SU(3) and Weak Interactions

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Assuming the existence of two more neutrinos ν_e' and ν_μ', it is suggested that the leptons fit in SU(3) triplets and that the lepton-lepton and lepton-baryon interactions are SU(3) symmetric. Possible existence of a SU(2) symmetry for these interactions is also considered. For either symmetry four neutrinos seem essential and we point out how experiment can test the existence of the two new neutrinos.

I. INTRODUCTION

AT present it is thought that the four Fermion weak interactions are of the (current)$\times$(current) type. In this picture a number of rules, deduced from experiment, are incorporated phenomenologically. These rules are:

(i) No neutral lepton currents.
(ii) No $\Delta Y=2$ decays leptonic or nonleptonic, where Y is the hypercharge.
(iii) Only $\Delta Q=\Delta Y$ leptonic decays of strange particles, with $\Delta Q=-\Delta Y$ decays absent.
(iv) $\Delta T=\frac{1}{2}$ rule for nonleptonic decays with $\Delta Y=1$.
(v) The $\Delta Y=0$ vector current for the baryons is conserved, i.e., the CVC hypothesis.
(vi) There is also the possibility of a $\Delta T=\frac{1}{2}$ rule for leptonic decays with $\Delta Y=1$.
(vii) There are two two-component left-handed neutrinos ν_e and ν_μ which are associated with the electron e and muon μ. Further, electron numbers and muon numbers are conserved.

In this note we concern ourselves with only the leptonic decays. We give a rationale, different from a (current)$\times$(current) picture, for writing down the lepton-lepton and lepton-baryon interactions. This rationale is obtained by requiring these interactions to be SU(3) symmetric and this automatically incorporates most of the above rules.

Classification of leptons in SU(3). We assign the leptons to the fundamental representation of SU(3).[1] To be able to do this we assume the existence of two more two-component left-handed neutrinos $\nu_e'(\neq\nu_e)$ and $\nu_\mu'(\neq\nu_\mu)$ in addition to ν_e and ν_μ differing from them in isospin and hypercharge. We can then form the SU(3) triplets [both $D^3(1,0)$]

$$l_e=(e^+,\bar{\nu}_e,\bar{\nu}_e') \quad \text{and} \quad l_\mu=(\mu^+,\bar{\nu}_\mu,\bar{\nu}_\mu')$$

in complete analogy to the Sakata triplet (p,n,Λ). We identify the isospin operator T_3 and hypercharge operator Y with $\sqrt{3}H_1$ and $2H_2$, respectively,[2] as is done in the case of the baryon and meson octets and assume the octet model of SU(3) for the strongly interacting particles of Gell-Mann[3] and Ne'eman.[4] To obtain integral charges for the leptons we define a new additive quantum number u, where $u=0$ for the baryons and mesons. For l_e and l_μ $u=+\frac{1}{3}$ and $u=-\frac{1}{3}$ for the conjugate representation (0,1). Note the number u is a kind of super-strangeness to be defined for the various SU(3) multiplets. The Gell-Mann–Nishijima relation is then

$$Q=u+T_3+Y/2.$$

TABLE I. Quantum numbers of the antileptons.

Antilepton	u	T_3	Y
l^+	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
$\bar{\nu}_l$	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$
$\bar{\nu}_l'$	$\frac{1}{3}$	0	$-\frac{2}{3}$

The values for u, Y, etc., for the leptons can be found in Table I.

In the table l denotes either μ or e. We note that Q, T_3, Y, and u are conserved in all interactions except for nonleptonic decays where Y and T_3 are not.

Lepton mass. Before discussing the weak interactions we point out how one could obtain a mass for the electron and the muon, which have been assumed to be massless in forming the triplets. This can be done by assuming that though electron e is massless in the triplet and is two-component, it is a superposition of both left- and right-handed parts, i.e., $e=e_L+e_R$. Thus, when we break the symmetry by the charge operator Q, the mass term for the electron will be $m_eTr(\bar{l}_eQl_e)=m_e(\bar{e}_Le_R+\bar{e}_Re_L)\neq0$. Exactly the same argument will apply for a nonzero muon mass term.

The consequences of SU(3), symmetric lepton-lepton and lepton-baryon interactions, for leptonic decays, are discussed below in Sec. II. This requirement as we show incorporates most of the extra assumptions (pointed out above) made in writing weak interactions. Further, it provides a clear and natural framework for the various models for the leptonic decays of baryons,[5,6] based on the octet model for baryons suggested recently.

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[1] Recently, an attempt has been made to fit the leptons into a SU(3) octet by R. E. Marshak, C. Ryan, T. K. Radha, and K. Raman, Phys. Rev. Letters **11**, 396 (1963).
[2] We follow the notation of R. E. Behrends, J. Dreitlein, C. Fronsdal, and W. Lee, Rev. Mod. Phys. **34**, 1 (1962).
[3] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[4] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[5] J. M. Cornwall and V. Singh, Phys. Rev. Letters **10**, 551 (1963).
[6] N. Cabbibo, Phys. Rev. Letters **10**, 531 (1963).

In Sec. III we point out the difficulties in our simple scheme and their possible resolution. In Sec. IV we consider the possibility of only a SU(2) symmetry for weak interactions of the leptons. Finally, it should be noted that the assumption of two more neutrinos $\nu_e'(\neq\nu_e)$ and $\nu_\mu'(\neq\nu_\mu)$ is essential to our considerations[7] and a reasonable success of our model. Consequently, we point out how one could test the existence of these two neutrinos experimentally.

II. LEPTON-LEPTON COUPLINGS

The simplest coupling which is SU(3) invariant is obtained by contracting the octet L_e got from $(\bar{l}_e l_e)$ with the octet L_μ from $(\bar{l}_\mu l_\mu)$. The interaction [with the usual space-time part $(1+\gamma_5)\gamma_\mu$ understood] is

$$(1/\sqrt{2})G_L(\bar{l}_e l_e)_8(\bar{l}_\mu l_\mu)_8 = G_L\,\mathrm{Tr}(L_e L_\mu)\,, \tag{1}$$

where, for example, $(\bar{l}_e l_e)_8 = L_e$ in 3×3 matrix form is

$$L_e=\begin{bmatrix} \frac{1}{3}(2\bar{e}^-e^- - \bar{\nu}_e\nu_e - \bar{\nu}_e'\nu_e') & (\bar{e}^-\nu_e) & (\bar{e}^-\nu_e') \\ (\bar{\nu}_e e^-) & -\frac{1}{3}(\bar{e}^-e^- - 2\bar{\nu}_e\nu_e + \bar{\nu}_e'\nu_e') & (\bar{\nu}_e\nu_e') \\ (\bar{\nu}_e' e^-) & (\bar{\nu}_e'\nu_e) & \frac{1}{3}(-\bar{e}^-e^- - \bar{\nu}_e\nu_e + 2\bar{\nu}_e'\nu_e') \end{bmatrix}. \tag{2}$$

Note the $(\bar{l}_e l_e)$ singlet coupling to $(\bar{l}_\mu l_\mu)$ singlet leads only to weak scattering of the leptons and is uninteresting. Apart from weak scattering the interaction (1) leads to the decays (which are experimentally indistinguishable)

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu\,, \tag{3a}$$

$$\mu^- \to e^- + \bar{\nu}_e' + \nu_\mu'\,. \tag{3b}$$

Since these are incoherent the Michel parameter will not be affected and from the muon rate we obtain

$$\sqrt{2}G_L = G = (1.023\pm0.002)\times10^{-5}m_p^{-2}\,. \tag{4}$$

Couplings of the type $(\bar{l}_e l_e)(\bar{l}_\mu l_e)$ immediately lead to two more muon-decays and as well as the unwanted $\mu\to3e$ mode. To forbid such interactions we invoke the conservation of muon and electron numbers.[8]

A. Lepton-Baryon Couplings

The lepton singlets and octets from $(\bar{l}_e l_e)$ or $(\bar{l}_\mu l_\mu)$ can couple only to the 1, $8(F)$ or $8(D)$ from $(\bar{B}B)$ to give SU(3) symmetric couplings. This automatically guarantees that the leptonic decays with $\Delta Q=\Delta S$ will only occur. The absence or smallness of $\Delta Q=-\Delta S$ decays is understood as arising from SU(3) violating lepton-baryon interactions. This also automatically guarantees that there are no $|\Delta S|=2$ leptonic decays. Denote the symmetric vector octet $(\bar{B}\gamma_\mu B)_{8(D)}$ by $\phi_D{}^V$, the antisymmetric axial pseudovector octet $(\bar{B}\gamma_\mu\gamma_5)_{8(F)}$ by $\phi_F{}^A$, etc. The elements of the 3×3 matrices $\phi_F{}^A$, $\phi_F{}^V$, etc., have been given by Gell-Mann.[9]

Assuming muon and electron number conservation, the most general coupling between the octets is

$$(G_1/\sqrt{2})[V_F\,\mathrm{Tr}(L_e\phi_F{}^V)+V_D\,\mathrm{Tr}(L_e\phi_D{}^V) - A_F\,\mathrm{Tr}(L_e\phi_F{}^A)-A_D\,\mathrm{Tr}(L_e\phi_D{}^A)] \tag{5}$$

with a similar term with L_e replaced by L_μ and different coupling constants in general. However, we will assume the interaction of L_e and L_μ to $(\bar{B}B)$ to be of identical form and strength. The coupling constants V_F, V_D, etc., are to be determined from experiment.

The beta-decay coupling from (5) comes out to be

$$(G_1/\sqrt{2})(\bar{e}^-[(1+\gamma_5)\gamma_\mu]\nu) \times(\bar{p}[\sqrt{2}(V_F+V_D)\gamma_\mu - \sqrt{2}(A_F+A_D)\gamma_5\gamma_\mu]n)\,. \tag{6}$$

From experiment,

$$V_F+V_D=1\,,\quad (A_F+A_D)=1.25 \tag{6a}$$

and $\sqrt{2}G_1=G$, i.e., $G_L=G_1$.

B. Test of the Existence of ν_l'

In the interaction (5) the lepton pair $(\bar{e}^-\nu_e)$ transforms[3,9] as (F_1-iF_2) and is coupled to the $\Delta Y=0$, $\Delta Q=1$ baryon current, while $(\bar{e}^-\nu_e')$ transforms as (F_4-iF_5) and is coupled to the $|\Delta Y|=1$, $\Delta Q=1$ baryon currents. Thus, ν_e or ν_μ will appear in $\Delta Y=0$ decays while ν_e' or ν_μ' will appear in $|\Delta Y|=1$ decays, so that we have

$$\pi^+\to l^+ + \nu_l\,, \tag{7}$$

$$K^+\to l^+ + \nu_l'\,. \tag{8}$$

The different couplings of ν_l and ν_l' provide a feasible experimental test of our four-neutrino hypothesis in the neutrino-beam experiments. For, in our scheme,

$$\nu_l + N \to N + l\,, \tag{9}$$

$$\nu_l' + N \to Y + l\,, \tag{10}$$

are allowed, where $N=$nucleon and $Y=\Sigma$ or Λ. But, we forbid the reactions (by hypercharge conservation)

$$\nu_l + N \nrightarrow Y + l\,, \tag{11}$$

$$\nu_l' + N \nrightarrow N + l\,. \tag{12}$$

[7] The possibility that $\nu_e'=\bar{\nu}_\mu$ and $\nu_\mu'=\bar{\nu}_e$ was suggested to us by the authors of Ref. 5. This model has the objection that the same particle, e.g., ν_e, has different transformation properties under SU(3) depending on whether it is in l_e or l_μ. Further, in addition to the consequences of our scheme it leads to neutrino flip which is in disagreement with experiment.

[8] Instead of invoking separate muon and electron number conservation, one can use only lepton number conservation to forbid such interactions if one defines e^-, μ^+ to be leptons together with ν_e and ν_e' (both left-handed) and ν_μ and ν_μ' (both right-handed) to be leptons.

[9] M. Gell-Mann, California Institute of Technology, Report CTSL-20, 1961 (unpublished).

Since the ν_μ form pion decay cannot lead to strange-particle production in a high-energy neutrino experiment, one could distinguish it from ν_μ' from K decay. In fact by varying the contamination of neutrinos for K decay in the neutrino beam from pions and studying the resulting strange-particle production, one could infer the existence of ν_μ', i.e., show $\nu_\mu \neq \nu_\mu'$.

C. Neutral Lepton Currents

The lepton-baryon coupling (5) also introduces neutral lepton currents of two kinds: (a) $\Delta Y=0$ currents which transform like F_3 and F_3; (b) $\Delta Y=1$ currents which transform like $(F_6 \pm iF_7)$.

(a) The $\Delta Y=0$ terms will lead to the decays (in lowest order)

$$\pi^0 \to e^+ + e^- \tag{13a}$$

$$\Sigma^0 \to \Lambda + e^+ + e^- \tag{13b}$$

$$\Lambda + \bar{\nu}_l + \nu_l \tag{13c}$$

$$\Lambda + \bar{\nu}_l' + \nu_l' \tag{13d}$$

The first two can take place electromagnetically, while the last two arise in our scheme. However, all these weak decays will be masked by the purely electromagnetic decays.[10] It may be pointed out that the decays (13a) and (13b) through a weak interaction will violate parity, while if purely electromagnetic they will preserve parity. In principle, therefore, the presence of (e^+e^-), etc., terms in the weak Lagrangian could be decided upon by an analysis of e^+, e^- angular distributions. However, such a test seems unfeasible at present.

(b) The $\Delta Y=\pm 1$ neutral lepton term is simply $(\bar{\nu}_l \nu_l')$ and leads to two neutrino, i.e., muon-like decays of strange particles. Of these, most of them turn out to be decays of a neutral particle into three other neutral particles. Experiment can best hope to measure the decays

$$\Sigma^+ \to p + \nu_l + \bar{\nu}_l', \tag{14}$$

$$K^+ \to \pi^+ + \bar{\nu}_l + \nu_l'. \tag{15}$$

[Note that $K^0 \to \nu_l + \bar{\nu}_l'$ or $\bar{K}^0 \to \bar{\nu}_l + \nu_l'$ are forbidden by angular momentum conservation, though formally allowed by the interaction (5).] Notice, however, that other neutral lepton couplings leading to say $\Sigma^+ \to p + e^+ + e^-$, etc., just do not arise in this scheme. Thus, their absence can be understood in a less *ad hoc* manner.

The rates for the muon-like decays of strange particles can be obtained from the usual decays into $(e\nu_e)$ by charge symmetry and putting the electron mass $m_e=0$. Thus, the rate for (14) should be equal to that for $\Sigma^- \to n + e^- + \bar{\nu}_e'$ and for (15) should be equal to that for $K^0 \to \pi^- + e^+ + \nu_e'$, with $m_e=0$. The present experimental evidence on (14) and (15) gives the branching ratios[11]

$$\frac{\Gamma(\Sigma^+ \to p + \nu + \bar{\nu})}{\Gamma(\Sigma^- \to n + e^- + \bar{\nu}_e)} \leqslant 1$$

and

$$\frac{\Gamma(K^+ \to \pi^+ + \nu + \bar{\nu})}{\Gamma(K^+ \to \pi^0 + e^+ + \nu_e)} = 10^{-1}. \tag{16}$$

These ratios should be a unity in our model and may prove to be a serious discrepancy with experiment. We suggest a way of resolving this difficulty in a later section.

D. Leptonic Decays of Baryons

Assuming the coupling (5) we determine the four unknowns V_F, V_D, A_F, and A_D and see whether one can fit the experimental data. The input data we use are:

$$V_F + V_D = 1, \tag{17}$$

$$A_F + A_D = 1.25, \tag{18}$$

from neutron beta-decay, the coupling constant G_1 being fixed from the rate. For $\Lambda \to p + e^- + \bar{\nu}_e'$ decay the effective V to A ratio is unity,[12] so that

$$\left|\frac{\sqrt{3}V_F + 1/\sqrt{3}V_D}{\sqrt{3}A_F + 1/\sqrt{3}A_D}\right| = 1, \tag{19}$$

since the relative sign of V to A is not known. The artio of the experimental rate[13] $\Gamma_{\text{Exp}}{}^\Lambda = 3\times 10^6$ sec^{-1} to the rate $\Gamma_{\text{UFI}}{}^\Lambda$ calculated from the universal Fermi interaction (UFI) hypothesis is given by

$$R = \frac{\Gamma_{\text{UFI}}{}^\Lambda}{\Gamma_{\text{Exp}}{}^\Lambda} = \frac{4}{\frac{1}{2}\{[\sqrt{3}V_F + (1/\sqrt{3})V_D]^2 + 3[\sqrt{3}A_F + (1/\sqrt{3})A_D]^2\}} \tag{20}$$

Equations (17)–(20) give four solutions

$$2V_F + 1 = \pm (6/R)^{1/2}, \quad 2V_F + 1 = \pm (2A_F + 1.25). \tag{21}$$

The $(++)$ solution seems the best and for this with $R=19.3$ we obtain $V_F=-0.22$, $V_D=1.22$, $A_F=-0.34$, and $A_D=1.59$. The type of interaction and the rates are given in Table II. In Table II the Γ_{UFI} have been taken

[10] M. A. B. Bég, Phys. Rev. **132**, 426 (1963).

[11] I am grateful to Dr. U. Nauenberg for the upper limit on $\Gamma(\Sigma^+ \to p + \nu + \bar{\nu})$ and to Dr. Byron Roe for the upper limit on $\Gamma(K^+ \to \pi^+ + \nu + \bar{\nu})$.

[12] C. Baglin, H. H. Bingham, R. T. Elliott, A. Haatuft, and C. Henderson *et al.*, Phys. Letters **6**, 186 (1963). These authors give the V/A ratio to be between 0 and 1. However, our conclusion that $\Sigma^- \to n + e^- + \bar{\nu}_e'$ is too large if $\Lambda \to p + e^- + \bar{\nu}_e'$ can be fitted is still valid for this range because V_F and V_D (and A_F and A_D) come out with opposite signs.

[13] W. Willis, R. Adair, H. Courant, H. Filthuth, P. Franzini *et al.*, Bull. Am. Phys. Soc. **8**, 349 (1963).

TABLE II. The type of interaction given in the table is for our best solution. Note other solutions could give a $V+A$ interaction for $\Lambda\to p+e^-+\bar\nu_e'$. The $\Gamma_{SU(3)}$ are the rates calculated on our theory. The Γ_{UFI} are taken from Okun and the experimental rates Γ_{Exp} are taken from Ref. 15 while the values marked (a) are taken from Willis *et al.* (Ref. 13).

Decay	Type of interaction	Γ_{UFI} (sec^{-1})	$\Gamma_{SU(3)}$ (sec^{-1})	Γ_{Exp} (sec^{-1})
$\Lambda\to p+e^-+\bar\nu_e'$	$0.229(V-A)$	5.8×10^7	3×10^6	3×10^6
$\Sigma^-\to\Lambda+e^-+\bar\nu_e$	$V-1.3A$	1.1×10^6	1.65×10^6	6.2×10^5(a)
$\Sigma^+\to\Lambda+e^++\nu_e$	$V-1.3A$	0.7×10^6	0.85×10^6	1.65×10^6(a)
$\Sigma^-\to n+e^-+\bar\nu_e'$	$(1.44V-1.93A)$	3.4×10^8	1.1×10^9	1.25×10^7
$\Xi^-\to\Lambda+e^-+\bar\nu_e'$	$(0.77V-1.06A)$	1.2×10^8	1.2×10^8	4.7×10^8
$\Xi^-\to\Sigma^0+e^-+\bar\nu_e'$	$(1/\sqrt2)(V-1.25A)$	1.4×10^7	10^7	?
$\Xi^0\to\Sigma^++e^-+\bar\nu_e'$	$(1/\sqrt2)(V-1.25A)$	1.4×10^7	10^7	?

from Okun[14] and the experimental rates have been taken from Barkas and Rosenfeld.[15] The calculation shows that though it is possible to fit all known rates within a factor of 2 or 3 except the decay $\Sigma^-\to n+e^-+\bar\nu_e'$ which comes out to be too large by a factor of 90! In fact, in a calculation of this type,[5,6] based on SU(3), once the over-all coupling constant G_1 is fixed from neutron decay it is not possible to fit both $\Sigma^-\to n+e^-+\bar\nu_e'$ and $\Lambda\to p+e^-+\bar\nu_e'$ simultaneously. Thus, the only way out of the dilemma is to have the over-all coupling constant for $\Delta Y=0$ and $\Delta Y=1$ currents different as was done by Cabbibo.[6] In the Cabbibo model $V_D=0$ so that the $\Delta Y=0$ vector current is conserved as required by CVC. Our calculation and that of Cornwall and Singh[5] suffer from the drawback that $V_D\neq0$ and consequently the $\Delta Y=0$ vector current is not a conserved current. In the next section we show how we can obtain the Cabbibo[6] model in the framework of a broken SU(3) symmetry.

Finally, we remark that the leptonic decay of the Ω^- in the decuplet, viz., $\Omega^-\to\Xi^0+l^-+\bar\nu_l'$, which is $\Delta Q=\Delta Y$ and $\Delta T=\frac12$, can be obtained by coupling the octet from 10×8 to the lepton octet L from $(\bar ll)$.

III. DIFFICULTIES AND THEIR RESOLUTION

In the simple SU(3) symmetric scheme two main difficulties arise: (i) muon-like decays of strange particles come out too large and (ii) with the coupling (5) the two decays $\Lambda\to p+e^-+\bar\nu_e'$ and $\Sigma^-\to n+e^-+\bar\nu_e'$ cannot together be made to agree with experiment.

To resolve (i) we could break the symmetry in a simple manner with the charge operator Q and write the baryon-lepton couplings with a typical term like

$$(G_1V_F/\sqrt2)\,\mathrm{Tr}(QL_e\phi_F{}^V)+\text{H.c.}, \tag{22}$$

where

$$Q=\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}.$$

In (22), $(\bar\nu_e\nu_e')$ terms get eliminated though a definite amount neutral lepton terms coupled to the $\Delta Y=0$ baryon current is still present.

To resolve (ii) we have to break the symmetry further with λ_8. So that the lepton-baryon coupling then has the typical term

$$(G_1V_F/\sqrt2)[\mathrm{Tr}(QL_e\phi_F{}^V)+\sqrt3\alpha\,\mathrm{Tr}(QL_e\lambda_8\phi_F{}^V)]+\text{H.c.}, \tag{23}$$

where α is to be determined and

$$\sqrt3\lambda_8=\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix}.$$

The breaking terms we envisage are of a very simple nature and transform as a combination of λ_3 and λ_8. In full the interaction (23) is then (for the simplest breaking):

$$\frac{G_1V_F}{\sqrt2}\left\{\left[\frac{(2\bar e^-e^- -\bar\nu_e\nu_e-\bar\nu_e'\nu_e')}{3}\left(\frac{\rho^0}{\sqrt2}+\frac{\omega^0}{\sqrt6}\right)\right](1+\alpha)+(\bar e^-\nu_e)\rho^-(1+\alpha)+(\bar e^-\nu_e')K^{-*}(1-2\alpha)\right\}+\text{H.c.}, \tag{24}$$

where we have *symbolically denoted the members of the* 1^- *octet* $\phi_F{}^V$ *by* $\rho^\pm$, ρ^0, ω^0, and K^{-*}, $\bar K^{0*}$, *etc.* Of course, in general, similar terms should be added to (24) for coupling to the $\phi_D{}^V$, $\phi_F{}^A$, etc. With the interaction (24) we can reproduce Cabbibo's results[6] by taking $V_F=1$, $V_D=0$, $A_F=0.30$, $A_D=0.95$ and with $\alpha=0.32$ and $G_1(1+\alpha)=G/\sqrt2$.

Another way of resolving the difficulty (ii) is to use (22) but assume that the baryon states which enter in (22) are in a frame F', which is related to the normal frame F of strong interactions by a rotation in SU(3) as suggested by Cabbibo.[16]

IV. IS THERE A SU(2) SYMMETRY?

We have noted that the SU(3) symmetry considered here has to be broken to obtain agreement with experiment. One may then ask whether a SU(2) symmetry could hold?

A. Leptons with Isospin

If one assumes that $(e^+,\bar\nu_e)$ and $\bar\nu_e'$ have $T=\frac12$, $Y=1$ and $T=0$, $Y=0$, respectively, like the nucleon doublet

[14] L. B. Okún, Ann. Rev. Nucl. Sci. 9, 61 (1959). Here and throughout by UFI is meant that the rate for the decay $B\to B'+e+\nu$ is calculated with the interaction $(G/\sqrt2)[\bar e(1+\gamma_5)\gamma_\mu\nu_e]\times[\bar B(1\pm\gamma_5)\gamma_\mu B']$.

[15] W. H. Barkas and A. H. Rosenfeld, University of California Radiation Laboratory Report UCRL-8030, 1963 (unpublished).

[16] N. Cabbibo, Phys. Rev. Letters 12, 62 (1964).

and the Λ, then one could write down isospin-invariant lepton-baryon interactions. But these again lead to muon-like decays, e.g., (14) or (15). To forbid these, one has to write the lepton-baryon interaction in analogy to electromagnetism. For example,

$$[\bar{\nu}_e'(e^-,\nu_e)]\left(\frac{1+\tau_3}{2}\right)[\bar{\Sigma}N]_{T=1/2}. \qquad (25)$$

Since each term like (25) may in general have a different coupling constant, one will have enough (too many, in fact) parameters to fit the leptonic decay modes. The interactions (25) now conserve Q, Y, T_3 (we do not need u) as is done by the electromagnetic interaction. In the case of isospin symmetry we only have to neglect the leptonic mass differences.

B. Leptons with *K* Spin

It is known that the electromagnetic interactions conserve K spin[17] [a SU(2) subgroup of SU(3)] for the strongly interacting particles. For these $K_3=\frac{1}{2}(2Y-Q)$ and the associated hypercharge $Y_K=-Q$. The baryon K-spin multiplets are given by Rosen[17] and in forming these one has to neglect all the baryon mass differences. Considerations of the above section suggest the weak interactions may, like electromagnetism, conserve K spin.

We define the l^+ to be a singlet with $K=0$, $Y_K=-1$ and $(\bar{\nu}_l,\bar{\nu}_l')$ to have $K=\frac{1}{2}$, $Y_K=0$, where $l=e$ or μ. The usual strangeness S for the leptons is then zero for $l^\pm$, zero for $\bar{\nu}_l$, and -1 for $\bar{\nu}_l'$. There are three kinds of K conserving interactions possible between the leptons and the baryons.

(i) Both $(\bar{l}l)$ and $(\bar{B}B)$ have $K=0$. These lead to weak scatterings and are uninteresting for our purposes.

(ii) Both $(\bar{l}l)$ and $(\bar{B}B)$ have $K=\frac{1}{2}$. These lead to the usual charged decays.

(iii) Both $(\bar{l}l)$ and $(\bar{B}B)$ have $K=1$. These lead to weak decays of strange particles like $\Sigma^+\to p+\nu_e+\bar{\nu}_e'$ etc., in particular. We assume that only the interactions of type (ii) which conserve K spin occur, while type (i) may be is absent but type (iii) is certainly absent. Note that in the electromagnetic case also one does not write all the possible K-conserving interactions. For the $[\bar{B}B\gamma]$ coupling since the photon γ is assumed to have $K=0$, $\bar{B}B$ must for a K-spin invariant, and three cases as above are possible. However, we only use the couplings in which both B and $\bar{B}$ have $K=\frac{1}{2}$.

The K-spin conserving lepton-baryon interactions of type (ii) give the weak-interaction Lagrangian,

$$\begin{aligned}(G_1/\sqrt{2})\{&(\bar{\nu}_e'O_\mu e^-)[-(\bar{\Xi}^-\Gamma_\mu^{(1)}\Lambda')+(\bar{\Lambda}'\Gamma_\mu^{(2)}p)\\&+(\bar{\Xi}^-\Gamma_\mu^{(3)}\Sigma'^0+\sqrt{2}\bar{\Sigma}^-\Gamma_\mu^{(3)}n)+(\sqrt{2}\bar{\Xi}^0\Gamma_\mu^{(4)}\Sigma^+ +\bar{\Sigma}^{0\prime}\Gamma_\mu^{(4)}p)]\\&+(\bar{\nu}_eO_\mu e^-)[(\bar{\Sigma}^-\Gamma_\mu^{(1)}\Lambda')-(\bar{\Lambda}'\Gamma_\mu^{(2)}\Sigma^+)\\&+(\sqrt{2}\bar{\Xi}^-\Gamma_\mu^{(3)}\Xi^0+\bar{\Sigma}^-\Gamma_\mu^{(3)}\Sigma'^0)\\&\qquad+(\bar{\Sigma}'^0\Gamma_\mu^4\Sigma^+ +\sqrt{2}\bar{n}\Gamma_\mu^{(4)}p)]\}, \qquad (26)\end{aligned}$$

[17] S. P. Rosen, Phys. Rev. Letters 11, 100 (1963).

where $O_\mu=(1+\gamma_5)\gamma_\mu$, $\Gamma_\mu^{(i)}=V_i\gamma_\mu-A_i\gamma_5\gamma_\mu$ and $\Sigma'^0=\frac{1}{2}(\sqrt{3}\Lambda-\Sigma^0)$, and $\Lambda'=\frac{1}{2}(\Lambda+\sqrt{3}\Sigma^0)$. The eight constants V_i and A_i, $i=1, 2, 3, 4$, have to be determined from experiment. The interaction automatically guarantees that the rules (i), (ii), and (iii) mentioned in the introduction are satisfied. The K-spin conserving lepton-lepton interaction is

$$(G_L/\sqrt{2})\{(\bar{\nu}_eO_\mu e^-)(\bar{\mu}^-O_\mu\nu_\mu)+(\bar{\nu}_e'O_\mu e^-)(\bar{\mu}^-O_\mu\nu_\mu')\}. \qquad (27)$$

From the experimental muon and beta-decay rates, we obtain (as before)

$$G_L=G_1=G/\sqrt{2} \quad\text{and}\quad V_4=1 \quad\text{and}\quad A_4=1.25. \qquad (28)$$

Since the decays actually take place for the Λ and Σ^0 we find that the interaction (26) divides the 12 decays into two sets of 6 decays; each set being determined by four parameters. The two sets are (omitting the lepton pair for brevity):

Set 1	Parameters	Set 2	Parameters
(1) $n\to p$		(7) $\Sigma^-\to n$	
(2) $\Lambda\to p$		(8) $\Sigma^-\to\Lambda$	
(3) $\Sigma^0\to p$	V_2, A_2	(9) $\Sigma^-\to\Sigma^0$	$V_1, A_1,$
(4) $\Sigma^0\to\Sigma^+$	V_4, A_4	(10) $\Xi^-\to\Lambda$	$V_3, A_3.$
(5) $\Sigma^+\to\Lambda$		(11) $\Xi^-\to\Sigma^0$	
(6) $\Xi^0\to\Sigma^+$		(12) $\Xi^-\to\Xi^0$	

Since the $\Lambda\to p$ and $\Sigma^-\to n$ decays are determined by independent parameters we should not have difficulty in fitting them simultaneously. Note also that $\Sigma^+\to\Lambda$ and $\Sigma^-\to\Lambda$ fall in different sets and their rates could be different in contrast with other models given in Refs. 5 and 6 and Sec. II D.

The requirement of the CVC hypothesis is that the vector part of the $\Delta Y=0$ current be given by

$$\sqrt{2}\bar{p}n-\sqrt{2}\bar{\Xi}^0\Xi^-+2(\bar{\Sigma}^0\Sigma^- -2\bar{\Sigma}^+\Sigma^0).$$

This implies that $V_1=V_2$, $V_3=-V_4$, and $V_1=\sqrt{3}V_4$. Since $V_4=1$ from beta decay, the vector part of $\Lambda\to p+e^-+\bar{\nu}_e'$ is then $(G_1/\sqrt{2})(1+\sqrt{2})(3/2)^{1/2}\gamma_\mu$. Thus even if its axial part was zero (which is not true) we get a rate of $(1+\sqrt{2})^2\,\frac{3}{4}\,\frac{1}{4}\Gamma_{\rm UFI}{}^{\Lambda}\simeq\Gamma_{\rm UFI}{}^{\Lambda}$ in contradiction with experiment. Thus, the hypothesis of K-spin-conserving weak interactions does not seem so attractive a possibility, unless we discard or modify the CVC hypothesis. Note that the CVC hypothesis requires $\Sigma^\pm\to\Lambda+e^\pm+\nu_e$ to be pure axial and this would be a good test of the hypothesis.

Finally, it should be noted that in the interaction (26) the pairs $(\bar{l}\nu_l)$ and $(\bar{l}\nu_l')$ are coupled to the $\Delta S=0$ and $\Delta S=1$ baryon currents. Consequently, one can test the existence of ν' as pointed out in Sec. II. B. It is worth emphasizing again that four neutrinos are essential to the K-spin SU(2) symmetry.[18]

[18] The assumption that e,μ have $K=0$ and (ν_e,ν_μ) form a K-spin doublet leads to both $\pi^+\to\mu^++\nu_e$ and $\pi^+\to\mu^++\nu_\mu$ and similarly, for the K-meson decays, in gross contradiction to experiment.

V. CONCLUSION

We have seen that the assumption of four neutrinos, ν_e, ν_e', ν_μ, and ν_μ', enables one to make the lepton-lepton and lepton-baryon weak interactions SU(3) symmetric. This simple scheme provides a rationale for writing down the weak interactions involving leptons, which is different from the (current)$\times$(current) one. The SU(3) symmetric coupling automatically guarantees $\Delta Q=\Delta Y$ rule, no $\Delta Y=2$ and $\Delta T=\frac{1}{2}$ rule for leptonic decays. However, it introduces neutral lepton currents (though only $(\bar{\nu}\nu')$ and this seems to disagree with experiment. For this reason a broken SU(3) symmetry is considered. Further, we have shown that the weak interactions may satisfy, together with the electromagnetic interaction, a SU(2) symmetry which conserves K spin. This SU(2) symmetric model, however, gets into trouble with experiment if the CVC hypothesis is assumed.

We have not considered nonleptonic decays here. We believe that these should be treated separately and maybe examples of broken SU(3) Yukawa interactions of mesons and baryons which transform like $(F_6 \pm iF_7)$ as has been suggested by various authors.[19]

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Neutrino Astronomy and Intermediate Bosons*

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Neutrino fluxes from strong radio sources are estimated, assuming the neutrino-production mechanism: $p+p \rightarrow$ nucleons+mesons $\rightarrow$ nucleons+electrons, gamma rays, and neutrinos. The neutrino fluxes calculated on the basis of this mechanism are too small to be easily detected unless there are resonances in neutrino processes associated with the production of intermediate bosons. It is shown that a resonance in the antineutrino-electron system, associated with the usually hypothesized W^- resonance, could be used to test, with standard experimental techniques, whether strong radio sources emit high-energy neutrinos in the quantities estimated in this paper. Two kinds of observational tests are described and counting rates are estimated. Observational tests of the kind we propose would provide important information about: (1) the mechanism for production of high-energy electrons in strong radio sources, and (2) the magnetic fields in such sources. Some comments concerning other logically possible neutrino resonances are also included.

I. INTRODUCTION

A DIRECT test of the theory of solar energy generation, based upon the observation of low-energy neutrinos from nuclear reactions occurring deep in the interior of the sun, has recently been shown to be feasible.[1,2] Because the sun is much closer to us than any other star, the solar neutrino flux completely dominates the low-energy (i.e., several MeV) neutrino flux reaching the earth. Hence the sun is the only main-sequence star from which one expects to observe neutrinos.

In this article, we explore the possibilities of observing high-energy neutrinos from strong radio sources,[3] which are believed to possess high-energy $(0.3-10^{+3}$ BeV) electrons. We begin by estimating neutrino fluxes from radio sources and show that these fluxes are too small to be easily detected unless there are resonances in neutrino processes associated with the production of intermediate bosons. Two types of resonances are logically possible with the kinds of targets available in the laboratory: (1) a resonance[4,5] in the neutrino-nucleon system that possesses both lepton and baryon number; and (2) a resonance,[6] the usually hypothesized W^--meson, in the antineutrino electron system that possesses neither lepton nor baryon number. Cowan[7] has tentatively interpreted results of an experiment on

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