

agreement it was necessary to adjust the inverse reaction cross-section parameters to fit low-energy data and to take proper account of the motion of the emitting nuclides. The agreement of the calculated and experimental energy spectra implies that it is not necessary to invoke a reduction of the Coulomb barrier at high-excitation energies to account for the relatively large number of sub-barrier α particles. The comparison does reveal that an appreciable fraction of the α particles emitted with energies greater than 25 MeV are probably associated with the cascade rather than the evaporation phase of the reaction. However, the total number of such α particles accounts for only about 10% of the

spectrum. It is of interest to note that although the evaporation calculation can only account for a fraction of the high-energy α particles, it does predict essentially the same F/B values as those observed. Once again, we attribute this fact to the motion of the emitting nuclides.

ACKNOWLEDGMENTS

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Electron and Muon Scattering from Nuclear Charge Distributions at Incident Momenta Between 50 and 183 MeV/c

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The comparison of theoretical elastic-scattering cross sections of positrons and electrons from Woods-Saxon (WS) and "wine-bottle" (WB) charge distributions of the nucleus of Au, carried out at 183 MeV in a previous paper by the authors, is extended to lower energies and repeated for muons of comparable incident momenta. It is found that, for momentum transfers of less than 1.5 F^{-1} , the percent change of the cross section corresponding to a change from the WS to the WB charge distribution is largest, of the order of 30% for incident momenta of $\sim 100 \text{ MeV}/c$, particularly for positrons. At an electron energy of 50 MeV the cross section depends mainly on the mean-square radius of the nucleus, and an accuracy better than 5% is needed in order to determine additional nuclear charge distribution parameters. The mean-square radii of the WS and WB charge distributions differ by 6.5% while the corresponding electron cross sections at 50 MeV differ by a maximum of 15%. A comparison with experimental elastic positron and electron scattering cross sections for Pb measured by Miller and Robinson is carried out, and a systematic discrepancy with theory is found for both e^+ and e^- cross sections for the 50–70-MeV energy range, while theory and experiment agree well at 87 MeV and higher energies. The calculation consists of a conventional numerical phase-shift analysis based on the Dirac equation, and the nuclei are assumed to be static, spherically symmetric extended charge distributions.

I. INTRODUCTION

THE desirability of using positrons as well as electrons for the determination of nuclear charge distributions by means of elastic-scattering experiments has been explored recently both experimentally^{1,2} and theoretically.^{3,4} Positrons are expected to yield infor-

mation independent of that obtained from electrons because the Coulomb repulsion for positrons reduces the wave function in the nuclear interior, enhancing the sensitivity to the "tail" of the charge distribution. The investigation presented by the authors in a previous note,⁴ denoted by RF in what follows, has been extended to lower energies,⁵ and it was found that electron cross sections continue to be sensitive to changes in the charge distribution at energies as low as 50 MeV. The usefulness of this result may be twofold. It serves to define the accuracy with which low-energy elastic-scattering experiments are to be carried out in order to yield information on the nuclear charge distribution,

* Research supported by a grant from the National Science Foundation.

† Contract monitored by the U. S. Air Force Office of Scientific Research.

¹ R. C. Miller and C. S. Robinson, *Ann. Phys. (N. Y.)* **2**, 129 (1957).

² J. Goldemberg, J. Pine, and D. Yount, *Phys. Rev.* **132**, 406 (1963).

³ R. Herman, B. C. Clark, and D. G. Ravenhall, *Phys. Rev.* **132**, 414 (1963).

⁴ G. H. Rawitscher and C. R. Fischer, *Phys. Rev.* **122**, 1330 (1961).

⁵ A preliminary report on some of this work is contained in *Bull. Am. Phys. Soc.* **8**, 57 (1963).

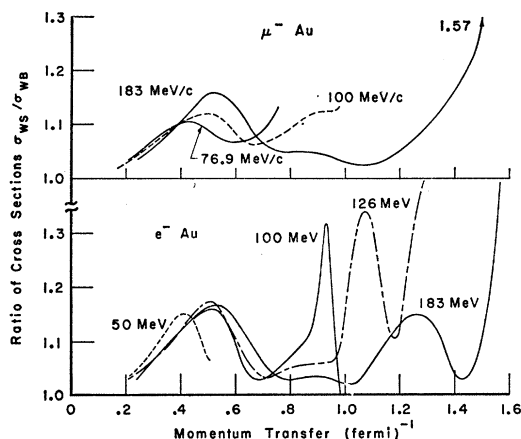


FIG. 1. Comparison of WS and WB cross sections for electron and negative muon scattering on Au. The unit of $(\text{fermi})^{-1}$ for the ordinate scale is obtained by dividing the momentum transfer by \hbar , where \hbar is Planck's constant divided by 2π .

and by encouraging the comparison between theory and experiment at low as well as at high energies, it may lead to⁶ the detection of effects⁷ which are not usually included in the calculation. These effects are due to the correlation between protons in the nucleus, the presence of nuclear magnetic moments and current distributions and the dynamic effects due to nuclear polarizability. Interpretation of muonic x-ray data which now yields accurate⁸ determination of rms nuclear radii, also suffers from as yet poorly known corrections arising from nuclear polarizability⁸ and would also benefit from accurate low-energy electron scattering studies. The results reported below also neglect such effects, and should therefore be looked upon only as a guide for future detailed comparison with experiment, in which more than the effect of variation of parameters describing static charge distributions is taken into consideration.

The notation is the same as that of the previous paper.⁴ The charge distribution is given by

$$\rho = \rho_0(1 + wr^2/c^2) / \{1 + \exp[(r-c)/z]\}, \quad (1)$$

where c , z , and w are the parameters which determine the r dependence of the nuclear charge distribution. The quantities $c_0 = cA^{-1/3}$ and $t = 4.40z$, where A denotes the mass number of the nucleus, are also commonly used.⁹

⁶ The possibility of detecting such effects from the electron-positron comparison has been considered in RF and was pointed out to the authors by Professor G. Breit.

⁷ U. Meyer-Berkhout, K. W. Ford, and A. E. S. Green, *Ann. Phys. (N.Y.)* **8**, 119 (1959), discuss some of these effects and point to the desirability of performing scattering experiments at various energies.

⁸ D. Quitman, R. Engfer, N. Hegel, P. Brix, G. Backenstoss *et al.*, *Nucl. Phys.* (to be published). This paper contains additional references to muonic x-ray work. The rms radius of the nucleus is determined from the muonic x-ray data with an accuracy of 1.5% for the medium heavy elements ($A \sim 50$ –100), while from electron scattering the nuclear surface thickness t is determined to $\sim 10\%$ and the half-way radius c to $\sim 3\%$. The notation is the same as that of RF.

⁹ R. Hofstadter, *Ann. Rev. Nucl. Sci.* **7**, 231 (1957).

TABLE I. Charge distribution parameters for Au.

	c^a	z^a	w	α	c'^a	z'_s
WS	6.38	0.535	0
WB	6.07	0.613	0.64
6P	6.07	0.613	0.64	0.08	7.283	0.613

^a In units of 10^{-13} cm.

II. ACCURACY

The calculations are based on a numerical phase-shift analysis as described in RF and were performed on IBM-709 and IBM-7090 digital computers. The accuracy is improved relative to the previous⁴ IBM-650 calculations by employing double precision arithmetic in places where large cancellations are expected to occur, for example, in the calculation of the point-nucleus Coulomb regular and irregular wave functions by means of series expansions. The error in the ratio of nuclear functions¹⁰ G/F at a matching point $x \sim 10$, which occurs for 183-MeV electron scattering on Au, is about $10^{-3}\%$ for the commonly used integration step $\Delta x \sim 0.1$. The corresponding error in the scattering cross section is less than 0.5% at angles less than 90° and increases to about 5% at 150° . At lower energies the matching point occurs at a smaller value of x , the number of angular momenta involved is smaller, and the accuracy increases.

III. COMPARISON OF WS AND WB CROSS SECTIONS FOR Au AT VARIOUS ENERGIES

The sensitivity of the cross section to the choice of the radial dependence of the charge distribution is again⁴ obtained by employing the Woods-Saxon (WS) and "wine-bottle" (WB) charge distributions already employed by Hahn *et al.*¹¹ in their analysis of electron scattering on Au. The parameters are given in Table I.

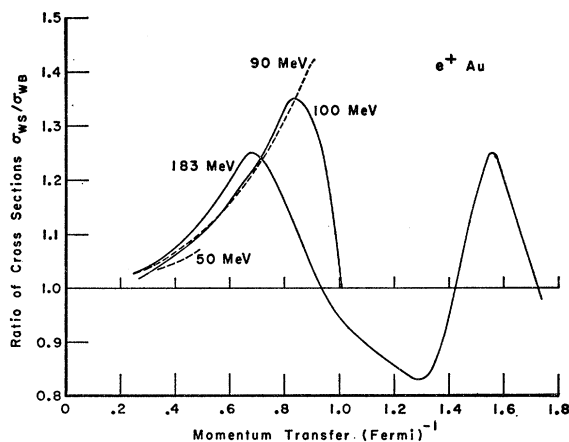


FIG. 2. Same as Fig. 1 for positrons.

¹⁰ The notation is the same as that of RF.

¹¹ B. Hahn, D. G. Ravenhall, and R. Hofstadter, *Phys. Rev.* **101**, 1131 (1956).

Figures 1, 2, and 3 show the comparison of the corresponding cross sections at the various energies.⁵ Figure 1 shows that for a given percent accuracy in the cross section, and at momentum-transfers of less than 1.4 F^{-1} electron scattering at 100 MeV could be used to differentiate between the WS and WB charge distributions as well as, if not better than, data at 183 MeV. For positrons the difference between WS and WB cross sections attains a clear maximum at 90 MeV as shown in Fig. 2, but at 50 MeV the difference is only $\sim 7\%$ which is a factor of 2 less than the 15% difference for electrons. The same effect occurs in the comparison of μ^+ and μ^- and is somewhat more pronounced than the effect in the positron-electron comparison. The WS and WB charge distributions differ both at small and large distances. The difference in the "tail" of the charge distributions beyond 6 F appears to be responsible for a great deal of the difference in the corresponding electron cross sections, particularly in the region beyond the first maximum, i.e., for momentum transfers larger than 0.8 F^{-1} . This can be seen by introducing a third charge distribution which differs from the WB distribution only in the "tail" region, and then comparing the resulting cross section to the WB cross section. This charge distribution is determined by six parameters and is given by the expression

$$\rho_{6P} = N\rho_{WB}/\{1 + \alpha \exp[(r - c')/z']\}, \quad (2)$$

where ρ_{WB} is the charge distribution determined by Eq. (1), α , c' , and z' are three additional parameters and N is a properly adjusted normalization constant. When the parameters have the values given in Table I, the resulting ρ_{6P} is nearly identical to ρ_{WB} for $r < 6 \text{ F}$; at 8 F $\rho_{WS} \approx \rho_{6P}$, and for $r > 8 \text{ F}$, $\rho_{6P} < \rho_{WS}$. The cross sections are compared in Fig. 4 in the form of ratios to the WB cross section.

For the 126-MeV case, the peaks in the WS/WB cross-section ratio are very similar to the peaks in the $6P$ /WB ratio for momentum transfers larger than 1.0

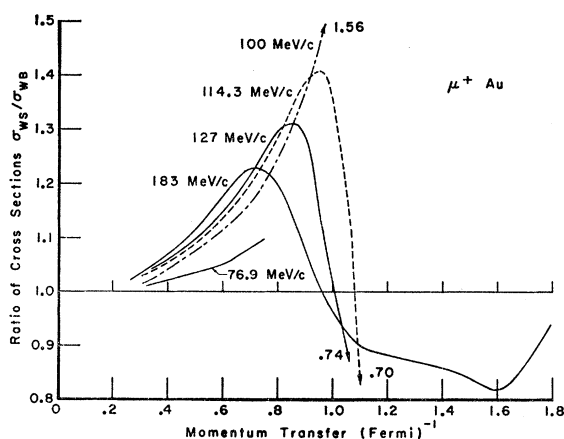


FIG. 3. Same as Fig. 1 for positive muons. The numbers near each curve indicate the incident momenta.

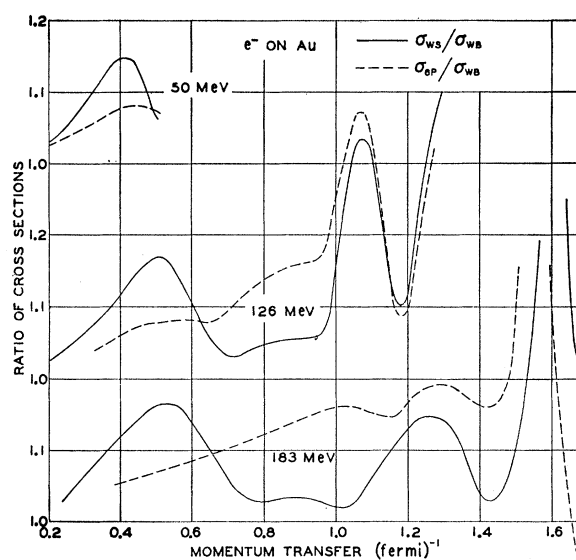


FIG. 4. Comparison of cross-section ratios σ_{WS}/σ_{WB} and σ_{6P}/σ_{WB} . The charge distributions $6P$ and WB have nearly the same central "wine-bottle" depression, and differ from each other at distances beyond 6 F. The $6P$ and WS charge distributions are nearly equal to each other in the "tail region" between 7 and 9 F. The similarity of the dashed and solid curves indicates that most of the difference between the WS and WB cross sections is due to the difference in the "tail" of these two charge distributions.

F^{-1} . This similarity, also noticeable in the 183-MeV case, is taken as an indication that the second and third peaks in the WS/WB cross section ratio are due to the difference between the WS and WB charge distributions for distances larger than 6 F, since the $6P$ and WB charge distributions differ only beyond that distance.

At low energies, the part of the elastic electron-scattering cross section which is due to the extended electric charge distribution is expected to yield information only on the rms radius of the nucleus.¹² The argument requires that $r/\lambda \ll 1$, where λ is the electron wavelength divided by 2π , and r is of the order of the nuclear dimensions. For the case of 50-MeV electrons, λ at large distances is $\sim 4 \text{ F}$ which is less than the half-density radius for Au, and it would therefore be reasonable to expect that radial moments of order higher than the second could be determined at this energy. The sensitivity of the cross section to the value of the rms radius was explored by selecting several combinations of charge distribution parameters c , t , and w , and comparing the corresponding cross sections to the WS cross section. The standard cross section has the values of c , t , and w listed for WS in Table I. Plots of the ratio of each cross section to the WS cross section as a function of scattering angle display either a single broad maximum or minimum which occurs in the vicinity of a scattering angle of 120° . If the maximum or minimum of each cross section ratio is plotted versus the mean-square radius of the corresponding

¹² H. Feshbach, Phys. Rev. **84**, 1206 (1951).

TABLE II. Elastic electron and positron scattering cross sections^a for the nucleus of Au.

Energy (MeV)	183		126		100		50	
	e^-	e^+	e^-	e^+	e^-	e^+	e^-	e^+
30	0.234(1)	0.390(1)	0.185(2)	0.204(2)	0.463(2)	0.417(2)	0.328(3)	0.211(3)
45	0.794(-1)	0.101(0)	0.755(0)	0.162(1)	0.326(1)	0.474(1)	0.532(2)	0.347(2)
60	0.558(-2)	0.719(-2)	0.678(-1)	0.155(0)	0.268(0)	0.738(0)	0.116(2)	0.887(1)
75	0.407(-3)	0.886(-3)	0.145(-1)	0.165(-1)	0.465(-1)	0.132(0)	0.282(1)	0.285(1)
90	0.115(-3)	0.619(-4)	0.186(-2)	0.237(-2)	0.142(-1)	0.256(-1)	0.744(0)	0.105(1)
105	0.645(-5)	0.844(-5)	0.263(-3)	0.574(-3)	0.352(-2)	0.529(-2)	0.217(0)	0.419(0)
120	0.364(-5)	0.304(-5)	0.126(-3)	0.184(-3)	0.686(-3)	0.117(-2)	0.752(-1)	0.173(0)
135	0.136(-5)	0.645(-6)	0.458(-4)	0.592(-4)	0.169(-3)	0.281(-3)	0.302(-1)	0.695(-1)
150	0.222(-6)	0.840(-7)	0.131(-4)	0.168(-4)	0.469(-4)	0.685(-4)	0.117(-1)	0.244(-1)

^a The units are 10^{-26} cm²/sr. The charge distribution is characterized by the WS parameters given in Table I.

^b Scattering angle in degrees.

nuclear charge distribution, the representative points scatter around a straight line. For example, the minimum values of the WB/WS and $6P$ /WS cross-section ratios are 0.87 and 0.94, respectively, and the ratios of the corresponding mean-square radii to the WS mean-square radius are 1.02 and 1.065, respectively. The value of the $6P$ /WS cross-section ratio would be equal to 0.96 if its representative point were to lie exactly on the straight line mentioned above. By reducing the value of t from 2.697 to 2.30 F and leaving

the values of the other WB parameters unchanged, the mean-square radius of the resulting charge distribution is 28.27 F², which is only 0.4% smaller than the mean-square radius of the WS charge distribution. The resulting cross section differs from the WS cross section by less than 1% at angles less than 120°, and the difference reaches a maximum of 6% at 180°. This comparison illustrates the accuracy of the cross-section measurement required to determine radial moments higher than the second, i.e., information additional to the mean-square radius. It is understood, of course, that before meaningful values of the rms radius are obtained from electron scattering measurements at the energies here examined, the various corrections mentioned above, as well as radiative corrections (real and virtual) have to be taken into account.

The slope of the line representing the change in maximum or minimum cross-section ratio as the mean-square radius is varied indicates that a 10% change in the cross section is obtained by a 5% change in the mean-square radius. The various charge distributions included in this survey are not drastically different from the WS and WB distributions listed in Table I. The values of c differ at most by 16% and the values of t by at most 7% from the values of the corresponding WS or WB parameters. The effect of a variation of the individual parameters is as follows: An increase of t by 12% for the WS charge distribution decreases the cross section by a maximum of 7%, and the corresponding mean-square radius increases by 3.5%. A decrease of t by 16%, or else a decrease of c_0 by 4% from the values these parameters have in the WB case, increases the cross section by a maximum of 15%, and the corresponding decrease in the mean-square radius is 7%. Table II lists electron and positron cross sections for the WS charge distribution at various energies, and Table III contains muon cross sections.

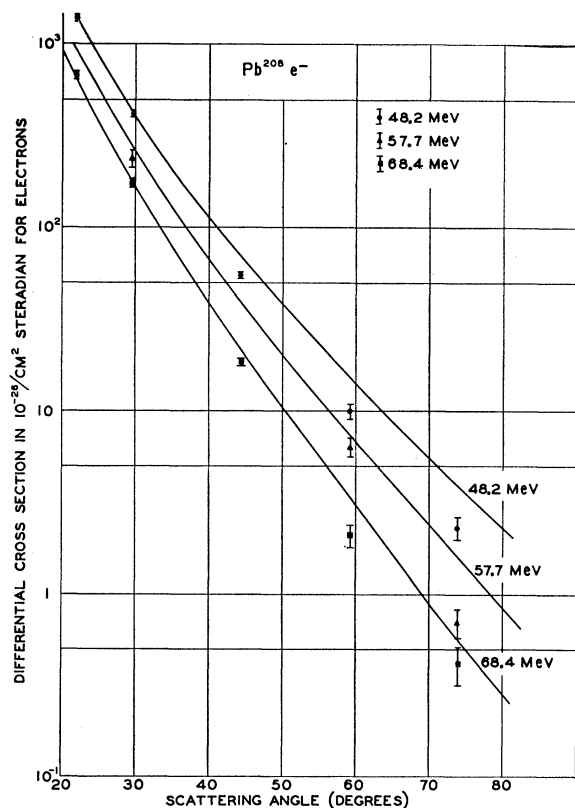


FIG. 5. Differential cross sections for scattering of electrons by Pb at 48.2, 57.7, and 68.4 MeV. The data are taken from Table I of the paper by Miller and Robinson, while the curves are calculated using the Woods-Saxon two-parameter charge distribution with $c=6.48$ and $z=0.535$.

IV. COMPARISON WITH EXPERIMENTAL CROSS SECTIONS OF MILLER AND ROBINSON

A. Comparison of Theory with Data

Miller and Robinson¹ have obtained absolute cross sections for elastic electron and positron scattering by

TABLE III. Elastic muon scattering cross sections^a for the nucleus of Au.

Energy (MeV) Momentum MeV/c θ^b	105.6 183.0		40 100.3		25 76.8	
	μ^-	μ^+	μ^-	μ^+	μ^-	μ^+
30	0.355(1)	0.678(1)	0.108(3)	0.127(3)	0.351(3)	0.332(3)
45	0.131(0)	0.222(0)	0.797(1)	0.194(2)	0.391(2)	0.634(2)
60	0.838(-2)	0.163(-1)	0.815(0)	0.440(1)	0.539(1)	0.190(2)
75	0.756(-3)	0.305(-2)	0.164(0)	0.124(1)	0.996(0)	0.744(1)
90	0.178(-3)	0.575(-3)	0.418(-1)	0.406(0)	0.284(0)	0.348(1)
105	0.115(-4)	0.113(-3)	0.877(-2)	0.151(0)	0.100(0)	0.188(1)
120	0.865(-5)	0.390(-4)	0.188(-2)	0.636(-1)	0.346(-1)	0.115(1)
135	0.325(-5)	0.305(-4)	0.822(-3)	0.308(-1)	0.116(-1)	0.790(0)
150	0.695(-6)	0.293(-4)	0.583(-3)	0.177(-1)	0.474(-2)	0.604(0)

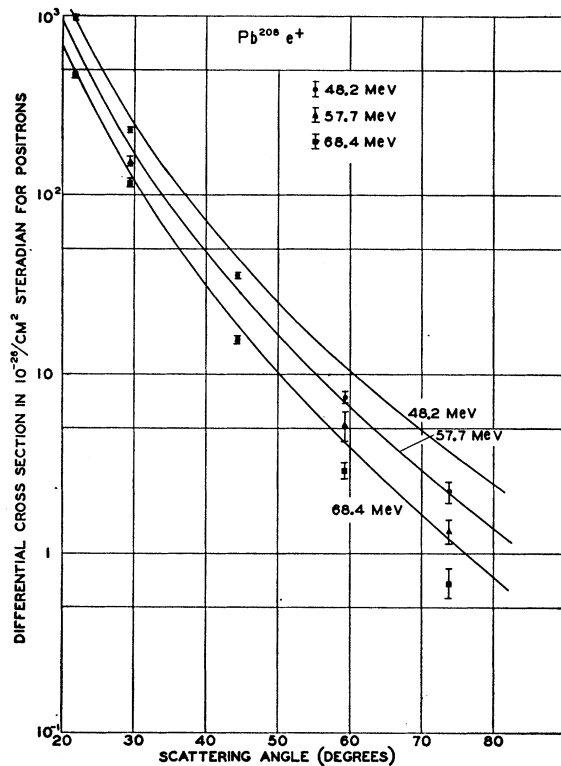
^a The units are 10^{-26} cm²/sr. The charge distribution is characterized by the WS parameters given in Table I.^b Scattering angle in degrees.

FIG. 6. Same as Fig. 5 except that positrons are used instead of electrons.

lead at angles between 21 and 74° and energies between 48.2 and 168.2 MeV. Some of their cross-section data are shown in Figs. 5–7. The smooth curves in Figs. 5–7 have been calculated for lead at the experimental energies by the method described above and correspond to the Woods-Saxon charge distribution with $c_0 = 1.097$ and $t = 2.35$.

These values for c_0 and t have been found by Hahn, Ravenhall, and Hofstadter¹¹ to fit the data for elastic scattering of electrons by gold at 183 MeV and lead to almost the same charge distribution as the one deter-

mined by Ford and Hill¹³ for lead.¹⁴ According to Hofstadter,^{9,14} for nuclei with atomic number greater than 20, the parameters

$$c_0 = (1.07 \pm 3)\%, \quad t = (2.45 \pm 10)\%,$$

fit elastic electron scattering data for a number of different elements up to an energy of 183 MeV. These results appear to be consistent with analysis of more

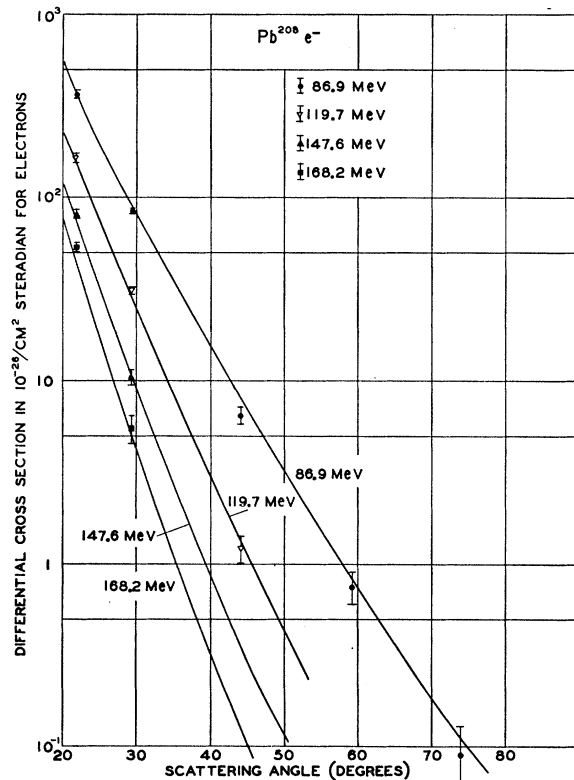


FIG. 7. Same as Fig. 5 but for electron energies of 86.9, 119.7, 147.6, and 168.2 MeV. A misprint in Miller and Robinson's paper has been corrected in plotting the above data. The point at $E = 86.9$ MeV and $\theta = 44.1^\circ$ should read $\sigma = 0.646 \times 10^{-26}$ cm²/sr.

¹³ K. W. Ford and D. L. Hill, Ann. Rev. Nucl. Sci. 5, 25 (1955).¹⁴ R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).

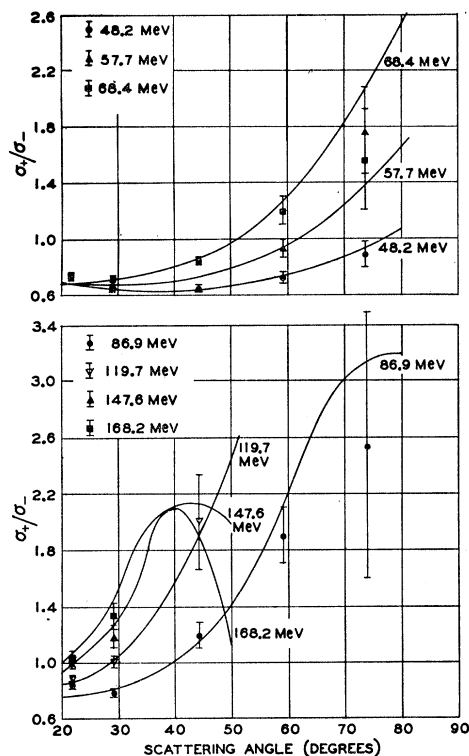


FIG. 8. Ratio of positron to electron scattering cross sections for Pb at energies between 48.2 and 168.2 MeV. The data are taken from Table V of the paper by Miller and Robinson while the curves are obtained by dividing the appropriate calculated cross-section values.

recent electron and positron scattering data,^{3,7,15,16} and also with muonic x-ray results.⁸

As the figures indicate, both electron and positron fits are poor at 48.2, 57.7, and 68.4 MeV, while much better fits are obtained for the higher energies starting at 86.9 MeV for both electrons and positrons. The graphs indicate that renormalizing the data will not substantially improve the lower energy fits as the theoretical curves agree with the data at the smaller scattering angles, but differ by as much as a factor of 2 at the larger angles.

The experimental procedure of Miller and Robinson involves the production of electrons and positrons in the target by the bremsstrahlung radiation from a betatron and measures the cross section only indirectly.¹⁷ The quantity measured is the product of the number dC/dE of electrons or positrons produced in the forward direction in the target times the differential scattering cross section. The fact that the ratio of positron to electron scattering cross sections σ_+/σ_- is in agreement with theory may be an indication that

¹⁵ L. R. B. Elton, *Nuclear Sizes* (Oxford University Press, London, 1961).

¹⁶ H. Crannell, R. Helm, H. Kendall, J. Oeser, and M. Yearian, *Phys. Rev.* **121**, 283 (1961).

¹⁷ The authors are indebted to Professor G. Breit for a discussion of various radiative corrections and to Dr. G. A. Peterson for conversations concerning the experimental techniques of Ref. 1.

the factor dC/dE is in error. These data, together with the ratios calculated using the same charge distribution as above, are shown in Fig. 8. The agreement between theory and experiment is good except for several of the 21.7° values and the 57.7- and 68.4-MeV points at 73.8°. Theory and experiment were also compared using the ratio $R = (\sigma_- - \sigma_+)/(\sigma_- + \sigma_+)$, which has been employed at Stanford.^{2,3} There was no disagreement between R calculated from the measurements of σ_- and σ_+ and the values calculated using the Woods-Saxon distribution. The large statistical errors for R in this case reduce the significance of this agreement. It is worthwhile to emphasize that in this case more information can be obtained from the comparison of the individual positron and electron cross sections with theory than from the comparison of the σ_+/σ_- ratio or the quantity R . This should, of course, be true in any case for which the first-order Born approximation is valid, since in such cases the information about the nucleus is contained in a form factor which cancels in the ratio of the cross sections.

B. Attempts to Improve the Fit

The parameters of the Woods-Saxon charge distribution were varied in an attempt to find a set consistent with the electron and positron cross sections of Miller

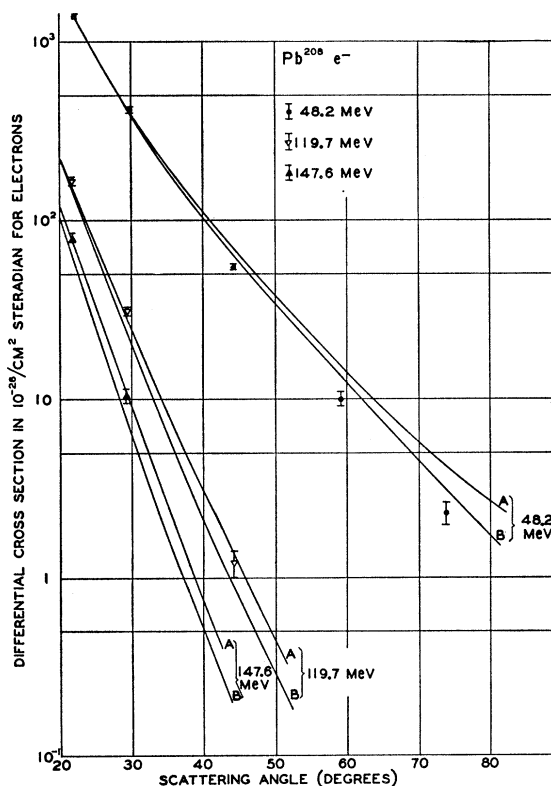


FIG. 9. Sensitivity of the electron scattering cross section to the parameter c for the Woods-Saxon distribution. The theoretical curves labeled A correspond to $c=6.48$ and $z=0.535$, while those labeled B correspond to $c=7.14$ and $z=0.535$.

and Robinson in the energy range 48.2–168.2 MeV. A fit to these data was also sought employing the “wine-bottle” distribution described in Sec. II of this paper.

Preliminary calculations showed that the cross sections, particularly at the low energies, are more sensitive to changes in the parameter c_0 than to changes in t and w . Therefore, considering the large discrepancy between experiment and theory at the low energies, only changes in c_0 were considered.

It was found that no fit to the low-energy positron data could be found by varying c_0 . This is a result of the positron's being influenced by the whole nuclear charge with negligible screening at low energy.¹ The parameter c_0 would have to be increased by at least 50% to bring the cross sections at $\theta = 70^\circ$ into agreement with the data, but this change would destroy the good agreement at the higher energies and conflicts with presently accepted values for the nuclear radius.^{8,9,13,14}

A somewhat less drastic change of c_0 is called for by the low-energy electron-scattering data. Increasing c_0 from 1.097 to 1.207 results in the shift in the curves from “A” to “B” as shown in Fig. 9. This change is in the right direction at low energies but causes disagreement at the higher energies and is sharply at variance with the result^{9,11} $c_0 = (1.097 \pm 2)\%$ for the Woods-Saxon distribution. Similar results were obtained when c_0 was varied in the “wine-bottle” distribution.

V. SUMMARY AND CONCLUSIONS

From the comparison of the WS and WB cross sections at various energies, it is found that the differ-

ence between the two cross sections reaches a maximum of about 30% at incident momenta of ~ 100 MeV/ c , provided that momentum transfers of more than 1.5 F⁻¹ are not included in the comparisons.

A calculation of electron and positron scattering cross sections employing the currently accepted static, spherically symmetric nuclear charge distribution disagrees by as much as a factor of 2 with the experimental values of Miller and Robinson in the 50–70-MeV energy range while good agreement is obtained in the 85–170-MeV energy range. It is concluded from the above considerations that additional measurements of electron or positron cross sections at energies between 50 and 180 MeV are desirable not only in order to clarify the discrepancy with the results of Miller and Robinson, but also because comparison with theory at various energies may yield information on the accuracy of the assumptions on which the calculations are based.

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Neutron Binding Energies in Heavy Nuclei

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Neutron binding energies in 53 nuclei of mass 81–209 were determined with 15 keV accuracy by measurements of Q values for (d,p) and (d,t) reactions. In seven cases, there are large discrepancies with previously accepted values; these are discussed in detail. There are strong indications of subshell closure at 56 neutrons in zirconium and at 64 neutrons in tin, but these indications are much weaker at 56 neutrons in molybdenum, and nonexistent at 64 neutrons in cadmium.

INTRODUCTION

NUCLEON binding energies have played an important part historically in nuclear structure physics, and their importance has hardly diminished up to the present. Only with the most recent developments in shell-model calculational techniques has it become possible to make reasonably accurate predictions of them,^{1,2} and the interest in these calculations continues

at a high level. The situation as regards experimental determinations of nucleon binding energies has been very satisfactory in the mass region $A \lesssim 70$ for some time, but until a year or two ago, measurements in heavier mass regions carried rather large errors. It was in order to improve this situation that the work herein described was undertaken.

In this paper, we report on determinations of neutron E . Baranger, M. Veneroni, M. Barnager, and J. V. Gillet, *Phys. Letters* **4**, 119 (1963).

² I. Talmi, *Rev. Mod. Phys.* **34**, 704 (1962).

¹ L. Kisslinger and R. A. Sorenson, *Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd.* **32**, No. 9 (1960). R. Arvieu,