}, -\frac{1}{2}$, respectively. This set is identified with the nucleon

$$N^{+,0}: \quad \langle 13| , \quad \langle 23| ,$$

and the corresponding mass displacement is

$$\langle M \rangle_N = m_1 .$$

The states with $a=3$ and $b=1, 2$ possess $Y=-1$, $T=\frac{1}{2}$ and $T_3=-\frac{1}{2}, \frac{1}{2}$, respectively. This is the cascade particle,

$$\Xi^{-,0}: \quad \langle 31| , \quad \langle 32| ,$$

and

$$\langle M \rangle_\Xi = m_2 .$$

Three states with quantum numbers $Y=0$, $T=1$ and $T_3=1, -1, 0$, the Σ-particle, are, respectively,

$$\Sigma^{+,-,0}: \quad \langle 12| , \quad -\langle 21| , \quad 2^{-1/2}(-\langle 11| + \langle 22|) ,$$

and

$$\langle M \rangle_\Sigma = 0 ,$$

which sets the mass origin for this calculation.

Finally, we consider the two states with $Y=T=0$,

$$2^{-1/2}(\langle 11| + \langle 22|) , \quad \langle 33| .$$

The submatrix of M for these states is

$$\begin{pmatrix} 2m_3 , & 2^{3/2} m_3 \\ 2^{3/2} m_3 , & m_1 + m_2 + 3m_3 \end{pmatrix} ,$$

where the second row and column refer to $\langle 33|$ and its adjoint. We shall denote the two mass eigenvalues of this matrix by $\Lambda-\Sigma$ and $Y^0-\Sigma$, where $Y^0>\Lambda$. Accordingly,

$$Y^0+\Lambda-2\Sigma=m_1+m_2+5m_3\,,$$
$$-(Y^0-\Sigma)(\Sigma-\Lambda)=2m_3(m_1+m_2)-2m_3{}^2$$

and, in the same kind of notation,

$$-(\Sigma-N)=m_1\,,\quad \Xi-\Sigma=m_2\,.$$

Then

$$\tfrac{1}{5}(Y^0+\Lambda-N-\Xi)=m_3\,,$$

and the implied relation among the masses of the five particles N, Λ, Σ, Ξ, and Y^0 is contained in

$$[\tfrac{1}{5}(Y^0+\Lambda-N-\Xi)]^2=\tfrac{1}{5}(Y^0+\Lambda-N-\Xi)(N+\Xi-2\Sigma)+\tfrac{1}{2}(Y^0-\Sigma)(\Sigma-\Lambda)\,.$$

This can be presented conveniently as the quadratic mass formula

$$(\tfrac{1}{5}M_b-\tfrac{7}{5}M_a)^2=2(\Sigma-\Lambda)M_a+M_a{}^2\,, \tag{5}$$

where

$$M_a=\tfrac{1}{2}(N+\Xi)-\tfrac{1}{4}(3\Lambda+\Sigma)$$

and

$$M_b=Y^0+2\Lambda-3\Sigma\,.$$

We shall add another physical requirement to the mass formula, which is in the nature of a stability condition. It is that N, the lowest mass of the multiplet, shall be stationary about a minimum value with respect to variations of the physical parameter represented by the mass Y^0, or equivalently that

$$dM_a/dM_b=0\,.$$

The consequences are the two relations

$$M_b-7M_a=0$$

and

$$M_a[M_a+2(\Sigma-\Lambda)]=0\,.$$

If the second factor were to vanish in the latter equation, implying that $M_b=-14(\Sigma-\Lambda)$, we should assert that

$$\Lambda-\Sigma=\frac{1}{11}(Y^0-\Lambda)>0\,,$$

in contradiction to the empirical order of the masses. We therefore conclude that both M_a and M_b vanish,

$$\tfrac{1}{2}(n+\Xi)=\tfrac{1}{4}(3\Lambda+\Sigma) \tag{6}$$

and

$$Y^0=3\Sigma-2\Lambda\,. \tag{7}$$

The latter result, in the form

$$\Sigma-\Lambda=\tfrac{1}{3}(Y^0-\Lambda)>0\,,$$

predicts the correct sequence of Λ and Σ. It follows, incidentally, from

$$\frac{d^2M_a}{dM_b{}^2}=\frac{1}{25}\frac{1}{\Sigma-\Lambda}>0$$

that N does assume a minimum value at the stationary point.

The mass relation (6) will be recognized as the Gell-Mann-Okubo mass formula,[8] which has been previously derived from an octuplet form of a unitary symmetry theory. It is well known to be accurately satisfied. Indeed, according to the mass values (in MeV)

$$N=939\,,\quad \Lambda=1115\,,\quad \Sigma=1193\,,\quad \Xi=1318\,,$$

we have

$$M_a=-6\text{ MeV}\,,$$
$$=-\frac{1}{13}(\Sigma-\Lambda)\,,$$

which represents an error of only 0.5 percent, in comparison with either term of M_a.

What name shall be assigned to the ninth baryon? According to (7), its mass should be approximately

$$Y^0\simeq 1349\text{ MeV}.$$

Is there a particle with $T=Y=0$ in the neighborhood of this mass? The obvious candidate is $Y_0{}^*$ at 1405 MeV. With this identification the latter is predicted to be a particle of the same spin and parity as the eight other baryons, $\frac{1}{2}^+$, which provides a crucial experimental test of this simple treatment. In other classifications[9] this particle has been assigned quantum numbers $\frac{1}{2}^-$ and $\frac{3}{2}^+$.

It may be that 56 MeV, or a 4% error, will seem an uncomfortably large discrepancy between theory and experiment for the mass of $Y_0{}^*$, in view of the high precision attained by the mass formula (6). But, remarkably enough, it is the deviation from the latter that is unacceptable since the experimental values imply a negative sign for the right-hand side of the quadratic mass formula (5).

$$2(\Sigma-\Lambda)M_a+M_a{}^2=-\left[\frac{5}{13}(\Sigma-\Lambda)\right]^2.$$

One would expect the difficulty to disappear when higher order perturbations are included. This raises the question whether small effects, of the magnitude suggested by the 6-MeV discrepancy in the mass formula (6), could suffice to produce the needed 56-MeV displacement in Y^0. This is not impossible since the stability condition for N asserts conversely ($dY^0/dN=\infty$) the great sensitivity of Y^0 to small perturbations.

When higher order perturbations are included, the two-dimensional matrix for the states with $Y=T=0$ becomes a general real symmetrical matrix. This can be conveyed by two additional contributions; a multiple, m_0, of the unit matrix, and an added term $2m$, associated with the $\langle 33|$ diagonal element. The mass formula (5)

[8] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962), and Ref. 5.

[9] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963); R. E. Behrends and L. F. Landovitz, *ibid.* **11**, 296 (1963).

is retained, with the substitutions

$$M_a \to M_a+m+\tfrac{3}{4}m_0,\quad M_b\to M_b-3m_0,$$
$$\Sigma-\Lambda\to\Sigma-\Lambda+m_0.$$

If m and m_0 are regarded as fixed numbers, the stability condition would imply the vanishing of the quantities that replace M_a and M_b, or that

$$m+\tfrac{3}{4}m_0=6\text{ MeV},$$
$$Y^0=1349+3m_0.$$

With $m=0$ we obtain an upward displacement of Y^0 by 24 MeV, which is an appreciable fraction of the discrepancy. But, instead, the problem will be intensified by setting $m_0=0$.

We must recognize that m is a dynamical parameter, subject to variation, in seeking the condition for stability. The latter now reads

$$\frac{1}{25}[M_b-7(M_a+m)](1-7dm/dY^0)=(\Sigma-\Lambda+M_a+m)(dm/dY^0)$$

so that neither M_b nor M_a+m are required to vanish. How great a dynamical variation of m, and how large a deviation in value from 6 MeV is needed to account for the empirical data? An accurate estimate of the latter is obtained from

$$M_a+m=\frac{M_b^2}{50(\Sigma-\Lambda)+14M_b}.$$

On inserting the numerical values $M_a=-6$ MeV, $M_b=56$ MeV, we obtain

$$m=6.67\text{ MeV}$$

and then

$$dm/dY^0=0.022.$$

This reassuring conclusion increases the plausibility of the claim that Y_0^* is the ninth baryon.

In the approximation that ignores these corrections, the mass constants in the perturbation matrix are given by

$$m_3=-\tfrac{1}{3}(m_1+m_2)=\tfrac{1}{2}(\Sigma-\Lambda)$$

and the two-dimensional submatrix for states with $Y=T=0$ becomes

$$(\Sigma-\Lambda)\begin{pmatrix}1 & 2^{1/2}\\ 2^{1/2} & 0\end{pmatrix}.$$

The two eigenvectors are

$$\Lambda:\quad 6^{-1/2}(\langle 11|+\langle 22|-2\langle 33|)$$

and

$$Y_0^*:\quad 3^{-1/2}(\langle 11|+\langle 22|+\langle 33|).$$

All nine states can be conveniently displayed by writing $\langle ab|$ in a square array, with rows and columns labeled by $a(\bar\psi)$ and $b(V)$, respectively, and using the particle symbol for the corresponding state,

$$\langle ab|=\begin{bmatrix}-2^{-1/2}\Sigma^0+6^{-1/2}\Lambda+3^{-1/2}Y_0^* & \Sigma^+ & N^+\\ -\Sigma^- & 2^{-1/2}\Sigma^0+6^{-1/2}\Lambda+3^{-1/2}Y_0^* & N^0\\ \Xi^- & \Xi^0 & -(\tfrac{2}{3})^{1/2}\Lambda+3^{-1/2}Y_0^*\end{bmatrix}. \tag{8}$$

When higher order perturbations are included, in the manner just described, the two states become

$$\begin{aligned}\Lambda:&\quad (5.17)^{-1/2}(\langle 11|+\langle 22|-1.78\langle 33|)\\ Y_0^*:&\quad (3.25)^{-1/2}(\langle 11|+\langle 22|+1.12\langle 33|)\end{aligned}\tag{9}$$

and the diagonal elements of the square array are changed into

$$\begin{aligned}\langle 11|&=-2^{-1/2}\Sigma^0+0.44\Lambda+0.55Y_0^*,\\ \langle 22|&=2^{-1/2}\Sigma^0+0.44\Lambda+0.55Y_0^*,\\ \langle 33|&=-0.78\Lambda+0.62Y_0^*.\end{aligned}\tag{10}$$

Mesons (1^-)

The known particles of this type will be tentatively related to the states generated by the operator products $\bar\psi_a\psi_b$, rather than the alternative $\bar V_aV_b$. With respect to such states the $\psi-V$ coupling term can be reduced to the structure symbolized by

$$M_{f'}{}^{(2)}=f'^2\psi_3\psi\bar\psi\bar\psi_3.$$

Within the unitary octuplet, the scalar $\psi\bar\psi$ can be effectively replaced by a constant. We shall adopt a more dynamical attitude toward this replacement, however, and conjecture that, in the evaluation of the perturbation matrix, all such operator unitary scalars are dominated by numbers, in the nature of vacuum expectation values. Then the states and the perturbation can be interpreted literally as

$$\langle ab|=\langle|\bar\psi_a\psi_b,|ab\rangle=\bar\psi_b\psi_a|\rangle,\quad M_{f'}{}^{(2)}=f'^2\bar\psi_3\psi_3,\tag{11}$$

apart from multiplicative constants, and the necessity of subtracting vacuum expectation values from the operators used in constructing the states. This viewpoint becomes significant when the unitary singlet state is added to form a set of nine such states. Although it is not assumed that the singlet state is degenerate with the other eight states[10] we do proceed to describe all the

[10] After completing this work I was made aware of a somewhat related contribution of S. Okubo, Phys. Letters **5**, 165 (1963) which is a perturbation treatment of nine degenerate states. As our discussion will show, better accord with experiment is reached if one includes a displacement of the unperturbed singlet state relative to the octuplet of states. *Note added in proof.* See also F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. (to be published).

states in terms of the set (11). This involves the complete neglect of V-field operators in the construction of the singlet state.

The structure of the perturbation matrix for the nine states is given in part by

$$\langle ab|M_{f'}^{(2)}|a'b'\rangle = f'^2\langle\bar\psi_a\psi_b\bar\psi_3\psi_3\bar\psi_{b'}\psi_{a'}\rangle$$
$$=\lambda(\delta_{a3}\delta_{a'3}\delta_{bb'}+\delta_{aa'}\delta_{b3}\delta_{b'3}).$$

The operators $\bar\psi$ and ψ have been paired to form invariant vacuum expectation values, while recalling that such terms have been removed in constructing the states, and noting that the vacuum expectation value of the perturbation can be discarded since it contributes only a common displacement of the multiplet. The symmetry of the matrix under the substitution $a\leftrightarrow b$, $a'\leftrightarrow b'$ assures hyperparity conservation. There is another perturbation term, assigning a constant λ' to the unitary singlet state, that represents the dynamical difference between the unitary singlet and octuplet. This contribution to the perturbation matrix is

$$\tfrac{1}{3}\lambda'\delta_{ab}\delta_{a'b'}.$$

Isotopic spin and hypercharge distinguish various submatrices. The states with $Y=1$, $T=\frac{1}{2}$, and $T_3=\frac{1}{2}$, $-\frac{1}{2}$ are

$$K^{*+,0}:\quad \langle 13|,\quad \langle 23|,$$

while the states

$$\bar K^{*-,0}:\quad \langle 31|,\quad \langle 32|$$

are characterized by $Y=-1$, $T=\frac{1}{2}$, and $T_3=-\frac{1}{2}$, $\frac{1}{2}$, respectively. For all these states we have

$$\langle M\rangle_{K^*}=\lambda.$$

The three states with $Y=0$, $T=1$ and $T_3=1,-1,0$ are

$$\rho^{+,-,0}:\quad \langle 12|,\quad -\langle 21|,\quad 2^{-1/2}(-\langle 11|+\langle 22|)$$

and

$$\langle M\rangle_\rho=0.$$

There are two states with $Y=T=0$,

$$2^{-1/2}(\langle 11|+\langle 22|),\quad \langle 33|.$$

The corresponding submatrix is

$$\begin{pmatrix} \frac{2}{3}\lambda', & \frac{2^{\frac{1}{2}}}{3}\lambda' \\ \frac{2^{\frac{1}{2}}}{3}\lambda', & \frac{1}{3}\lambda'+2\lambda \end{pmatrix}.$$

The two eigenvalues of this matrix will be denoted by $\omega-\rho$ and $\phi-\rho$, with $\phi>\omega$. Then we have

$$\phi+\omega-2\rho=\lambda'+2\lambda,$$
$$(\phi-\rho)(\omega-\rho)=\tfrac{4}{3}\lambda'\lambda,$$

where, in this notation

$$K^*-\rho=\lambda.$$

We obtain, successively,

$$\phi+\omega-2K^*=\lambda'$$

and the mass relation for odd parity, unit spin mesons:

$$(\phi-\rho)(\omega-\rho)=\tfrac{4}{3}(K^*-\rho)(\phi+\omega-2K^*).\qquad(12)$$

It is a well-known aspect of field spectral properties that the squared mass plays the same role for Bose fields as does the mass with Fermi fields. Hence the particle symbols in (12) are to be interpreted as squared masses.[11] The experimental values are (in MeV)

$$\rho^{1/2}=755\pm5,\quad \omega^{1/2}=781\pm0.8,$$
$$K^{*1/2}=888\pm3,\quad \phi^{1/2}=1019\pm0.5,$$

or (in BeV^2)

$$\rho=0.570\pm0.008,\quad \omega=0.610\pm0.001,$$
$$K^*=0.789\pm0.005,\quad \phi=1.038\pm0.001.$$

Both sides of Eq. (12) are quite small, owing to the respective factors

$$\omega-\rho=0.040\pm0.009,$$
$$\phi+\omega-2K^*=0.070\pm0.010,$$

and they are equal within the experimental errors,

$$0.019\pm0.004=0.020\pm0.003.$$

These errors are largely associated with ρ. It is desirable, therefore, to isolate ρ in the mass formula (12), as given by

$$(\rho+\tfrac{1}{6}(\phi+\omega)-\tfrac{4}{3}K^*)^2=K^{*2}-\omega\phi+K^*(\phi+\omega-2K^*)$$
$$+\frac{1}{36}(\phi+\omega-2K^*)^2.\qquad(13)$$

The effect of errors in K^* is minimized on the right-hand side, in virtue of the approximate equality between $\phi+\omega$ and $2K^*$. Even more striking is the near cancellation of K^{*2} and $\omega\phi$, according to

$$(\omega\phi)^{1/2}-K^*=0.007\pm0.005.$$

The square root of the right-hand side in (13) has the numerical value 0.211 ± 0.001. If we choose its sign to be negative, ρ is predicted to be

$$\rho=0.566\pm0.007$$

or

$$\rho^{1/2}=752\pm5\text{ MeV}.$$

The agreement with the measured value is amazing, particularly in view of the seemingly crude dynamical assumptions involved.

[11] A comment to this effect is generally ascribed to R. P. Feynman (unpublished).

Incidentally, a simplified version of (13), from which some very small terms are omitted is

$$K^* = \tfrac{1}{4}[3\rho + \tfrac{1}{2}(\phi+\omega) + 3(\phi\omega)^{1/4}(\phi^{1/2} - \omega^{1/2})].$$

It is still fulfilled within experimental error.

The parameters of the perturbation matrix are related by

$$\frac{\lambda'}{\lambda} = \frac{\phi+\omega-2K^*}{K^*-\rho} \simeq \tfrac{1}{3}.$$

Thus, the two-dimensional submatrix is

$$\frac{2}{9}\lambda \begin{pmatrix} 1, & 2^{-1/2} \\ 2^{-1/2}, & 19/2 \end{pmatrix}$$

and its eigenvectors are

$$\begin{aligned} \omega&: \quad (2.013)^{-1/2}[\langle 11| + \langle 22| - 0.116\langle 33|], \\ \phi&: \quad (1.003)^{-1/2}[0.058(\langle 11| + \langle 22|) + \langle 33|], \end{aligned} \tag{14}$$

which differ very little from $2^{-1/2}(\langle 11| + \langle 22|)$ and $\langle 33|$. Using the latter, the nine states are displayed in the square array

$$\langle ab| = \begin{bmatrix} -2^{-1/2}\rho^0 + 2^{-1/2}\omega, & \rho^+, & K^{*+} \\ -\rho^-, & 2^{-1/2}\rho^0 + 2^{-1/2}\omega, & K^{*0} \\ \bar{K}^{*-}, & \bar{K}^{*0}, & \phi \end{bmatrix}, \tag{15}$$

where row and column are labeled by $a(\bar{\psi})$ and $b(\psi)$. The diagonal elements of the array are given somewhat more precisely by

$$\begin{aligned} \langle 11| &= -2^{-1/2}\rho + 0.705\omega + 0.058\phi, \\ \langle 22| &= 2^{-1/2}\rho + 0.705\omega + 0.058\phi, \\ \langle 33| &= \phantom{-2^{-1/2}\rho} - 0.081\omega + 0.997\phi. \end{aligned} \tag{16}$$

In order to exhibit the couplings among the baryons and the mesons we shall introduce phenomenological or secondary fields that possess the transformation properties of the corresponding particle states. Thus, we have a $\frac{1}{2}^+$ baryon spinor field

$$\Psi_{ab} \sim \bar{\psi}_a V_b,$$

with its adjoint

$$\bar{\Psi}_{ab} \sim \bar{V}_a \psi_b,$$

and a 1^- meson vector field

$$U_{ab} \sim \bar{\psi}_a \psi_b.$$

As an initial approximation we construct such couplings to be invariant under the two independent groups of U_3 transformations associated with the ψ and V primary fields (should this be called W_3 invariance?). The unitary structure required for the coupling of baryons and mesons is illustrated by

$$\mathcal{L}_{U\Psi} = g_{U\Psi} \sum_{abc} U_{ab}\Psi_{bc}\bar{\Psi}_{ca} = g_{U\Psi}\,\mathrm{Tr}\,U\Psi\bar{\Psi}.$$

The irreducibility of the summation expresses the essential equivalence of all nine meson states. When written out, using particle symbols for the corresponding fields, this becomes, approximately,

$$\begin{aligned} \mathcal{L}_{U\Psi}/g_{U\Psi} = {} & 2^{-1/2}\omega\left[\bar{N}N + \bar{\Sigma}\Sigma + \tfrac{1}{3}\bar{\Lambda}\Lambda + \tfrac{2}{3}\bar{Y}Y + \frac{2^{\frac{1}{2}}}{3}(\bar{\Lambda}Y + \bar{Y}\Lambda)\right] + \phi\left[\bar{\Xi}\Xi + \tfrac{2}{3}\bar{\Lambda}\Lambda + \tfrac{1}{3}\bar{Y}Y - \frac{2^{\frac{1}{2}}}{3}(\bar{\Lambda}Y + \bar{Y}\Lambda)\right] \\ & + 2^{-1/2}\rho[-\bar{N}2tN - \bar{\Sigma}t\Sigma + \bar{\Sigma}(3^{-1/2}\Lambda + (\tfrac{2}{3})^{1/2}Y) - (3^{-1/2}\bar{\Lambda} + (\tfrac{2}{3})^{1/2}\bar{Y})\Sigma^g] \\ & + (\bar{N}K^*)(-(\tfrac{2}{3})^{1/2}\Lambda + 3^{-1/2}Y) + (6^{-1/2}\bar{\Lambda} + 3^{-1/2}\bar{Y})(\Xi K^*) + 2^{1/2}\bar{\Sigma}^g(\Xi t K^*) \\ & + (-(\tfrac{2}{3})^{1/2}\bar{\Lambda} + 3^{-1/2}\bar{Y})(\bar{K}^*N) + (\bar{K}^*\bar{\Xi})(6^{-1/2}\Lambda + 3^{-1/2}Y) - 2^{1/2}\Sigma(\bar{K}^*t\bar{\Xi}). \end{aligned} \tag{17}$$

Here t represents the appropriate isotopic spin matrices,

$$\rho t = \rho^0 t_3 + \rho^- 2^{-1/2}(t_1 - it_2) - \rho^+(t_1 + it_2)$$

and similarly for Σt, while Σ^g and $\bar{\Sigma}^g$ indicate the effect of the hyperparity operation (omitting spin matrices):

$$\begin{aligned} \bar{\Sigma}^g &= e^{\pi i t_2}\Sigma = (\Sigma^-, \Sigma^+, -\Sigma^0), \\ \Sigma^g &= e^{\pi i t_2}\bar{\Sigma} = (\bar{\Sigma}^+, \bar{\Sigma}^-, -\bar{\Sigma}^0). \end{aligned}$$

It will be noted that ω is not coupled precisely to the total phenomenological current of nucleonic charge nor is ρ coupled precisely to the total phenomenological current of isotopic spin.[12] But what is most significant at the moment is the essential absence of interaction between ϕ and the nucleon. This is consistent with the marked weakness of ϕ production relative to ω production in $\pi-N$ collisions,[13] and indicates that 1^- mesons have been assigned correctly to the $\bar{\psi}_a\psi_b$ states. The alternative assignment would reverse the roles of N and $\bar{\Xi}$. If the linear combinations (16) are used, the most important effect is the substitution $2^{-1/2}\omega \to 0.705\omega + 0.058\phi$. Then the intrinsic ratio of ϕ to ω production in pion-nucleon interactions is presumably of the magnitude indicated by $(0.058/0.705)^2 = 0.69\times 10^{-2}$. This could easily be increased by a factor ~ 1.5 in view of the relatively large uncertainty in $\phi+\omega-2K^*$. There is an experimental upper limit to this ratio measured in the reaction $\pi^- + p \to \pi^- + p + \omega,\ \phi$. When corrected for phase-space factors, it is $0.016/0.55 = 2.9\times 10^{-2}$.

[12] Compare J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).

[13] Y. Y. Lee, W. Moebs, B. Rose, D. Sinclair, and J. Vander Velde, Phys. Rev. Letters **11**, 508 (1963); M. Abolins, R. Lander, W. Mehlhop, N. Xuong, and P. Yager, *ibid.* **11**, 381 (1963).

Mesons (0^-)

If one assumes that 0^- mesons are also related to $\bar{\psi}_a\psi_b$ states, the preceding discussion can be applied to these particles. States that describe particles in the rest system will be obtained by using γ_5 instead of the matrices required for 1^- particles, γ_k, $k=1, 2, 3$. The algebraic similarity of these matrices implies that the constant λ associated with the second-order $\psi - V$ coupling is the same for both types of mesons. That is, when one follows the procedure of inserting vacuum expectation values for unitary and space-time scalars, the two calculations symbolized by

$$\lambda_1\delta_{kl}=f'^2\langle\bar{\psi}_1\gamma_k\psi_3\bar{\psi}_3\psi_3\bar{\psi}_3\gamma_l\psi_1\rangle$$
$$=f'^2\langle\psi_3\bar{\psi}_3\rangle\langle\psi_3\bar{\psi}_3\rangle\langle\bar{\psi}_1\gamma_k\gamma_l\psi_1\rangle$$

and

$$\lambda_0=f'^2\langle\bar{\psi}_1\gamma_5\psi_3\bar{\psi}_3\psi_3\bar{\psi}_3\gamma_5\psi_1\rangle$$
$$=f'^2\langle\psi_3\bar{\psi}_3\rangle\langle\psi_3\bar{\psi}_3\rangle\langle\bar{\psi}_1\gamma_5\gamma_5\psi_1\rangle,$$

will yield the same values for λ_0 and λ_1. The experimental 0^- meson masses are (in MeV)

$$\pi^{1/2}=138.04\pm0.05,\quad K^{1/2}=496.0\pm0.3,\quad \eta^{1/2}=548.5\pm0.6$$

and (BeV^2)

$$\pi=0.01904,\quad K=0.2460+0.0005,\quad \eta=0.3001\pm0.0006.$$

Thus

$$\lambda_0=K-\pi=0.2270\pm0.0005 \tag{18}$$

is to be compared with

$$\lambda_1=K^*-\rho=0.219\pm0.009.$$

They are indeed equal, within experimental error.[14]

A ninth 0^- meson has not yet been identified. That presumably means that the unitary singlet is displaced far above the octuplet. In the limit of infinite singlet mass the G-M-O formula is obtained for the known mesons,

$$K=\tfrac{1}{4}(3\eta+\pi).$$

This relation is obeyed moderately well; the right-hand side equals 0.2298 ± 0.0005, which is a discrepancy of 7%. It is significant that K exceeds the value computed in this way. That enables the discrepancy to be accounted for by a ninth meson. We shall designate this unknown fourth type of 0^- meson as δ. The mass relation

$$(\delta-\pi)(\eta-\pi)=\tfrac{4}{3}(K-\pi)(\delta+\eta-2K),$$

or

$$\delta-\pi=\frac{(K-\pi)(2K-\eta-\pi)}{K-\frac{1}{4}(3\eta+\pi)} \tag{19}$$

predicts

$$\delta=2.45\pm0.1,\quad \delta^{1/2}=1560\pm30 \text{ MeV}. \tag{20}$$

Without taking this number too seriously, it does seem reasonable to anticipate that the 0^- meson δ exists somewhere in the vicinity of 1.5 BeV.

The two parameters of the perturbation matrix for these particles are related by

$$\lambda_0'/\lambda_0=(\delta+\eta-2K)/(K-\pi)\simeq10.$$

The two dimensional submatrix for $Y=T=0$ is

$$\frac{20}{3}\lambda\begin{pmatrix}1, & 2^{-1/2}\\ 2^{-1/2}, & \frac{4}{5}\end{pmatrix}$$

and the eigenvectors are

$$\begin{aligned}\eta:&\quad(4.66)^{-1/2}[\langle11|+\langle22|-1.63\langle33|],\\ \delta:&\quad(3.51)^{-1/2}[\langle11|+\langle22|+1.23\langle33|].\end{aligned} \tag{21}$$

The diagonal elements of a square array are given by

$$\begin{aligned}\langle11|&=-2^{-1/2}\pi^0+0.465\eta+0.533\delta,\\ \langle22|&=2^{-1/2}\pi^0+0.465\eta+0.533\delta,\\ \langle33|&=-0.755\eta+0.654\delta.\end{aligned} \tag{22}$$

A rough approximation to these numerical coefficients is used for simplicity in writing

$$\langle ab|=\begin{bmatrix}-2^{-1/2}\pi^0+\frac{1}{2}\eta+\frac{1}{2}\delta, & \pi^+, & K^+\\ -\pi^-, & 2^{-1/2}\pi^0+\frac{1}{2}\eta+\frac{1}{2}\delta, & K^0\\ \bar{K}^-, & \bar{K}^0, & -2^{-1/2}\eta+2^{-1/2}\delta\end{bmatrix}. \tag{23}$$

The same approximation will be used for the secondary pseudoscalar field that is associated with these particles,

$$\Phi_{ab}\sim\bar{\psi}_a\psi_b.$$

The unitary structure required for an initial approximation to the coupling between the 0^- mesons and the baryons is given by

$$\mathcal{L}_{\Phi\Psi}=g_{\Phi\Psi}\,\mathrm{Tr}\Phi\Psi\bar{\Psi},$$

while couplings among the mesons are illustrated by

$$\mathcal{L}_{U\Phi^2}=g_{U\Phi^2}\,\mathrm{Tr}U\Phi\Phi$$

and

$$\mathcal{L}_{\Phi U^2}=g_{\Phi U^2}\,\mathrm{Tr}\Phi UU.$$

[14] This property of the data has been noticed by S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

The baryon coupling term is

$$\mathcal{L}_{\Phi\Psi}/g_{\Phi\Psi}=\tfrac{1}{2}\eta\left[\bar{N}N+\bar{\Sigma}\Sigma-2^{1/2}\bar{\Xi}\Xi-\frac{2^{3/2}-1}{3}\bar{\Lambda}\Lambda+\frac{2-2^{1/2}}{3}\bar{Y}Y+\frac{2+2^{1/2}}{3}(\bar{\Lambda}Y+\bar{Y}\Lambda)\right]$$
$$+\tfrac{1}{2}\delta\left[\bar{N}N+\bar{\Sigma}\Sigma+2^{1/2}\bar{\Xi}\Xi+\frac{2^{3/2}+1}{3}\bar{\Lambda}\Lambda+\frac{2+2^{1/2}}{3}\bar{Y}Y-\frac{2-2^{1/2}}{3}(\bar{\Lambda}Y+\bar{Y}\Lambda)\right]$$
$$+2^{-1/2}\pi[-\bar{N}2tN-\bar{\Sigma}t\Sigma+\bar{\Sigma}(3^{-1/2}\Lambda+(\tfrac{2}{3})^{1/2}Y)-(3^{-1/2}\bar{\Lambda}+(\tfrac{2}{3})^{1/2}\bar{Y})\Sigma^{g}]$$
$$+(\bar{N}K)(-(\tfrac{2}{3})^{1/2}\Lambda+3^{-1/2}Y)+(6^{-1/2}\bar{\Lambda}+3^{-1/2}\bar{Y})(\Xi K)+2^{1/2}\Sigma^{g}(\Xi tK)$$
$$+(-(\tfrac{2}{3})^{1/2}\bar{\Lambda}+3^{-1/2}\bar{Y})(\bar{K}N)+(\bar{K}\bar{\Xi})(6^{-1/2}\Lambda+3^{-1/2}Y)-2^{1/2}\Sigma(\bar{K}t\bar{\Xi}). \quad (24)$$

The explicit space-time structure is obtained by extending the known pion-nucleon coupling, which we use in the pseudovector form

$$(f_{\pi N}/\mu_{\pi})\Phi_{\pi}{}^{\lambda}\bar{\Psi}_{N}i\gamma_5\gamma_\lambda 2t\Psi_N.$$

Thus, the pion coupling between Y_0^* and Σ is given by

$$(\tfrac{2}{3})^{1/2}(f_{\pi N}/\mu_\pi)\Phi_\pi{}^\lambda[-\bar{\Psi}_\Sigma i\gamma_5\gamma_\lambda\Psi_Y+\bar{\Psi}_Y i\gamma_5\gamma_\lambda\bar{\Psi}_{\Sigma^g}].$$

From this we calculate the single-pion emission contribution to the width of Y_0^*,

$$\Gamma_\pi(Y_0^*)=\frac{4}{3}\frac{f_{\pi N}^2}{4\pi}\frac{p_\pi^3}{\mu_\pi^2}=30\text{ MeV}, \quad (25)$$

where p_π is the pion momentum, and

$$f_{\pi N}^2/4\pi=0.1$$

has been used. This is somewhat smaller than the experimentally indicated width[15] of $\sim$50 MeV.

The complete absence of $\pi-\Xi$ coupling should be noted. It supports the identification of the 0^- mesons as $\bar{\psi}_a\psi_b$ states. Also of interest is the lack of $\bar{N}K\Sigma$ interaction, which is not altered by including π-mesons. However, the relation between Λ and Σ couplings should be sensitive to the effect of the symmetry-destroying coupling, particularly if it is amplified by the small pion mass.

The coupling between the vector field U and the pseudoscalar field Φ can be given the following space-time structure:

$$\mathcal{L}_{U\Phi^2}=g_{U\Phi^2}\sum_{abc}U_{ab}{}^\lambda(1/2i)(\Phi_{bc}\Phi_{\lambda ca}-\Phi_{\lambda bc}\Phi_{ca}).$$

One should observe that the bilinear Φ factor is traceless and that terms with $a=b=c$ are not present. The unitary structure is written out as

$$\mathcal{L}_{U\Phi^2}/g_{U\Phi^2}=(\phi-2^{-1/2}\omega)\bar{K}K+2^{-1/2}\rho(\bar{\pi}t\pi+\bar{K}2tK)$$
$$+2^{1/2}\pi\bar{K}tK^*+2^{1/2}\bar{K}^*tK\pi+(2^{-1/2}-\tfrac{1}{2})$$
$$\times(\delta\bar{K}K^*+\bar{K}^*K\delta)-(2^{-1/2}+\tfrac{1}{2})$$
$$\times(\eta\bar{K}K^*+\bar{K}^*K\eta), \quad (26)$$

where

$$\bar{\pi}=-e^{\pi i t_2}\pi=(-\pi^-,-\pi^+,\pi^0).$$

This phenomenological interaction describes three known decay processes, $\rho\to\pi+\pi$, $K^*\to K+\pi$, $\phi\to K+\bar{K}$, and gives a first approximation to their rates in terms of a common coupling constant. We find that

$$\Gamma(\rho,K^*,\phi)=(1,\tfrac{3}{4},1)\frac{1}{3}\left(\frac{g^2}{4\pi}\right)\frac{p^3}{\mu^2},$$

where p is the relative momentum of the decay products and μ is the unstable particle mass in each reaction. The theoretical values for the ratios are

$$\Gamma(K^*)/\Gamma(\rho)=1/3.5,\quad \Gamma_K(\phi)/\Gamma(\rho)=1/50. \quad (27)$$

The experimental results for the widths are (in MeV)

$$\Gamma(\rho)=120\pm10,\quad \Gamma(K^*)=50\pm10,\quad \Gamma(\phi)=3.1\pm1.0.$$

From the first of these we obtain

$$g_{U\Phi^2}{}^2/4\pi\sim5.$$

The widths predicted for K^* and ϕ are then

$$\Gamma(K^*)=35\pm3,\quad \Gamma_K(\phi)=2.4\pm0.2. \quad (28)$$

The latter should not be compared with the measured total width until we have estimated the contribution of the alternative process $\phi\to\rho+\pi$.

This is described by the phenomenological coupling term

$$\mathcal{L}_{\Phi U^2}=g_{\Phi U^2}\sum_{abc}\Phi_{ab}\tfrac{1}{4}\epsilon_{\mu\nu\lambda\kappa}U_{bc}{}^{\mu\nu}U_{ca}{}^{\lambda\kappa},$$

which is symmetrical in the two U factors. On using the approximate forms of the square arrays it is written out as

$$\mathcal{L}_{\Phi U^2}/g_{\Phi U^2}=\tfrac{1}{2}\delta[\omega\omega+\bar{\rho}\rho+(1+2^{1/2})\bar{K}^*K^*+2^{1/2}\phi\phi]$$
$$+\tfrac{1}{2}\eta[\omega\omega+\bar{\rho}\rho+(1-2^{1/2})\bar{K}^*K^*-2^{1/2}\phi\phi]$$
$$-2^{1/2}\pi\bar{K}^*tK^*+2^{1/2}\bar{\pi}\rho\omega+2^{-1/2}\omega\bar{K}^*K$$
$$+2^{-1/2}\bar{K}K^*\omega+\phi\bar{K}^*K+\bar{K}K^*\phi$$
$$-2^{1/2}\rho\bar{K}^*tK-2^{1/2}\bar{K}tK^*\rho, \quad (29)$$

which contains no $\pi\rho\phi$ coupling term. The more precise

[15] A general summary of experimental information is given by Matts Roos, Rev. Mod. Phys. 35, 314 (1963).

version obtained by the substitution (16),

$$2^{-1/2}\omega \rightarrow 0.705\omega + 0.058\phi\,,$$

gives a small coupling[16] which implies an almost forbidden $\phi \rightarrow \rho+\pi$ decay.

The rate for the latter process depends upon the coupling constant $g_{\Phi U^2}$ which cannot be inferred from any similar decay rate. It can, however, be related to the partial width for $\omega \rightarrow \pi^+ + \pi^- + \pi^0$, if this reaction is viewed as proceeding through the stages $\omega \rightarrow \rho+\pi \rightarrow 3\pi$. We shall temporarily adopt a value quoted in the literature,[17] which asserts that

$$(\mu_\omega g_{\Phi U^2})^2/4\pi \sim 6\,.$$

With the definition

$$g_{\pi\rho\phi} = 0.116 g_{\Phi U^2}$$

we find that

$$\Gamma_\pi(\phi) = [(\mu_\phi g_{\pi\rho\phi})^2/4\pi](p^3/\mu^2)_\phi = 1.0 \text{ MeV}\,.$$

The theoretical branching ratio

$$\Gamma_\pi(\phi)/\Gamma_K(\phi) = 0.42$$

is consistent with a measurement[18] that gives 0.35 ± 0.2. The total width

$$\Gamma(\phi) = 3.4 \text{ MeV}$$

agrees well with the observed value.[19]

L'ENVOI

To avoid a paper of excessive length, these elementary studies of particle phenomena will be continued in other publications. But we cannot refrain from a retrospective glance, for it is necessary to emphasize the distinction between the general dynamical theory that has been proposed and the specific implications that have been obtained. The latter also involve a dynamical conjecture, that the effect of the symmetry-destroying interaction can be discussed by perturbation theory, which requires an approximate phenomenological validity for the underlying symmetry group W_3. This need not be true. It is conceivable that the $\psi - V$ coupling term is sufficiently strong that its major effect is the establishment of a dynamical regime governed by the common symmetry group SU_3, while only secondarily destroying that symmetry. In this light, the crucial experimental spin-parity determination for Y_0^* (1405 MeV) will test whether W_3 or SU_3 is the more realistic underlying symmetry group.[20] Should this trial of the W_3 hypothesis fail, such of its successes as the interpretation of the weak ϕ production in $\pi - N$ collisions will demand new explanations.

Note added in proof. It has been suggested [for example, I. S. Gerstein and K. T. Mahanthappa, Phys. Rev. Letters **12**, 570 (1964)] that the $\frac{3}{2}^+$ baryon resonances belong to a $3\times 6 = 18$ dimensional representation of W_3. This presumably is not correct since the representation that contains protons and positive pions is the $3\times 15 = 45$ dimensional one that is mentioned (for that purpose) in the text. An equivalent remark has been published by R. E. Cutkosky, Phys. Rev. Letters **12**, 530 (1964). The 45-dimensional W_3 representation contains SU_3 representations of dimensionality 10, 8, and 27. The experimental situation with regard to the 8- and 27-fold representations is unclear. There are some resonances below 2 BeV that may be $\frac{3}{2}^+$, but it is likely that most of these states are more massive. It seems that the mechanism which destroys W_3 symmetry is more effective in separating the various SU_3 multiplets than in splitting the individual ones.

If the separation of SU_3 multiplets in a W_3 representation is large enough, a new possibility appears, which has been noted by P. Freund and Y. Nambu, Phys. Rev. Letters **12**, 714 (1964). Fermions need not have a common parity. Thus, the known $\frac{3}{2}^-$ baryon resonances might belong to the same 45-dimensional W_3 representation as the tenfold $\frac{3}{2}^+$ resonances, and a $\frac{1}{2}^- Y_0^*$ (1405) could be the ninth baryon. With this interpretation, the pseudovector pion coupling (24) gives a width for Y_0^* of the observed magnitude. A resonance that may belong to a 27-fold representation has been found [R. Alvarez, Z. Bar-Yam, W. Kern, D. Luckey, L. S. Osborne, S. Tazzari, and R. Fessel, Phys. Rev. Letters **12**, 710 (1964)]. Perhaps this particle will help to establish the larger pattern of W_3 symmetry.

A uniform description of nine 1^- mesons and nine 0^- mesons is given in the text. It unites unitary singlet and octuplet states while retaining a mass displacement between the multiplets. This procedure is successful for the 1^- mesons. It is somewhat justified by the small singlet-octuplet displacement implied by the proximity of the ω and ρ masses. The situation is not so favorable for the 0^- mesons π, K, η, δ since the singlet must be considerably more massive than the octuplet. That should imply some difference in structure for the two types of states. One could attempt to represent this relative distortion by including an "overlap" factor $\alpha(\alpha^2<1)$ in the matrix element, of the symmetry destroying perturbation, that connects singlet and

[16] If the production processes $\pi+N \rightarrow \pi+N+(\omega,\phi)$ are dominated by the $\pi\rho(\omega,\phi)$ coupling, one again estimates the intrinsic ratio to be in the order of one percent.

[17] See, for example, G. Feinberg and H. S. Mani, Phys. Rev. Letters **11**, 448 (1963).

[18] P. L. Connolly, E. Hart, K. Lai, G. London, G. Moneti, *et al.*, Phys. Rev. Letters **10**, 371 (1963).

[19] N. Gelfand, D. Miller, M. Nussbaum, J. Ratau, J. Schultz, *et al.*, Phys. Rev. Letters **11**, 438 (1963).

[20] I am grateful to S. F. Tuan for reminding me of the possibility that the ninth baryon Y^0 is an unknown metastable particle. The $\pi+\Sigma$ threshold is centered at 1331 MeV. It is quite conceivable that higher order perturbations displace Y^0 downward from 1349 MeV below this nearby threshold. Such a particle would decay radiatively into Λ and could be confused with Σ^0 in high-energy production experiments. This alternative would permit the more usual assignment $\frac{1}{2}^-$ for Y_0^* (1405) but raises the problem of identifying the other members of the nonuplet to which Y_0^* should belong. Or is this particle a member of the triplet generated by ψ_a?

octuplet states. The modified (mass)^2 formula is

$$(\eta-\pi)(\delta-\pi)=\tfrac{4}{3}(K-\pi)(\eta+\delta-2K)+8/9(1-\alpha^2)(K-\pi)^2$$

or

$$\delta-\pi=\frac{K-\pi}{K-\frac{1}{4}(3\eta+\pi)}\times[2K-\eta-\pi-\tfrac{2}{3}(1-\alpha^2)(K-\pi)]\geq\tfrac{4}{3}(K-\pi).$$

The lower limit, $\delta^{1/2}\geq 567$ MeV, is reached for $\alpha=0$. The maximum value $\delta^{1/2}\leq 1560$ MeV occurs for $|\alpha|=1$. The actual magnitude of α will be determined by considerations of dynamical stability. Thus, increasing α raises δ, which then tends to decrease α, the overlap factor. The δ mass should be some reasonable average of the two unattainable extremes: 567 MeV$<\delta^{1/2}<$1560 MeV.

A new meson that decays into $\pi^++\pi^-+\eta$ has been announced [G. R. Kalbfleisch, L. W. Alvarez, A. Barbaro-Galtieri, O. I. Dahl, P. Eberhard, *et al.*, Phys. Rev. Letters **12**, 527 (1964); M. Goldberg, M. Gundzik, S. Lichtman, J. Leitner, M. Primer *et al.*, *ibid.* **12**, 546 (1964)]. Its properties are consistent with the quantum numbers $T=Y=0$, $J^P=0^-$, and $G=+1$, which are those of the δ meson. The mass observed for the particle is 959±2 MeV. The value of the overlap parameter required to represent this mass is $|\alpha|=0.53$. If we accept the identification of the new meson with δ, the ratio of the two perturbation parameters is reduced to $\lambda'/\lambda=3.2$, and the particle states with $T=Y=0$ become

$$\eta=(3.89)^{-1/2}[\langle 11|+\langle 22|-1.37\langle 33|],$$
$$\delta=(4.13)^{-1/2}[\langle 11|+\langle 22|+1.46\langle 33|].$$

This change improves the accuracy of the approximate square array, Eq. (23).

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$K^+\Lambda$ Photoproduction from Hydrogen at 1200 MeV*

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Measurements of the angular distribution of the cross section for the photoproduction of the $K^+\Lambda$ system from hydrogen have been made in the c.m. angular interval from 15° to 85° at a photon energy of 1200 MeV. The reaction was identified by detecting the K^+ mesons with a magnet spectrometer and a velocity selection system consisting of two Čerenkov counters. The angular distribution at this energy is very similar to that at lower energies in that it is peaked forward and is easily fit with a quadratic in $\cos\theta_{\text{c.m.}}$. Special emphasis was placed on the forward direction in an attempt to find evidence for the one-K-exchange pole. A Taylor-Moravcsik analysis of the data is presented, but the results are inconclusive.

I. INTRODUCTION

SINCE the original investigations[1] of the photoproduction of K^+ mesons, improvements in both detection schemes and electron synchrotron beam intensities have resulted in a significant increase in our knowledge of these reactions. Of the several possible associated photoproduction reactions, the simplest experimentally is

$$\gamma+p\rightarrow K^++\Lambda\quad[\text{threshold: 911 MeV}]$$

and the experiment reported here concerns this process only. Recent work at Cornell[2,3] has yielded several angular distributions of the cross section for this reaction in the energy range from 934 to 1160 MeV. Furthermore, the polarization of the Λ at 90° in the center of mass for several energies up to 1160 MeV has been reported by the same group.[4] As was clear from the early measurements, the production near threshold is consistent with the $K^+\Lambda$ system being in an S state. However, at 1000 MeV the appearance of polarization of the Λ and a gentle rise of the angular distribution in the K^+ forward direction indicate that higher partial waves are beginning to be excited. As the photon energy is increased, the Λ polarization and the 90° cross section remain essentially constant[4] while the angular distribution of the cross section becomes more peaked[2,3] in the forward K direction. The results of the present investigation of the photoproduction of the $K^+\Lambda$ system from hydrogen show that this behavior of the cross section also obtains at a photon energy of 1200 MeV.

The K^+ photoproduction experiments are of some theoretical interest as they provide, in principle at

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] P. L. Donoho and R. L. Walker, Phys. Rev. **112**, 981 (1958); B. D. McDaniel, A. Silverman, R. R. Wilson, and G. Cortellessa, *ibid.* **115**, 1039 (1959); H. M. Brody, A. M. Wetherell, and R. L. Walker, *ibid.* **119**, 1710 (1960).

[2] R. L. Anderson, E. Gabathuler, D. Jones, B. D. McDaniel, and A. J. Sadoff, Phys. Rev. Letters **9**, 131 (1962).

[3] A. J. Sadoff, R. L. Anderson, E. Gabathuler, and D. Jones, Bull. Am. Phys. Soc. **9**, 34 (1964).

[4] H. Thom, E. Gabathuler, D. Jones, B. D. McDaniel, and W. M. Woodward, Phys. Rev. Letters **11**, 433 (1963).