# Electric Dipole Moment of a Nucleon*

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A violation of time-reversal invariance of any of the elementary particle interactions would imply the existence of a static electric-dipole moment for all these particles. We make here a theoretical estimate of the electric-dipole moment ($\mu_e$) of a bare nucleon, assuming the existence of an intermediate vector boson ($W$) and a small violation of $T$ invariance for the weak interactions. With the existing experimental upper limit for $T$ violation from polarized neutron decay, we find that $\mu_e \lesssim e/m_N \times 10^{-7}$. Since a value for $\mu_e$ somewhat smaller than this upper limit seems to be experimentally observable, such an experiment would provide a good test for $T$ invariance as well as for the existence of $W$ mesons.

## INTRODUCTION

IT is well known that the electric-dipole moment of a nucleon should vanish if all its interactions are time-reversal invariant ($T$).[1] While it is fairly well established by now that weak interactions violate invariance under charge conjugation ($C$) and parity ($P$), they may still be invariant under $CP$ and hence (by the $CPT$ theorem) under $T$. However, observations on the correlations between the neutron spin vector and the proton and electron momentum vectors in the decay of polarized neutrons do not rule out the possibility of an appreciable breakdown of $T$ invariance.[2] In a recent paper,[3] Schiff has discussed the measurability of nuclear electric-dipole moments and has shown that moments somewhat less than $e/m_N \times 10^{-7}$ are expected to be within the scope of experimental measurements being performed by Fairbank.[4]

It is our aim here to estimate theoretically the electric-dipole moment of a nucleon based on the existence of intermediate vector bosons[5] and on the assumption of a small violation of time-reversal invariance through the weak interaction vertices. In order to cause the sum of all Feynman graphs, which are of first order in the weak-interaction coupling constant $g^2$ but of arbitrary order in $\alpha$, to be finite, we use the $\xi$-limiting formalism of Lee and Yang.[6] In this formalism, a cutoff $\Lambda$ is introduced in a gauge-invariant way as $\Lambda = \xi^{-1/2} m_W$, where $m_W$ is the mass of the $W$ meson. We evaluate the electric-dipole moment only for a bare nucleon since there is as yet no consistent way of introducing the strong interaction effects accurately and since we are interested only in estimating the order of magnitude of the electric-dipole moment.

We find that the electric-dipole moment does not depend explicitly on the mass of the $W$ meson and is of the order of $10^{-7}\ e/m_N$ where we have taken for the violation of $T$ invariance, the maximum value that is consistent with the experiments of Ref. 2.

## CALCULATIONS

Assuming the existence of weak vector bosons $W$, the Feynman diagrams to be evaluated for the calculation of the electric-dipole moment of the bare nucleons is given in Figs. 1 and 2.[7] We are interested only in the static moment $\mu_e$, so we take the limit of zero photon energy ($k=0$).

We do not take into account the contribution from Fig. 1(b), for a proton, since in the absence of strong interactions (and hence of the anomalous magnetic moment terms) there is no direct $n$-$n$-$\gamma$ interaction in the limit $k \to 0$. Thus we consider only Fig. 1(a) for the evaluation of $\mu_e$ for the proton.

The evaluation of $\mu_e$ for the neutron will be discussed later.

The interaction between $W^{\pm}$ and the nucleons is written as

$$ig(V_\lambda + e^{i\Phi} A_\lambda) W_\lambda + \text{H.c.}, \qquad (1)$$

where $V_\lambda$ and $A_\lambda$ are given at low-energies approxi-

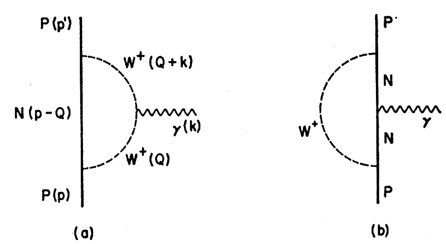


FIG. 1. Diagrams contributing to electric-dipole moment of a proton assuming the existence of intermediate weak vector bosons ($W$).

[7] We do not include diagrams where the intermediate nucleon is replaced by a strange particle ($\Lambda$ or $\Sigma$) since it is known that the strange-particle weak decays are down by a factor of order 20 compared to $\beta$ decay and since we are interested only in an order-of-magnitude estimate of the electric-dipole moment.

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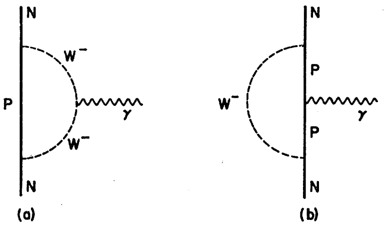


FIG. 2. Diagrams contributing to electric-dipole moment of a neutron assuming the existence of intermediate weak vector bosons.

mately by

$$V_\lambda = \bar{p}\gamma_\lambda n \quad \text{and} \quad A_\lambda = \bar{p}\gamma_\lambda\gamma_5 n. \tag{2}$$

We have not taken into account the strong interaction renormalization of the axial vector coupling constant; i.e., we take $|G_A/G_V|$ equal to 1 instead of 1.2. The violation of time-reversal invariance is introduced through the phase factor $e^{i\Phi}$ between $A_\lambda$ and $V_\lambda$. The coupling constant $g$ is related to the universal four-Fermion interaction coupling constant $G$ through the relation

$$g^2/m_W^2 = G/\sqrt{2} \approx 10^{-5}/\sqrt{2}m_p^2. \tag{3}$$

If the $W$ mesons do not exist, the diagrams to be considered for the proton is given in Fig. 3. It is easily seen that the contribution from Fig. 3 is smaller by a factor of order $G$ compared to Fig. 1(a), and so is too small to be measured by the experiment under consideration.[4] However, we show below that Fig. 1(a) gives a value for $\mu_e$ which is within the range of experimental possibilities.

The matrix element for Fig. 1(a) can be written as[8]

$$\frac{g^2}{(2\pi)^4}\int d^4Q\bar{u}_{p'}\gamma_\alpha(1+e^{-i\Phi}\gamma_5)\frac{1}{-i(p-Q)-m_N} \times\gamma_\beta(1+e^{i\Phi}\gamma_5)u_p[S(Q+k)V_\mu(Q+k,Q)S(Q)]_{\alpha\beta}, \tag{4}$$

where $S(Q+k)$ is the propagator for the virtual $W$ meson with four momentum $(Q+k)$ and $V_\mu$ is the three-point $W$-$W$-$\gamma$ vertex interaction. In the $\xi$-limiting formalism they are given by

$$S(p) = \frac{-i}{p^2+m_W^2}\left[1+\frac{(1-\xi)p\tilde{p}}{\xi p^2+m_W^2}\right] \tag{5}$$

and

$$V_\mu(p',p) = ie[(p'+p)_\mu + (\xi+K)(\epsilon_\lambda\tilde{p}+p'\tilde{\epsilon}_\lambda) - (1+K)(\epsilon_\lambda\tilde{p}'+p\tilde{\epsilon}_\lambda)], \tag{6}$$

where the magnetic moment of $W$ meson is given by

$$(e/2m_W)(1+K). \tag{7}$$

We are interested in calculating only the electric-dipole moment, which is the coefficient of the term $(\bar{u}_p\gamma_5\sigma_{\mu\nu}k_\nu u_p)$ in the limit $k\to 0$. So picking out only the $\gamma_5$ terms in (4) we have

$$\frac{ig^2}{(2\pi)^4}\int d^4Q\frac{[S(Q+k)V_\mu(Q+k,Q)S(Q)]_{\alpha\beta}}{Q^2-2Q\cdot p}$$
$$\times\bar{u}_{p'}[\gamma_5\gamma_\alpha(-\cos\Phi+i\sin\Phi)(\boldsymbol{p}-\boldsymbol{Q}+im_N)\gamma_\beta$$
$$+\gamma_\alpha(\cos\Phi+i\sin\Phi)(\boldsymbol{p}-\boldsymbol{Q}+im_N)\gamma_\beta\gamma_5]u_p$$
$$=\frac{-2ig^2}{(2\pi)^4}\int d^4Q\frac{[S(Q+k)V_\mu(Q+k,Q)S(Q)]_{\alpha\beta}}{Q^2-2Q\cdot p}$$
$$\times\bar{u}_{p'}[\gamma_5\gamma_\alpha\{\cos\Phi(\boldsymbol{p}-\boldsymbol{Q})+m_N\sin\Phi\}\gamma_\beta]u_p. \tag{8}$$

However, we know that as $\Phi\to 0$, time-reversal invariance holds and the electric-dipole moment has to vanish. Thus the contribution from the $\cos\Phi$ term should be zero. These terms can also be shown explicitly to vanish.[9] Thus we are interested only in the following term:

$$\frac{-2ig^2}{(2\pi)^4}m_N\sin\Phi\int d^4Q[\bar{u}_{p'}\gamma_5\gamma_\alpha\gamma_\beta u_p] \times[S(Q+k)V_\mu S(Q)]_{\alpha\beta}\frac{1}{Q^2-2Q\cdot p}. \tag{9}$$

Lee and Bernstein[10] have shown that in order to explain the electromagnetic form factors of the neutrino as calculated on the basis of the $\xi$-limiting process, the magnetic moment of the $W$ meson should be equal to $e/2m_W$; so the parameter $K$ in (7) has to be zero. However, for completeness we consider both cases.

*Case (1): K=0*

The contribution from Fig. 1(a) turns out to be

$$[\mu_e]_{K=0} = a_0\ln\xi + c_0, \tag{10}$$

where

$$a_0 = -\frac{1}{8\pi^2}\frac{g^2}{m_W^2}em_N\sin\Phi$$

and

$$c_0 = -\frac{1}{8\pi^2}\frac{g^2}{m_W^2}em_N\sin\Phi\left[\frac{m_W^2}{m_N^2}-2+\left(1-\frac{5m_W^2}{m_N^2}+\frac{m_W^4}{2m_N^4}\right)\right.$$
$$\times\ln\frac{m_N^2}{m_W^2}+\frac{5m_N^2-m_W^2}{m_N^2}\frac{2m_N^2-m_W^2}{m_N^2}\frac{m_W}{(4m_N^2-m_W^2)^{1/2}}$$
$$\left.\times\tan^{-1}\frac{(4m_N^2-m_W^2)^{1/2}}{m_W}\right].$$

[8] We use the same notation as in Ref. (6).

[9] From the $\cos\Phi$ term in Eq. (8) the only nonvanishing contribution to the electric-dipole moment term, after integration, turns out to be proportional to $\bar{u}_{p'}\gamma_5\gamma_\alpha\boldsymbol{p}\gamma_\beta u_p$. However, in the limit as $k\to 0$, $p\to p'$, so that we may replace $p$ by $(p+p')/2$; we then find that the contribution to $\mu_e$ from this term also vanishes. Thus the $\cos\Phi$ term cannot contribute at all. The authors are grateful to Dr. F. Chilton for helpful discussion of this point.

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In calculating $c_0$, we have assumed that $2m_N > m_W$; the calculation of $a_0$ is independent of this assumption.

Equation (10) gives only the contribution from the lowest order diagram. To include all radiative corrections we evaluate the contribution from all diagrams of first order in $g^2$ but of arbitrary order in $\alpha$. However for these diagrams we do not evaluate the terms independent of $\xi$ since they are of order $\alpha$ compared to $c_0$. Thus including only the most singular parts of the higher order graphs we find

$$[\mu_e]_{K=0} = c_0 + a_0 \ln\xi + \sum_{n=1}^{\infty} a_n \left(\frac{\alpha}{\xi}\right)^n, \qquad (11)$$

where $a_n$'s are numbers independent of $\alpha$ and $\xi$.

Following Ref. (5), we assume that the entire sum (11) leads to a finite result in the limit $\xi \to 0$. Equation (11) can be rewritten as

$$[\mu_e]_{K=0} = c_0 + a_0 \ln\alpha - a_0 \ln x + \sum_{n=1}^{\infty} a_n x^n$$

$$= c_0 + a_0 \ln\alpha + f(x),$$

where $x = \alpha/\xi$ and for small $x$, the function $f(x)$ can be represented by its series expansion

$$f(x) = -a_0 \ln x + \sum_{n=1}^{\infty} a_n x^n.$$

Taking the limit $\xi \to 0$ and keeping $\alpha$ fixed [(i.e.) $x \to \infty$] we find

$$[\mu_e]_{K=0} \to c_0 + a_0 \ln\alpha + f(\infty),$$

where $f(\infty)$ is finite and independent of $\alpha$. So mathematically, as $\alpha \to 0$ we have the expression for the electric-dipole moment given by

$$[\mu_e]_{K=0} \cong a_0 \ln\alpha. \qquad (12)$$

Since it is believed that $m_W$ is of the same order as $m_N$, even the term $\ln(m_N^2/m_W^2)$ in $c_0$ is expected to be of smaller order as compared to $\ln\alpha$ and as $f(\infty)$ does not contain any log terms, $c_0$ and $f(\infty)$ can be neglected.

*Case (2): $K \neq 0$*

In this case it is known that the power series in $\alpha$ is much more divergent than in case (1). This comes about because at a virtual $W$-$W$-$\gamma$ vertex, we have as shown by Lee[6]

$$\bar{p}' V_\lambda(p',p) p = ieK[k^2 p_\lambda - k_\lambda (k \cdot p)] + ie\xi[p^2 p_\lambda' + p'^2 p_\lambda]. \qquad (13)$$

Remembering the fact that the cutoff for the momentum is equal to $\xi^{-1/2} m_W$, the presence of $\xi$ in the second term suppresses two powers of $p$ and so the first term dominates over the second. However, in the case $K=0$, the first term vanishes and so only the second term exists,

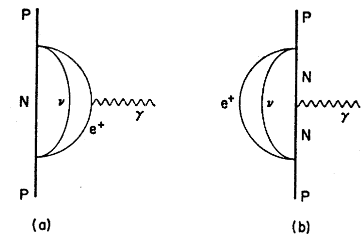


FIG. 3. Diagrams contributing to the electric-dipole moment of a proton in the absence of $W$ mesons.

thus reducing the degree of divergence in case (1) as compared to case (2).

We find in this case the power series expansion for the electric-dipole moment term to be[11]

$$[\mu_e]_{K\neq 0} = a_0' \ln\xi + \sum_{n=1}^{\infty} a_n' (\alpha K^2/\xi^2)^n, \qquad (14)$$

where

$$a_0' = (1/8\pi^2)(g^2/m_W^2) e m_N (1-K) \sin\Phi.$$

Writing $x = \alpha K^2/\xi^2$ and taking the limit $\xi \to 0$ we find

$$[\mu_e]_{K\neq 0} \to \tfrac{1}{2} a_0' \ln(\alpha K^2). \qquad (15)$$

## RESULTS

To make an estimate of the order of magnitude of the electric-dipole moment we take the expressions for $\mu_e$ given by Eq. (12) for $K=0$ and by Eq. (14) for $K \neq 0$. Thus we have

$$[\mu_e]_{K=0} = (1/8\pi^2)(g^2/m_W^2) e m_N |\sin\Phi \ln\alpha| \qquad (16)$$

and

$$[\mu_e]_{K\neq 0} = (1/8\pi^2)(g^2/m_W^2) e m_N \tfrac{1}{2}(1-K) |\sin\Phi \ln\alpha K^2|.$$

It is interesting to note that $g^2$ enters these formulas only in the form $g^2/m_W^2$. Using Eq. (3) we have

$$[\mu_e]_{K=0} = [10^{-5}/\sqrt{2}(8\pi^2)] |\sin\Phi \ln\alpha| (e/m_N). \qquad (17)$$

Thus we find that the mass of the $W$ meson does not occur explicitly in the expression for $\mu_e$, and so we do not require the experimental value of $m_W$ for the determination of $\mu_e$. Writing in Schiff's notation[12] $\mu = e\gamma/m_N$, we find for the constant $\gamma$

$$\gamma_{K=0} \simeq 5 \times 10^{-7} \sin\Phi \qquad (18)$$

and

$$\gamma_{K\neq 0} \approx 10^{-7} (\sin\Phi) \tfrac{1}{2}(1-K) \ln(\alpha K^2). \qquad (19)$$

[11] It is to be noted that unlike Bernstein and Lee's result for the neutrino-charge form factor, which contains a quadratically divergent term even in the lowest order, in our case such a quadratic divergence does not occur in the lowest order. Fig. 1(a) does have a quadratic divergence in the lowest order; however, we are interested only in the electric-dipole moment term which is proportional to $\sigma_{\alpha\beta}\gamma_5$. It so happens that this term is only logarithmically divergent in the lowest order.

[12] It should be mentioned that we have expressed the electric-dipole moment of a bare nucleon in terms of the mass of a physical nucleon. We believe we are justified in doing so, since we are interested only in an order of magnitude estimate which is not expected to be altered significantly when strong interaction effects are included.

The experimental upper limit for $\sin\Phi$ is 0.26 (Ref. 2).

Putting this value in (18), we find as a typical maximum value for $\gamma$

$$(\gamma_{K=0})_{\max} \approx 1.3 \times 10^{-7}. \qquad (20)$$

In the case of the neutron we have to consider Figs. 2(a) and 2(b) for the evaluation of the electric-dipole moment. However, we find that the contribution to the $\ln\xi$ term from Fig. 2(b) vanishes. Thus we expect the electric-dipole moments of the neutron and proton to be of the same order of magnitude.

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# Longitudinal Behavior of Electromagnetic Showers*

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The need to establish recognition patterns for high-energy photons and electrons in cosmic-ray work, and more recently, around high-energy accelerators, has stimulated theoretical and experimental investigations into the properties of electromagnetic cascades. The most recent results of statistical computations, for longitudinal development and for lateral and angular spread, are in large part inconsistent with earlier published data. In order to study the longitudinal behavior of electron-induced showers, measurements have been made with a monoenergetic electron beam (energies 100–1000 MeV) at the 1.5-BeV CIT electron synchrotron. Buildup and energy dissipation were investigated, through the sampling of showers generated in lead of variable thicknesses, by means of a Lucite Čerenkov counter. Average numbers of shower particles with energies above 10 MeV are given for these incoming energies and penetration depths up to 10 radiation lengths; also, shower fluctuations are presented for the same points. The results of this experiment can readily be compared with the data recently computed by Crawford and Messel. Agreement appears to be satisfactory.

## INTRODUCTION

The buildup and decay of electromagnetic cascade showers, initiated by high-energy photons or electrons, has long been studied in connection with cosmic-ray work. More recently, the cascading properties have been used in increasing measure for the selective detection of showering particles in work around high-energy particle accelerators.

Although the equations describing the fundamental processes involved in buildup and decay of showers are well established, analytical solutions of the shower distributions taking into account the full amount of physical phenomena are impossible to obtain. Various approaches and approximations have been put forward, notably separating the problems of longitudinal, lateral, and angular structure. More recently, calculations using statistical models have been made to obtain numerical data for various sets of input parameters. In this approach, it is easier to take all of the important cross-section data into account and not to introduce too many oversimplifications. The most recent data of Messel *et al.*[1] and Crawford and Messel[2] differ appreciably from earlier results obtained by Wilson[3] for the longitudinal development of showers, as well as from approximate analytical solutions as presented by Belenkii and Ivanenko[4] and others.[5] There has been no experimental check so far on the validity of Messel's data. Earlier work, e.g., by Lal and Subramanian,[6] has too many inherent uncertainties to allow for a quantitative confirmation.

The structure of electromagnetic showers (longitudinal, lateral, and angular distributions) is of vital interest for the identification of particles in heavy backgrounds, either singling out high-energy photons and electrons in the presence of copious $\pi$, $K$, $p\cdots$ production from targets around high-energy accelerators, or discriminating against them; and contingent knowledge may help to establish recognition patterns for incoming photons and/or electrons of given energies. Therefore,

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