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Phenomenological Analysis of Reactions Such as $K^-+p \rightarrow \Lambda+\omega$†

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The angular correlation function describing the decay products of Λ and ω in the reaction $K^-+p \rightarrow \Lambda+\omega$ is analyzed and expressed in terms of production matrix elements. It is shown that, with unpolarized target, aside from two phase factors all production matrix elements can be determined from angular correlations. With polarized target, the production matrix elements may be completely determined up to an over-all phase factor. In addition, with polarized target the $K-\Lambda$ relative parity may be directly measured.

I. INTRODUCTION

EVENTS such as

$$\begin{array}{cccc} K^-+p\rightarrow & \Lambda & +\omega & \\ & \downarrow & & \searrow \\ & \pi^-+p & & \pi^++\pi^-+\pi^0 \end{array} \tag{1}$$

are observed copiously in bubble chambers.[1] Owing to the fact that the ω is a 1^- particle and to parity violation in Λ decay, an extensive amount of information is available in these pictures. The decays of ω and Λ are almost[2] perfect analyzers of their respective density matrices. Furthermore, the observation of both decays allows for determination of spin correlations. Consequently, aside from two phase factors, the matrix elements for

$$K^-+p\rightarrow\Lambda+\omega \tag{2}$$

for all spin orientations of p, Λ, and ω may be obtained from angular correlation measurements in (1). A polarized target is needed to measure one of these phase factors. (In addition, with polarized protons the K-Λ parity may be directly measured.[3]) Information concerning the remaining phase factor (over-all phase) is obtainable only when interference terms between (2) and, e.g., events $K^-+p\rightarrow\Lambda+\pi^++\pi^-+\pi^0$ with the pions in states other than 1^- (background events) are measured.

In our analysis,[4] we shall assume that these background events may be neglected. Our results show certain consistency requirements which may be violated by such background events, and which provide, therefore, checks on the correctness of this assumption (Tables IV and V).

In Sec. II, we show that the angular correlations in (1) are given by a simple expression involving twelve parameters F_{ij}, and show that from the values F_{ij} one may obtain the matrix elements for (2). Section III indicates the derivation of the results presented in Sec. II and contains further discussion of the matrix elements. In Sec. IV, we discuss (1) with polarized proton target and show how the phase not measurable with unpolarized protons and the $K\Lambda$ parity may be measured. Our discussions also apply, with minor modifications, to other reactions like (2); e.g., $K^-+p\rightarrow\Lambda+\varphi$. This case is discussed in Sec. V.

For simplicity we use noncovariant language, however, no nonrelativistic approximations are made in this paper.

II. THE ANGULAR CORRELATION FUNCTION F

The reaction (1), summed over the Dalitz plot for ω decay,[5] is described by an angular correlation function F defined by

$$d\sigma=R_\Lambda R_\omega(\Lambda/K)\left[F\frac{d\Omega_p}{4\pi}\frac{d\Omega_k}{4\pi}\right]d\Omega\,, \tag{3}$$

where $d\sigma$ is the differential cross section in the c.m. frame of (2) for the production of Λ into solid angle $d\Omega$ when the proton from Λ decay is omitted into solid angle $d\Omega_p$ in the Λ rest frame and the normal $\mathbf{k}$ to the plane of ω decay is in the element of solid angle $d\Omega_k$ in the ω rest frame; F is a Lorentz invariant function. Our notation is

$\mathbf{k}=\mathbf{p}_+\times\mathbf{p}_0$: (λ,μ,ν) are direction cosines of $\mathbf{k}$,

$\mathbf{p}_{\substack{+\\0}}$ = momentum of $\pi^{\substack{+\\0}}$ in ω rest frame,

$\mathbf{p}$ = momentum of decay proton in Λ rest frame: (x,y,z) are direction cosines of $\mathbf{p}$,

† Work supported in part by the National Science Foundation.

[1] S. M. Flatté, R. W. Huff, D. O. Huwe, F. T. Solmitz, and M. L. Stevenson, Bull. Am. Phys. Soc. 8, 603 (1963).

[2] A complete determination of the spin orientation of Λ is afforded by its decay. All but three parameters of the density matrix in spin space describing ω can be measured in the angular distribution of its decay products, $\pi^+\pi^0\pi^-$; the three parameters describing the polarization vector of ω are not measurable in $\omega\rightarrow3\pi$. The remaining five parameters can be determined from the angular distribution of the normal to the plane of decay.

[3] Stephen L. Adler and Alfred S. Goldhaber, Phys. Rev. Letters **10**, 217 (1963); and S. M. Bilenky, Nuovo Cimento **10**, 1049 (1958). These authors report similar effects in other reactions.

[4] Analyses of (1) have also been made by Robert W. Huff, Phys. Rev. **133**, B1078 (1964), and R. J. Oakes and S. M. Berman, Phys. Rev. (to be published).

[5] The differential cross section measured as a function of the energies of π^+ and π^0 may also be used to define F. It has the product form

$$d\sigma=(gdE_+dE_0)R_\Lambda R_\omega(\Lambda/K)\left[F\frac{d\Omega_p}{4\pi}\frac{d\Omega_k}{4\pi}\right]d\Omega,$$

where gdE_+dE_0 represents the Dalitz distribution for ω decay.

$\mathbf{K}$=momentum of K^- in c.m. of (2),
$\mathbf{\Lambda}$=momentum of Λ in c.m. of (2),
$\mathbf{n}=\mathbf{K}\times\mathbf{\Lambda}=K\Lambda\sin\theta\hat{n}$,
R_Λ=branching ratio for $\Lambda\to\pi^-+p$,
R_ω=branching ratio for $\omega\to\pi^++\pi^0+\pi^-$.

For fixed $\mathbf{K}$ and $\mathbf{\Lambda}$ and unpolarized proton target, F depends only on the direction of cosines of $\mathbf{k}$ and $\mathbf{p}$. It is a homogeneous quadratic function of the direction cosines of $\mathbf{k}$ (λ,μ,ν). (This follows from the fact that ω is a 1^- particle and the matrix elements for $\omega\to\pi^++\pi^0+\pi^-$ are, aside from a common factor, given by the three components of $\mathbf{k}$.) Owing to parity violation in Λ decay, F is linear function of the direction cosines of $\mathbf{p}$ (x,y,z). Therefore, F has the form[6]

$$F=F_0+\alpha_\Lambda(xF_1+yF_2+zF_3)\,, \tag{4}$$

where α_Λ is the Λ decay asymmetry parameter[7]; and F_0, F_1, F_2, F_3 are homogeneous quadratic functions of λ, μ, ν:

$$F_i=F_{i1}\lambda^2+F_{i2}\mu^2+F_{i3}\nu^2+F_{i4}\lambda\mu+F_{i5}\lambda\nu+F_{i6}\mu\nu\,. \tag{5}$$

Twelve of these twenty-four F_{ij} vanish when the normal $\mathbf{n}$ to the production plane is a coordinate axis (see Table I) if parity is conserved in (2) and the ω decay. These are represented by zeros in Table II. To see why

TABLE I. The direction cosines of $\mathbf{p}$ and $\mathbf{k}$ defined with respect to the triad $\mathbf{n}$, $\mathbf{K}$, and $\mathbf{n}\times\mathbf{K}$. All vectors are defined in text; $\mathbf{n}$ is normal to the production plane. The form of the matrices displayed in Tables II and IV would be the same with any other choice of axes in the production plane.

	$\hat{n}$	$\hat{K}$	$\hat{n}\times\hat{K}$
$\mathbf{p}$	z	x	y
$\mathbf{k}$	ν	λ	μ

the indicated entries vanish consider, for example, F_{05}; if

$$\langle\lambda\nu\rangle=\int\frac{d\Omega_p}{4\pi}\frac{d\Omega_k}{4\pi}F\lambda\nu \tag{6}$$

from (4) and (5) we have

$$\langle\lambda\nu\rangle=(1/15)F_{05}\,. \tag{7}$$

So F_{05} is proportional to the average of $\mathbf{k}\cdot\mathbf{K}\mathbf{k}\cdot(\mathbf{K}\times\mathbf{\Lambda})$, a pseudoscalar quantity. It must therefore vanish, since parity is violated only in Λ decay. Similarly, if

$$\langle\lambda\mu x\rangle=\int\frac{d\Omega_p}{4\pi}\frac{d\Omega_k}{4\pi}F\lambda\mu x\,, \tag{8}$$

one has from (4) and (5)

$$\langle\lambda\mu x\rangle=(\alpha_\Lambda/45)F_{14}\,. \tag{9}$$

[6] The functions F_1, F_2, F_3 are often denoted by the vector $I\mathbf{P}_\Lambda$ and the expression (4) written as $I+\alpha_\Lambda I\mathbf{P}_\Lambda\cdot\hat{p}$.

[7] James W. Cronin and Oliver E. Overseth, Phys. Rev. **129**, 1795 (1963).

TABLE II. Vanishing and nonvanishing elements of the matrix which yields the angular correlation function F [see Eqs. (4) and (5)] for unpolarized target. As explained in the text, the vanishing of the entries shown here is due to parity conservation. This table applies for both even and odd K-Λ parity. It also applies when contaminations of the type $K^-+p\to\Lambda+\pi^++\pi^-+\pi^0$ are included in the data; in this case additional terms have to be included in (5).

	λ^2	μ^2	ν^2	$\lambda\mu$	$\lambda\nu$	$\mu\nu$
F_0	√	√	√	√	0	0
F_1	0	0	0	0	√	√
F_2	0	0	0	0	√	√
F_3	√	√	√	√	0	0

Only pseudoscalar quantities proportional to α_Λ can be different from zero if parity is conserved in (2) and ω decay. Therefore, F_{14} must vanish. Similarly, one may show that all the zeros in Table II follow from parity conservation. The form of the matrix in Table II (and Table IV) is invariant with respect to the choice of directions in the production plane.

The F_{ij} may all be expressed as averages such as (6) and (8). For $j=4$, 5, 6, the formulas are identical to (7) and (9). For $j=1$, 2, 3, one has formulas of the form

$$F_{01}=(3/2)(4\langle\lambda^2\rangle-\langle\mu^2\rangle-\langle\nu^2\rangle) \tag{10}$$

$$F_{11}=(9/2\alpha_\Lambda)(4\langle\lambda^2x\rangle-\langle\mu^2x\rangle-\langle\nu^2x\rangle)\,. \tag{11}$$

Note that if the F_{ij} are evaluated directly from data containing background events by the use of expressions such as (7), the background events $K^-+p\to\Lambda+\pi^++\pi^0+\pi^-$ will not contribute to the zeros in Table II. The contamination due to background may be visible in a Dalitz plot of the energies of the pions from ω decay. It may also violate certain constraints which the nonvanishing F_{ij} must satisfy owing to the fact that they are the bilinear forms displayed in Table IV.

For fixed energy and angle of production, the twelve nonzero entries in Table II are determined by ten real numbers; the magnitudes of, and relative phases within, the two sets of amplitudes (a_+,b_+,c_+) and (a_-,b_-,c_-) defined in Table III. Owing to parity conservation, the

TABLE III. The six nonvanishing amplitudes for (2) with odd K-Λ parity; e.g., a_+ is the amplitude for producing Λ with spin (in its rest frame) "up" and ω with spin component zero along $\mathbf{n}$ (in its rest frame) from a proton with spin "up" in the laboratory. The spin quantization axis is normal to the production plane; i.e., "up" means parallel to $\mathbf{n}=\mathbf{K}\times\mathbf{\Lambda}$. These amplitudes depend upon the energy and production angle of (2). If K-Λ parity is even, we define these amplitudes as above with *all* arrows in the *last* column reversed.

	Λ spin	ω spin	p spin
a_+	↑	$\hat{n}$	↑
b_+	↓	$\hat{K}$	↑
c_+	↓	$\hat{n}\times\hat{K}$	↑
a_-	↓	$\hat{n}$	↓
b_-	↑	$\hat{K}$	↓
c_-	↑	$\hat{n}\times\hat{K}$	↓

TABLE IV. The nonzero entries of Table II evaluated in terms of the amplitudes defined in Table III. Notice that Tables III and IV are applicable for both signs of K-Λ parity. [The common factor $\frac{3}{2}$ arises from definition (3); a factor $\frac{1}{2}$ comes from averaging initial spins, the factor of 3 arises because in describing ω decay R_ω includes integration over $d\Omega_k$.]

	λ^2	μ^2	ν^2	$\lambda\mu$	$\lambda\nu$	$\mu\nu$
$(2/3)F_0$	$\|b_+\|^2+\|b_-\|^2$	$\|c_+\|^2+\|c_-\|^2$	$\|a_+\|^2+\|a_-\|^2$	$2\,\mathrm{Re}(b_+c_+^*+b_-c_-^*)$	0	0
$(2/3)F_1$	0	0	0	0	$+2\,\mathrm{Re}(a_+b_+^*+a_-^*b_-)$	$+2\,\mathrm{Re}(a_+c_+^*+a_-^*c_-)$
$(2/3)F_2$	0	0	0	0	$-2\,\mathrm{Im}(a_+b_+^*+a_-^*b_-)$	$-2\,\mathrm{Im}(a_+c_+^*+a_-^*c_-)$
$(2/3)F_3$	$-\|b_+\|^2+\|b_-\|^2$	$-\|c_+\|^2+\|c_-\|^2$	$\|a_+\|^2-\|a_-\|^2$	$-2\,\mathrm{Re}(b_+c_+^*-b_-c_-^*)$	0	0

remaining six amplitudes vanish.[8] The nonzero entries of Table II are evaluated in Table IV. (A discussion of the amplitudes and the evaulation is given in Sec. III.) Table IV shows the constraints which measured values of F_{ij} must satisfy when background contamination is negligible. In this case, the magnitudes and relative phases of the sets (a_+,b_+,c_+) and (a_-,b_-,c_-) may be uniquely determined from the values of the F_{ij} measured at fixed energy and production angle: the F_{0j} and F_{3j} for $j=1$, 2, 3 yield the six magnitudes; the F_{1j} and F_{2j} for $j=5$, 6 then give solutions for the relative phases. The complex numbers $a_+b_+^*$ and $a_-^*b_-$ have known magnitudes. Their sum is given by F_{15} and F_{25}. Thus there are two solutions for $a_+b_+^*$ and $a_-^*b_-$. Similarly there are two solutions for $a_+c_+^*$ and $a_-^*c_-$. Only one of these sets of solutions will yield the observed value of F_{04}: then the observed value of F_{34} serves as an additional check.

If there is not enough data for obtaining the F_{ij} by the averaging procedure (7), an alternative method is to use Table II (or Table IV) and search for the set of F_{ij} (or amplitudes) with maximum likelihood.

Table II also applies to data averaged over production angle and/or energy. However, the entries in Table IV must be correspondingly averaged and consequently there are twelve independent quantities obtainable from the data.[9] The constraints on the F_{ij} go over into inequalities in such a process.

Some information regarding the dependence of the F_{ij} on energy and production angle can be deduced from general considerations. For example, for $\theta=0$ or π;

$$\text{all } F_{ij}=0 \quad \textit{except} \quad F_{01},\quad F_{02}=F_{03},\quad F_{25}=-F_{34}. \tag{12}$$

These relations follow from the axial symmetry of the events which requires

$$a_+=-a_-,\quad b_+=b_-,\quad c_+=-c_-,\quad a_-=ic_+. \tag{13}$$

Equations (13) also follow from (25) below which show how the amplitudes behave as $\theta\to 0$ and π, and also near threshold.

[8] Aage Bohr, Nucl. Phys. **10**, 486 (1959); A. A. Cheshkov and Yu. M. Shirokov, Zh. Eksperim. i Teor. Fiz. **42**, 144 (1962) [English transl.: Soviet Phys.—JETP **15**, 103 (1962)].

[9] After such averaging, the density matrix describing the Λ-ω system has eighteen independent parameters. Experiments such as the one described here can measure only twelve of them.

III. DERIVATION OF F AND DISCUSSION OF AMPLITUDES

The amplitudes defined in Table III are convenient for describing reactions such as (2) with unpolarized target. They correspond to S-matrix elements for which the spin quantization axis (z axis) is normal to the production plane. These S-matrix elements have been discussed by Bohr[8] and, for the relativistic case, by Cheshkov and Shirokov.[8] Let $T_{bm;a}$ be the amplitude for producing Λ and ω with z component of spins b and m, respectively, from protons with z component of spin a. Then for each value of $\mathbf{K}$ and $\mathbf{\Lambda}$, owing to parity conservation, one has

$$T_{bm;a}=\eta_g(-1)^{b+m-a}T_{bm;a}, \tag{14}$$

where

$$\eta_g=(\eta_\Lambda\eta_\omega/\eta_p\eta_K)(-1)^{S_\Lambda+S_\omega-S_p-S_K}. \tag{15}$$

The η are the parity factors[10] giving the intrinsic parities of the particles and the s are their spins. If the K-Λ parity is odd the six nonvanishing amplitudes satisfying (14) are linear combinations of the amplitudes defined in Table III; for example,

$$\begin{aligned} T_{\uparrow 0\uparrow}&=a_+\\ T_{\downarrow 1\uparrow}&=-(b_++ic_+)/\sqrt{2}. \end{aligned} \tag{16}$$

If the K-Λ parity is even, the amplitudes in Table III are redefined by reversing the proton spin in every case in accordance with (14) so that the nonvanishing amplitudes are $T_{\uparrow 0\downarrow}=a_+$, $T_{\downarrow 1\downarrow}=-(b_++ic_+)/\sqrt{2}$, etc.

To evaluate (4) in terms of the amplitudes in Table III, let ρ_Λ be the density matrix in spin space for Λ. For convenience, we take Trace $\rho_\Lambda=(2/3)F_0$ (see caption to Table IV). Then

$$\rho_\Lambda=(1/3)(F_0+\boldsymbol{\sigma}\cdot\mathbf{F}), \tag{17}$$

where $\boldsymbol{\sigma}$ are Pauli spin matrices. The Lorentz-invariant matrix element for $\omega\to\pi^++\pi^0+\pi^-$ is proportional to $\mathbf{k}\cdot\mathbf{n}$ if, in its rest frame, the ω spin component along $\mathbf{n}$ is zero (here $\mathbf{n}$ is any arbitrary direction). Consequently, the amplitude corresponding to a_+ for $K^-+p\to\Lambda$

[10] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959). The $T_{bm;a}$ are linearly related to the helicity amplitudes of Jacob and Wick (see, e.g., Cheshkov and Shirokov, Ref. 8); (14) follows from Eq. (44) of Jacob and Wick.

TABLE V. The terms in F which arise when target is polarized with polarization $\mathbf{P}$ normal to incident beam direction. (See Sec. V for definition of ϕ.) $F^0{}_{ij}$ are F_{ij} values for unpolarized target; they are given in Table IV. These terms must be added to those in Table IV if the K-Λ parity is odd and subtracted if the K-Λ parity is even.

	λ^2	μ^2	ν^2	$\lambda\mu$	$\lambda\nu$	$\mu\nu$
F_0	$+F^0{}_{31}P\sin\phi$	$+F^0{}_{32}P\sin\phi$	$-F^0{}_{33}P\sin\phi$	$+F^0{}_{34}P\sin\phi$	$3\,\mathrm{Im}(a_+b_-^*+b_+a_-^*)$ $\times P\cos\phi$	$3\,\mathrm{Im}(a_+c_-^*+c_+a_-^*)$ $\times P\cos\phi$
F_1	$3\,\mathrm{Im}b_+b_-^*$ $\times P\cos\phi$	$3\,\mathrm{Im}c_+c_-^*$ $\times P\cos\phi$	$3\,\mathrm{Im}a_+a_-^*$ $\times P\cos\phi$	$3\,\mathrm{Im}(b_+c_-^*+c_+b_-^*)$ $\times P\cos\phi$	$-3\,\mathrm{Re}(b_+a_+^*-a_-b_-^*)$ $\times P\sin\phi$	$-3\,\mathrm{Re}(c_+a_+^*-a_-c_-^*)$ $\times P\sin\phi$
F_2	$-3\,\mathrm{Re}b_+b_-^*$ $\times P\cos\phi$	$-3\,\mathrm{Re}c_+c_-^*$ $\times P\cos\phi$	$3\,\mathrm{Re}a_+a_-^*$ $\times P\cos\phi$	$-3\,\mathrm{Re}(b_+c_-^*+c_+b_-^*)$ $\times P\cos\phi$	$-3\,\mathrm{Im}(b_+a_+^*-a_-b_-^*)$ $\times P\sin\phi$	$-3\,\mathrm{Im}(c_+a_+^*-a_-c_-^*)$ $\times P\sin\phi$
F_3	$F^0{}_{01}P\sin\phi$	$F^0{}_{02}P\sin\phi$	$-F^0{}_{03}P\sin\phi$	$F^0{}_{04}P\sin\phi$	$3\,\mathrm{Im}(a_+b_-^*-b_+a_-^*)$ $\times P\cos\phi$	$3\,\mathrm{Im}(a_+c_-^*-c_+a_-^*)$ $\times P\cos\phi$

$+\omega \rightarrow \Lambda+\pi^++\pi^-+\pi^0$ is $\alpha_+=a_+\nu$, and

$$\rho_\Lambda = \mathcal{S}\rho_i\mathcal{S}^\dagger, \tag{18}$$

where

$$\mathcal{S}=\mathcal{S}_{\mathrm{odd}}=\begin{pmatrix}\alpha_+ & \beta_- \\ \beta_+ & \alpha_-\end{pmatrix} \quad \text{if } K\text{-}\Lambda \text{ parity is odd} \tag{19}$$

$$\mathcal{S}=\mathcal{S}_{\mathrm{even}}=\begin{pmatrix}\beta_- & \alpha_+ \\ \alpha_- & \beta_+\end{pmatrix} \quad \text{if } K\text{-}\Lambda \text{ parity is even} \tag{20}$$

$$\alpha_-=a_-\nu\,, \quad \beta_+=b_+\lambda+c_+\mu\,, \quad \beta_-=b_-\lambda+c_-\mu \tag{21}$$

and $\rho_i=1+\boldsymbol{\sigma}\cdot\mathbf{P}$, where $\mathbf{P}$ is the target polarization referred to the axes of Table I ($P_3=\mathbf{P}\cdot\hat{n}$, $P_1=\mathbf{P}\cdot\hat{K}$, etc.); (17) and (18) give for unpolarized target

$$F_0=(3/2)\,\mathrm{Trace}\,\mathcal{S}^\dagger\mathcal{S}\,,\quad F_1=(3/2)\,\mathrm{Trace}\,\sigma_1\mathcal{S}^\dagger\mathcal{S}\,,\ \text{etc.} \tag{22}$$

Note that

$$\mathcal{S}_{\mathrm{even}}=\mathcal{S}_{\mathrm{odd}}\sigma_1=\mathcal{S}_{\mathrm{odd}}\boldsymbol{\sigma}\cdot\hat{K}\,. \tag{23}$$

Consequently, (19) and (20) yield identical results for unpolarized target.

Expectations regarding the angular dependence and magnitudes near threshold of the amplitudes in Table III may be obtained by considering the form of the S matrix in spin space of (2). Evaluating Dirac spinors for Λ and p in the c.m. of (2), one may express the most general result consistent with odd K-Λ parity and the assignment 1^- for ω as an axial vector whose components are 2×2 matrices. The matrix elements are simply related to the amplitudes of Table III. The matrix must have the form

$$\alpha\boldsymbol{\sigma}+\beta\mathbf{n}+\mathbf{K}(\gamma\boldsymbol{\sigma}\cdot\mathbf{K}+\epsilon\boldsymbol{\sigma}\cdot\mathbf{y})+\mathbf{y}(\rho\boldsymbol{\sigma}\cdot\mathbf{K}+\tau\boldsymbol{\sigma}\cdot\mathbf{y})\,, \tag{24}$$

where $\boldsymbol{\sigma}$ are Pauli spin matrices, $\mathbf{y}=\mathbf{n}\times\mathbf{K}$, and α, β, γ, etc. are functions of energy and production angle. Setting $|\mathbf{n}|=\Lambda K\sin\theta$, and evaluating the amplitudes of Table III from (24), one finds

$$\begin{aligned}
a_+&=\alpha+\beta K\Lambda\sin\theta\,,\\
a_-&=-\alpha+\beta K\Lambda\sin\theta\,,\\
b_+&=\alpha+\gamma K^2+i\epsilon K^3\Lambda\sin\theta\,,\\
b_-&=\alpha+\gamma K^2-i\epsilon K^3\Lambda\sin\theta\,,\\
c_+&=i\alpha+\rho K^3\Lambda\sin\theta+i\tau K^4\Lambda^2\sin^2\theta\,,\\
c_-&=-i\alpha+\rho K^3\Lambda\sin\theta-i\tau K^4\Lambda^2\sin^2\theta\,.
\end{aligned} \tag{25}$$

Equations (25) display the behavior of these amplitudes near threshold ($\Lambda=0$) and near $\theta=0$ or π. The functions α, β, γ, etc., may be expected to have a regular behavior in these regions.

IV. POLARIZED TARGET

If the proton target is polarized normal to the incident beam direction, (3) still applies provided one understands that in this case F depends also on the azimuthal angle of $\mathbf{\Lambda}$. Let ϕ be the azimuthal angle of $\mathbf{\Lambda}$ measured in the c.m. of (2) from $\mathbf{P}$ with polar axis chosen to be the incident beam direction and $\mathbf{P}$ the target polarization. Then F may be calculated from (17)–(21) with $\mathbf{P}$ expressed in the coordinate frame of Table I; i.e., $P_1=0$, $P_2=P\cos\phi$, $P_3=-P\sin\phi$. The ϕ-independent terms are displayed in Table V. From (23) one sees that, except for over-all sign, identical results are obtained for these terms with (19) and with (20). The terms in Table V must be added to those in Table IV if the K-Λ parity is odd and subtracted if the K-Λ parity is even.

To measure the K-Λ parity, one need only measure, e.g., the right-left asymmetry of (2). Defining N_R to be the number of Λ's produced to the right when looking along the beam with $\mathbf{P}$ up, and N_L those produced to the left; from Table V, one has

$$\begin{aligned}
(N_R-N_L)/(N_R+N_L)&=-\eta P(2/\pi)\\
&\quad\times(F_{33}{}^0-F_{31}{}^0-F_{32}{}^0)/(F_{03}{}^0+F_{01}{}^0+F_{02}{}^0)\,,
\end{aligned} \tag{26}$$

where $\eta=+1$ if K-Λ parity is odd, and $\eta=-1$ if K-Λ parity is even. The $F^0{}_{ij}$ are values measured with unpolarized target. A similar expression can be derived for the right-left asymmetry of the Λ polarization normal to the production plane F_3.

Terms proportional to $\cos\phi$ appear in Table V in places corresponding to the zeros of Table II. This is because $P\cos\phi=\mathbf{P}\cdot[\hat{\Lambda}-\hat{K}(\hat{K}\cdot\hat{\Lambda})]/\sin\theta$ is a pseudoscalar quantity. These terms may be used to determine, for example, the relative phase of a_+ and a_-.

V. $K^-+p\to\Lambda+\varphi$

The process

$$\begin{array}{lll} K^-+p\to & \Lambda & +\varphi \\ & \downarrow & \qquad\searrow \\ & \pi^-+p & \qquad K+\bar{K} \end{array} \tag{27}$$

is also described by an angular correlation function F defined as in (3). In this case, $\mathbf{k}$ is the momentum of K in the φ rest frame. Because φ is a 1^- particle, the matrix element for $\varphi\to K+\bar{K}$ is also proportional to $\mathbf{k}\cdot\hat{n}$ when the φ spin component along $\hat{n}$ is zero. Therefore, reinterpreting $\mathbf{k}$ accordingly, one may apply all the discussions and results concerning F of the previous sections to (27).

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Proof of a Conjecture of S. Weinberg*

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A proof is given of Weinberg's conjecture[1] that the irreducible N-particle kernel is of Hilbert-Schmidt type if the potentials, which describe the pair interactions, are square integrable.

INTRODUCTION

THIS note merely serves as an appendix to a recent paper by S. Weinberg on multiparticle scattering problems.[1] In the framework of nonrelativistic quantum mechanics, Weinberg has developed an integral equation for the full Green's function of a system of N pairwise-interacting particles which is free of the inadequacies of Lippman-Schwinger type equations. This equation is of the form

$$G(W)=D(W)+I(W)G(W)\,, \tag{1}$$

where W is the complex energy parameter, $D(W)$ the disconnected part of the Green's function $G(W)$, and $I(W)$ an integral operator whose kernel is called the irreducible N-particle kernel. Both $D(W)$ and $I(W)$ are known explicitly in terms of the Green's functions of all subsystems with a smaller number of particles. Weinberg has conjectured that $I(W)$ is of Hilbert-Schmidt type (HS type), if all the pair interactions are described by square-integrable potentials, and he has given a proof for the cases $N=2$, 3. Since his conclusions are based essentially on this conjecture, a proof for arbitrary N is certainly desirable. To furnish such a proof is the only objective of this note.

A NEW REPRESENTATION OF $I(W)$

If not defined otherwise, our terminology is that of Weinberg. We consider an arbitrary decomposition D of the system of particles $(1\cdots N)$ into clusters. The Hamiltonian H_D of this decomposed system is then the full Hamiltonian minus all interactions between different clusters. Weinberg has given an explicit expression for $I(W)$ in terms of all Green's functions $G_D=(W-H_D)^{-1}$. However, in this form $I(W)$ appears as a sum of operators which are not even completely continuous, and the connectedness is not evident. In order to prove that $I(W)$ is of HS type, we have to put it in a different form.

We consider all possible sequences $S=(D_1,D_2,\cdots,D_N)$ of cluster decompositions with the following properties: D_1 is the trivial decomposition into one cluster $(1\cdots N)$, and D_k is obtained from D_{k-1} by splitting one of the clusters of D_{k-1} into two parts. Therefore, each sequence S has N terms and ends with the finest possible decomposition $D_N=(1)(2)\cdots(N)$. By aid of these sequences we can write the connected part $C(W)$ of $G(W)$ in the form

$$C(W)=\sum_{\text{all }S}G_{D_N}V_{D_ND_{N-1}}G_{D_{N-1}}\cdots G_{D_2}V_{D_2D_1}G_{D_1}$$
$$=I(W)G(W)\,, \tag{2}$$

where $V_{D_kD_{k-1}}$ is the sum of all interactions which are dropped in the transition from $H_{D_{k-1}}$ to H_{D_k}: $V_{D_kD_{k-1}}=H_{D_{k-1}}-H_{D_k}$. Obviously $G_{D_1}=G$, so that one gets from (2) an expression for $I(W)$ simply by omitting the last factors G_{D_1} in the sum. In this form the connectedness of $C(W)$ and $I(W)$ is evident. We do not want to prove here the equivalence of (2) with Weinberg's expression—the reader may easily check it for $N=2$, 3, 4.

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[1] S. Weinberg, Phys. Rev. **133**, B232 (1964).