Evaporation of Alpha Particles in the Interaction of High-Energy Protons with AgBr†

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A calculation of the energy spectra, angular distributions, and multiplicities of alpha particles evaporated in the interactions of AgBr with 1- and 2-GeV protons has been performed. A Monte Carlo evaporation calculation was combined with the results of the cascade calculation of Metropolis *et al.* to obtain these data. Special attention has been paid to the motion of the evaporating nuclei and the calculated spectra are obtained in the laboratory system. Consequently, a direct comparison with experiment is possible. The results are compared with the data of Katcoff and co-workers and good over-all agreement is obtained. In particular, the agreement of the calculated and experimental energy spectra below 15 MeV indicates that it is not necessary to invoke the principle of barrier reduction at high-excitation energies to account for the emission of low-energy α particles. The calculation predicts too few α particles with energies above 25 MeV, and it is concluded that the cascade process accounts, in part, for the high-energy portion of the spectrum.

I. INTRODUCTION

INVESTIGATIONS of the interactions of high-energy protons with heavy emulsion nuclei have greatly contributed to the understanding of nuclear reactions. Information has been obtained[1–20] on the emission probability, energy spectrum, and angular distribution of a variety of emitted particles and heavy fragments.

The study of alpha-particle emission in these reactions is of special interest because of the information it can provide about the evaporation process at high excitation energies. Previous comparisons of alpha-particle emission with evaporation theory have indicated two serious discrepancies between experiment and calculation. The emission of sub-barrier particles has been found to be far more probable than expected as has also been that of very-high-energy α particles. The first phenomenon has been attributed[21,22] to the reduction of the Coulomb barrier at high-excitation energy resulting from thermal expansion of the nucleus or from surface oscillations. The second effect has been explained in terms of the emission of α particles during the intranuclear cascade.[15] These explanations have usually not taken complete account of the motion of the emitting nucleus. It is thus conceivable that the emission of both very-low and very-high-energy α particles in the laboratory system can be accounted for in terms of the partial cancellation or addition of the velocities of the emitted α particle and the emitting nucleus. In fact, an approximate calculation by Baker *et al.*[13] indicates that center-of-mass motion effects may indeed partly account for these discrepancies.

In recent years it has become possible to perform fairly realistic calculations of nuclear evaporation in high-energy reactions. The Monte Carlo cascade calculations of Metropolis *et al.*[23] appear to have been moderately successful in predicting the distribution of residual nuclei and excitation energies following cascades initiated by high-energy protons. These calcula-

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tions have been extended by Porile[24] to include the distribution of recoil momenta of the residual nuclei. These results may be coupled with Monte Carlo evaporation calculations that take into account the recoil velocity of the residual nucleus at each step of the evaporation process.[25] In this fashion, it is possible to obtain energy spectra and angular distributions that are directly comparable with those observed in the laboratory. We report in this paper the results of such a calculation for α particles evaporated in the interaction of silver and bromine with 1- and 2-GeV protons. These results are directly comparable with the recent data of Katcoff and co-workers[13,17] and should permit a more definitive answer to the questions of barrier reduction and cascade emission than has heretofore been possible.

II. THE CALCULATION

The starting nuclei for the evaporation calculation were obtained from the Monte Carlo cascade calculation of Metropolis *et al.*[23] The most useful data for the purposes of this calculation were those for 0.96- and 1.84-GeV protons incident on Ru^{100}. The residual nuclei resulting from these interactions were shifted in charge and mass number to correspond to targets of Ag^{107}, Ag^{109}, Br^{79}, and Br^{81}. In addition, the excitation energies of the residual nuclei were increased or decreased by 5–15% in order to take account of the calculated[23] variation of average excitation energy with target mass number. It has been previously demonstrated[25] that this shifting procedure does not introduce any significant distortions into the calculated distributions. The results for the silver targets were weighted by a factor of 1.2 relative to those for the bromine targets because of the larger total reaction cross section of silver. The results of approximately 800 cascades for Ru^{100} were available at 0.96 GeV and those of 400 cascades at 1.84 GeV.

The velocity components of the residual nuclei were taken from Porile's calculation.[24] It has been previously pointed out[25] that, on the average, this calculation overestimates the value of V_y, one of the transverse components of velocity, by about 40%. This overestimate results from the random choice of the sign of the y component of momentum of the cascade particles. The calculation of Metropolis *et al.*[23] had not kept track of this sign so that a somewhat arbitrary choice became necessary. The values of V_y, obtained from Porile's calculation,[24] were accordingly reduced by 40%, although the effect of this change on the final results is minor.

The evaporation calculation consisted of an adaptation of the Monte Carlo calculation due to Dostrovsky *et al.*[26] The modifications introduced in order to keep account of the kinematics of the evaporation process have been described elsewhere.[25] Briefly, the calculated channel energy is aportioned between the emitted particle and the residual nucleus by conserving the linear momentum of the system. The choice of two random numbers determines the direction of motion of the evaporated particle in the system of the emitting nucleus on the assumption of isotropic evaporation. The energy and direction of the evaporated particle in the laboratory system are determined by means of vectorial addition of its velocity to that of the emitting nucleus. The latter is obtained by vectorial addition of the velocity imparted in the cascade process and that resulting from all prior evaporation steps. The velocity in the laboratory system of the nth evaporated particle V_{L_n}, is thus given by the expression

$$\mathbf{V}_{L_n}=\left[\frac{2M_{R_n}E_n}{M_{P_n}(M_{R_n}+M_{P_n})}\right]^{1/2}\mathbf{u}+\sum_{i=1}^{n-1}\mathbf{v}_i+\mathbf{v}_c\,, \qquad (1)$$

where M_{R_n} is the mass of the residual nucleus resulting from the evaporation of n particles, M_{P_n} is the mass of the nth evaporated particle, E_n is the channel energy for the nth evaporation step, $\mathbf{v}_i$ is the recoil velocity due to the evaporation of the ith particle, $\mathbf{v}_c$ is the velocity of the residual nucleus following the cascade process, and $\mathbf{u}$ is a unit vector.

The emission probabilities and channel energies for charged particles depend strongly on the parameters used to approximate the inverse reaction cross section. The formalism developed by Dostrovsky *et al.*[26] uses the expression

$$\sigma_c=\sigma_g(1+c)[1-(kV/\epsilon)] \qquad (2)$$

to obtain this cross section. In this expression σ_g is the geometric cross section, V is the classical Coulomb barrier, and ϵ is the channel energy. The values of the constants c and k were determined by fitting this expression to continuum theory values. Calculations using Eq. (2) have been shown to give fairly good agreement with experimental cross sections for reactions involving charged particle emission.[26,27] On the other hand, comparisons with energy spectra of evaporated protons and alpha particles have indicated gross discrepancies.[28] These can be attributed, at least in part, to the sharp cutoff to the emission of low-energy particles inherent in the use of Eq. (2). We have attempted to correct this shortcoming of the calculation by adjusting the values of c and k while retaining the form of Eq. (2). In order to make this adjustment, we have attempted to fit the energy spectra of α particles obtained by Sherr and Brady[29] in their study of (p,α) reactions on medium

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TABLE I. Parameters in Eq. (2) for α-particle cross sections.

Z	Dostrovsky *et al.* (Ref. 26)		Present work	
	c	k	c	k
30	0.10	0.91	0.0	0.70
50	0.08	0.97	0.0	0.70

weight elements at 17.5 MeV. These energy spectra have been shown to be in good agreement with evaporation calculations[29] and the angular distributions in the center-of-mass system are essentially isotropic. Accordingly, any calculation of the evaporation process in high-energy reactions should first be able to account for the less complicated situation of compound nuclear reactions at low bombarding energies.

The calculated differential energy spectra of alpha particles in the center-of-mass system are compared in Figs. 1 and 2 with the corresponding results of Sherr and Brady[29] for the (p,α) reactions on Fe^{56} and Rh^{103}. Calculated spectra are shown for both the inverse reaction cross-section parameters given by Dostrovsky *et al.*[26] and the adjusted values used in this work. It is seen that the calculated spectra based on the parameters of Dostrovsky *et al.*[26] are in poor agreement with experiment. The calculated peak energies are some 2–4 MeV higher than the experimental values, while the minimum α-particle energies are displaced to higher values by about 3–5 MeV. Suitable adjustments in the values of c and k lead to the calculated spectra given by the solid curves in Figs. 1 and 2. The peak energies predicted by this calculation agree to within 0.5 MeV with the experimental values, and the discrepancies at the low-energy end of the spectra are greatly reduced. It may also be noted that the calculated spectra based on the adjusted parameters agree more closely with the experimental values in magnitude as well as in shape. This fact is not particularly significant since the magnitude of the cross section depends strongly on the pairing energy parameter[30] and small adjustments in the latter could change the situation significantly.

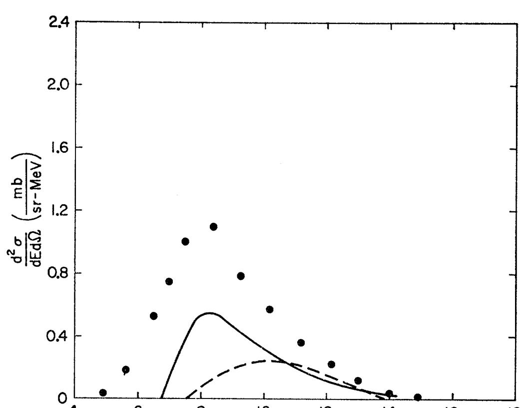

FIG. 1. Comparison of the energy spectrum of α particles emitted in the reaction $Fe^{56}(p,\alpha)$ ($E_p=17.5$ MeV) with calculation. The experimental points of Sherr and Brady (Ref. 29) are plotted as a function of α-particle energy in the center-of-mass system. (Reference 29 gives this plot in terms of the channel energy.) Solid curve—calculated spectrum obtained with the present set of inverse reaction cross-section parameters. Dashed curve—calculated spectrum obtained with the corresponding set of parameters due to Dostrovsky *et al.* (Ref. 26).

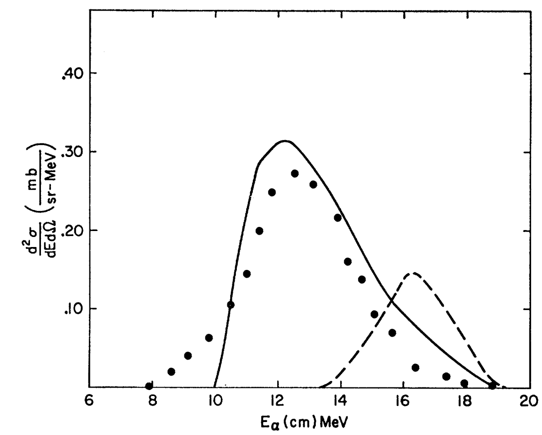

FIG. 2. Comparison of the energy spectrum of α particles emitted in the reaction $Rh^{103}(p,\alpha)$ ($E_p=17.5$ MeV) with calculation. The various curves have the same meaning as in Fig. 1.

The values of the parameters used in the above calculation are compared with the values of Dostrovsky *et al.*[26] in Table I. In both calculations the nuclear radius parameter r_0 was taken as 1.5 F, the level density parameter a was given by $a=A/20$, and the pairing energies were taken from Cameron.[31] The values of c and k used in the computation of He^3 emission were obtained from our values for α particles in the manner outlined by Dostrovsky *et al.*[26] The parameters for the emission cross sections of neutrons, protons, deuterons, and tritons were the same as those used by Dostrovsky *et al.*[26]

The evaporation calculation for AgBr was performed, then, with the adjusted constants for the inverse reaction cross section for He nuclei. The level density parameter was taken as $A/10$, since this value has been shown to give the best agreement for high-energy reactions.[32] In some cases the calculation was also performed with $a=A/20$ in order to determine the sensitivity of the results to the value of a. In order to improve the statistical accuracy of the results, three evaporation calculations were performed for each starting nuclide.

The evaporation calculation normally considers the emission of the six lightest particles through He^4. In the present calculation the evaporation of heavier particles was included, since the results of Baker and

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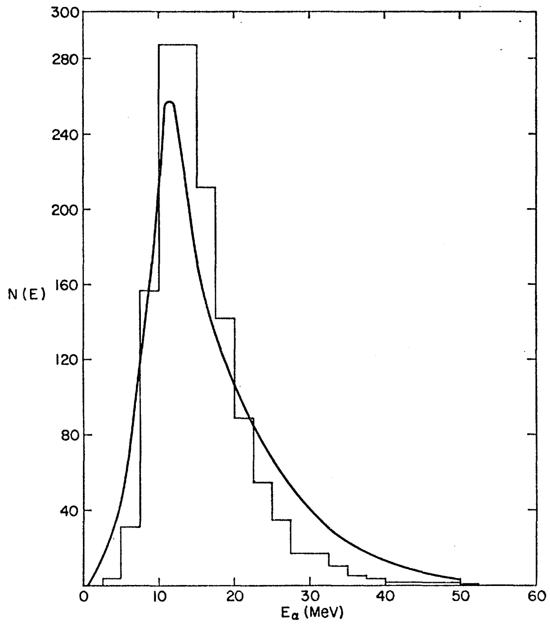

FIG. 3. Comparison of the calculated energy spectrum of α particles emitted in the interaction of AgBr with 1-GeV protons with the experimental spectrum of Katcoff and co-workers (Refs. 13, 17). Both spectra are in the laboratory system and have been normalized to the same area. Solid curve—experimental spectrum; Histogram—calculated spectrum.

Katcoff[17] indicate an appreciable probability for the emission of fragments with $Z=3-6$ in high-energy reactions. Since the calculation of the emission probabilities of all nuclides with $Z=3-6$, both in their ground and bound excited states, was found to be prohibitive in terms of computer time, a simplifying procedure was adopted. This procedure has been described in detail by Porile and Tanaka.[25] It consists, briefly, of replacing all particles with $A=6-10$ by Li^7 in its ground state. The emission probability of the latter is multiplied by the average ratio of the summed emission probabilities of the $A=6-10$ particles in their ground and bound excited states to that of Li^7 in its ground state. The average value of this ratio was found to be 4.5 on the basis of several cascades in which all the emission probabilities were computed. The calculation of heavy particle emission probabilities was based on the treatment of Dostrovsky *et al.*[33]

III. COMPARISON WITH EXPERIMENT

Katcoff and co-workers[13,17] have measured the multiplicity, energy spectrum, and angular distribution of α particles emitted in the reactions of AgBr with 1-, 2-, and 3-GeV protons. Their results, in fact, also include the contribution of other particles with $Z=2$. Although a preliminary calculation showed that He^6 emission is negligible compared to that of He^4, there is a significant contribution due to He^3. Accordingly, our calculation includes results for both He^3 and He^4. The results of Katcoff and collaborators are divided into three groups according to whether only He nuclei, He nuclei and light fragments, or He nuclei and fission fragments are emitted. Since fission is not taken into account in our calculation, a comparison is only made with events falling in the first two categories.

The experimental energy spectra obtained at 1 and 2 GeV are compared with the calculated values at 0.96 and 1.84 GeV in Figs. 3 and 4, respectively. It is seen that the calculated peak energies are in excellent agreement with the experimental values. It is of interest to note that the most probable α-particle energy is about 12 MeV while the classical Coulomb barrier against α-particle emission from bromine, for instance, is 13.5 MeV. Both the experimental and calculated spectra extend down to about 2 MeV, although it appears that the relative number of very-low-energy α particles is somewhat underestimated by the calculation. This underestimate is consistent with the failure to account for the lowest energy α particles in the (p,α) experiments at 17.5 MeV. It may be concluded that although a large fraction of the emitted α particles have sub-barrier energies, this fact in no way requires the postulate of barrier reduction at high-excitation energies.

We attribute the emission of low-energy α particles to the following three factors. First, the evaporation process actually favors the emission of α particles having energies somewhat below the Coulomb barrier. Although the cross section for α-particle capture decreases sharply below the Coulomb barrier, the calculated energy spectrum is determined by the product of a number of terms, among which is the level density of the residual nucleus. The latter has an inverse exponential dependence on the evaporation energy

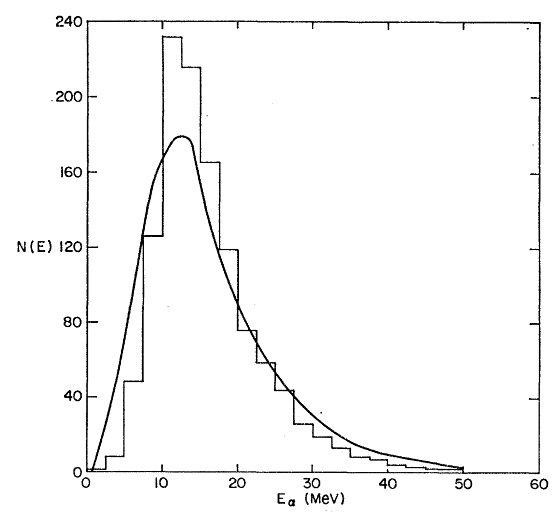

FIG. 4. Comparison of calculated and experimental spectra of α particles emitted in the interaction of AgBr with 2-GeV protons. See Fig. 3 for details.

[33] I. Dostrovsky, Z. Fraenkel, and P. Rabinowitz, Phys. Rev. **118**, 791 (1960).

and thus favors sub-barrier emission. The results of Sherr and Brady,[29] shown in Figs. 1 and 2, thus indicate that α particles with energies as low as 5 and 8 MeV are evaporated from cobalt and palladium compound nuclei, respectively. Second, the probability for α-particle evaporation remains fairly large down to rather low-excitation energies in the mass range of interest. Evaporation energies as low as those obtained in the $Fe^{56}(p,\alpha)$ experiment can therefore be expected. Third, the motion of the evaporating nuclei leads to a broader energy spectrum of α particles in the laboratory system than in the moving frame of reference. This is shown in Fig. 5 in which the laboratory spectrum of α particles is compared with that obtained in the system of the moving nuclei, i.e., the system in which the evaporating nuclides are at rest. The full widths at half-maximum of these two spectra thus are about 11 and 6 MeV, respectively. It is interesting to note that although the relative yield of low-energy α particles ($E<10$ MeV) is nearly a factor of 2 larger in the laboratory system, the peak energy is practically the same in both frames of reference.

The data in Figs. 3 and 4 indicate that the calculation tends to underestimate the number of α particles emitted with energies above 25 MeV. In order to obtain the most meaningful estimate of this discrepancy, it seemed reasonable to renormalize the calculated spectra so that they would be in agreement with experimental values at very low energies. If this is done, we find that the relative number of α particles with energies between 25 and 50 MeV observed by Baker *et al.*[13,17] exceeds the corresponding calculated number by factors of 2.5 ± 0.5 at 1 GeV and 1.3 ± 0.3 at 2 GeV. It is reasonable to attribute this excess of high-energy α particles to the cascade process. We do not feel, however, that quantitative estimates of this contribution are warranted as the above ratios will be shown to depend on the value of a. It should be pointed out that the experimental results have a cutoff at 50 MeV and that α particles having higher energies are known to be emitted.

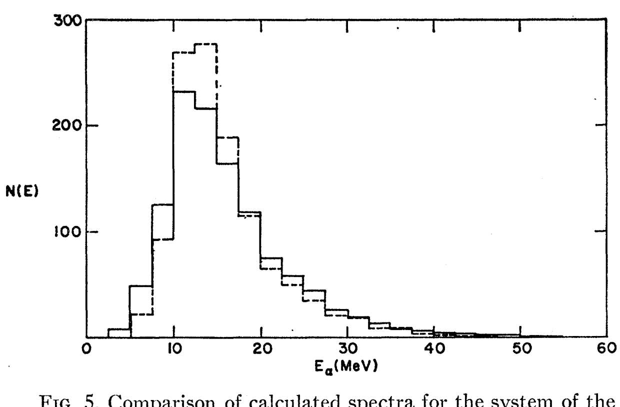

FIG. 5. Comparison of calculated spectra for the system of the moving nuclei (dashed histogram) and for the laboratory system (solid histogram). Both spectra are for 1.8-GeV incident protons.

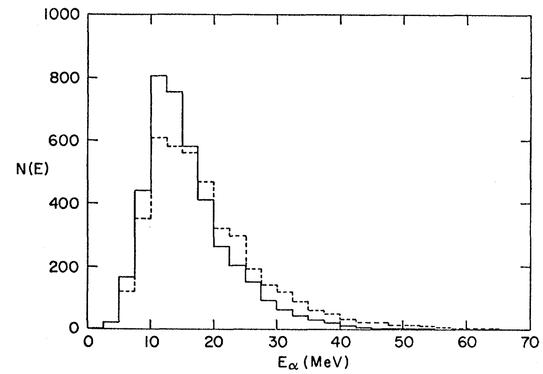

FIG. 6. Effect of the level density parameter on the calculated α-particle spectrum at 1.8 GeV. Solid histogram—$a=A/10$; dashed histogram—$a=A/20$.

Skjeggestad and Sorensen,[10] for instance, report that the probability for the emission of 100-MeV α particles in high-energy interactions with heavy emulsion nuclei is about 25% that of 50-MeV α particles. The calculated evaporation spectra indicate that the emission of α particles above 50 MeV is very unlikely, so that the latter can be attributed almost exclusively to the cascade process.

The above conclusions concerning the emission of high-energy α particles are fairly sensitive to the value of the level density parameter used in the calculation. A comparison of spectra calculated with $a=A/10$ and $a=A/20$ is given in Fig. 6. It is seen that the spectrum obtained with the smaller value of a is shifted to higher energies. Although the value of the most probable α-particle energy remains essentially unchanged, it is seen that the relative number of 25–50-MeV α particles increases by nearly a factor of 2. Even the choice of $a=A/20$, however, cannot account for the evaporation of 100-MeV α particles. Also, the disagreement with experiment at the low-energy end of the spectrum becomes considerably more pronounced. We therefore believe that the discrepancy between experiment and calculation at the high-energy end of the spectrum is real and indicates the emission of α particles in the cascade process.

The experimental and calculated angular distributions of α particles are compared in Figs. 7 and 8. The calculated distributions are seen to be in very good agreement with experiment at both energies. The effect of the predominantly forward motion of the residual nuclei following the cascade can be seen in the forward peaking of the angular distributions. The magnitude of this effect is most easily seen from the values of the ratio of forward-to-backward emission, F/B. These values are summarized in Table II. It is seen that the calculation predicts a 20% deviation from symmetry about 90° due to the cascade-induced recoil velocity, in very good agreement with experiment.

Although the effect of the cascade process on the

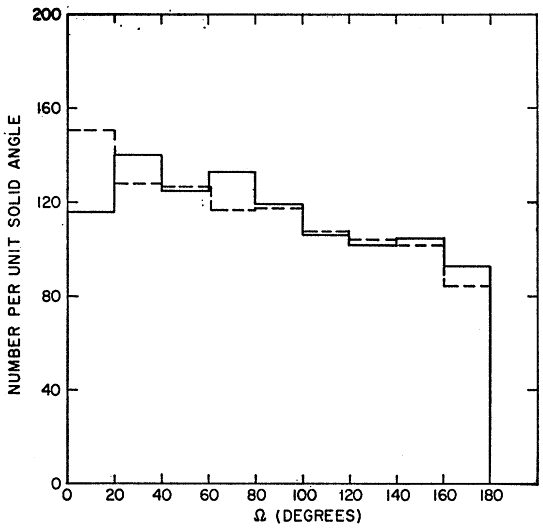

FIG. 7. Comparison of the calculated and experimental angular distributions of α particles for 1-GeV protons. Solid histogram—experimental data; dashed histogram—calculated results. The curves have been normalized to the same area.

TABLE II. Calculated and experimental F/B ratios and multiplicity values.

Proton energy		Experimental[a]	Calculated
1 GeV	F/B	1.15±0.09	1.20±0.04
	F/B (E_α<10 MeV)	0.65±0.10	0.48±0.04
	F/B (E_α>25 MeV)	2.1 ±0.5	3.09±0.43
	α/α event	1.08±0.06	1.61±0.07 (1.35±0.05)[b]
	α/event	...	0.59±0.03 (0.51±0.02)[b]
2 GeV	F/B	1.15±0.09	1.20±0.04
	F/B (E_α<10 MeV)	0.69±0.11	0.48±0.04
	F/B (E_α>25 MeV)	2.7 ±0.6	2.53±0.31
	α/α event	1.62±0.09	1.85±0.08 (1.58±0.07)[b]
	α/event	...	0.93±0.04 (0.79±0.03)[b]

[a] From Refs. 13 and 17 and unpublished data of Baker and Katcoff.
[b] These values were obtained with the inverse reaction cross section constants of Dostrovsky *et al.* (Ref. 26).

angular distribution of α particles is, on the average, fairly minor, much larger effects may be noted if α particles with particular energies are selected. For instance, the calculated values of F/B for low-energy α particles are only about 0.5, in fairly good agreement with experiment. This result confirms the conclusion that low-energy α particles arise in large measure from the partial cancellation of the velocity of the emitted α particle and that of the forward moving emitting nucleus. On the other hand, the calculated F/B values for 25–50-MeV α particles are close to 3, again in fairly good agreement with experiment. These large values of F/B imply that up to approximately 25% of the energy of the α particles is due to the forward motion of the emitting nuclei. This result is in qualitative agreement with the data presented in Fig. 5. The comparison of the calculated and experimental energy spectra has indicated that a substantial fraction of the high-energy α particles are emitted in the cascade. In view of this fact, the agreement of the calculated and experimental F/B values is probably somewhat fortuitous. It should, perhaps, be pointed out that F/B values of 3 are quite reasonable for low-energy particles emitted in the cascade process. The calculations of Metropolis *et al.*[23] thus indicate that the F/B values for 30–90-MeV protons emitted in cascades initiated by 1.8-GeV protons range from 2 for U to 3 for Al.

The experimental and calculated multiplicity values are compared in Table II. The experimental values refer only to interactions in which at least one α particle is emitted. The calculated values are given both for events of this type as well as for all events. The calculation has been performed with both sets of inverse reaction cross-section parameters listed in Table I. The choice of the parameters due to Dostrovsky *et al.*[26] leads to values that are about 20% smaller than those obtained with the present set. In both cases, the calculation predicts that only about 50% of the interactions lead to the evaporation of α particles. The calculated multiplicities are in good agreement with experiment at 2 GeV, but are too high at 1 GeV. This fact is somewhat surprising as the Monte Carlo cascade calculations of Metropolis *et al.*[23] tend to give better agreement with other types of experiments[34] for proton energies below 1 GeV.

In summary, we have shown that a Monte Carlo cascade-evaporation calculation can account for most of the features of α-particle emission from heavy emulsion nuclei at 1 and 2 GeV. In order to obtain this

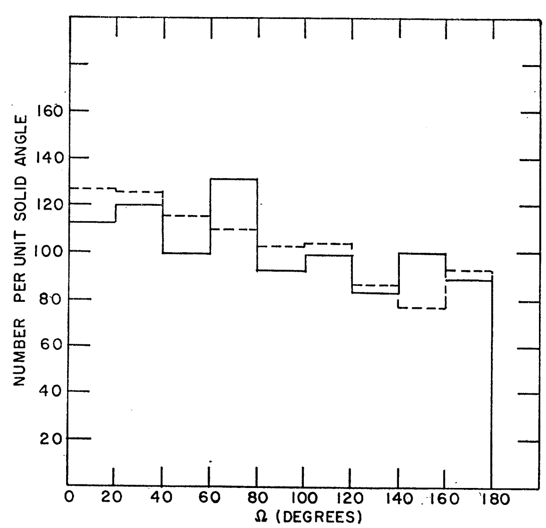

FIG. 8. Comparison of the calculated and experimental angular distributions of α particles for 2-GeV protons. See Fig. 7 for details.

[34] N. T. Porile, Phys. Rev. **125**, 1379 (1962).

agreement it was necessary to adjust the inverse reaction cross-section parameters to fit low-energy data and to take proper account of the motion of the emitting nuclides. The agreement of the calculated and experimental energy spectra implies that it is not necessary to invoke a reduction of the Coulomb barrier at high-excitation energies to account for the relatively large number of sub-barrier α particles. The comparison does reveal that an appreciable fraction of the α particles emitted with energies greater than 25 MeV are probably associated with the cascade rather than the evaporation phase of the reaction. However, the total number of such α particles accounts for only about 10% of the spectrum. It is of interest to note that although the evaporation calculation can only account for a fraction of the high-energy α particles, it does predict essentially the same F/B values as those observed. Once again, we attribute this fact to the motion of the emitting nuclides.

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Electron and Muon Scattering from Nuclear Charge Distributions at Incident Momenta Between 50 and 183 MeV/c

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The comparison of theoretical elastic-scattering cross sections of positrons and electrons from Woods-Saxon (WS) and "wine-bottle" (WB) charge distributions of the nucleus of Au, carried out at 183 MeV in a previous paper by the authors, is extended to lower energies and repeated for muons of comparable incident momenta. It is found that, for momentum transfers of less than 1.5 F^{-1}, the percent change of the cross section corresponding to a change from the WS to the WB charge distribution is largest, of the order of 30% for incident momenta of $\sim$100 MeV/c, particularly for positrons. At an electron energy of 50 MeV the cross section depends mainly on the mean-square radius of the nucleus, and an accuracy better than 5% is needed in order to determine additional nuclear charge distribution parameters. The mean-square radii of the WS and WB charge distributions differ by 6.5% while the corresponding electron cross sections at 50 MeV differ by a maximum of 15%. A comparison with experimental elastic positron and electron scattering cross sections for Pb measured by Miller and Robinson is carried out, and a systematic discrepancy with theory is found for both e^+ and e^- cross sections for the 50–70-MeV energy range, while theory and experiment agree well at 87 MeV and higher energies. The calculation consists of a conventional numerical phase-shift analysis based on the Dirac equation, and the nuclei are assumed to be static, spherically symmetric extended charge distributions.

I. INTRODUCTION

THE desirability of using positrons as well as electrons for the determination of nuclear charge distributions by means of elastic-scattering experiments has been explored recently both experimentally[1,2] and theoretically.[3,4] Positrons are expected to yield information independent of that obtained from electrons because the Coulomb repulsion for positrons reduces the wave function in the nuclear interior, enhancing the sensitivity to the "tail" of the charge distribution. The investigation presented by the authors in a previous note,[4] denoted by RF in what follows, has been extended to lower energies,[5] and it was found that electron cross sections continue to be sensitive to changes in the charge distribution at energies as low as 50 MeV. The usefulness of this result may be twofold. It serves to define the accuracy with which low-energy elastic-scattering experiments are to be carried out in order to yield information on the nuclear charge distribution,

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