# Possible Method for Isolating the K\*-Exchange Contribution to the Reaction $\gamma + p \rightarrow K^+ + \Lambda$

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It is pointed out that for the reaction  $\gamma + p \rightarrow K^+ + \Lambda$  induced by linearly polarized photons the differential cross section is independent of the  $K^+$ -exchange amplitude when the plane of scattering (the plane containing the initial and final momenta) is at 90° to the plane of polarization (the plane containing the photon polarization and momentum). Measurement and analysis of this cross section thus provides a relatively direct means of studying the unknown effects of the vector  $(K^*)^+$  and of extracting information on the vector and tensor form factors associated with its coupling to  $(p,\Lambda)$ . A detailed study is made of this possibility within the framework of a particular model for the reaction. The result concerning the differential cross section in the 90° plane is general; for example, in the reaction  $\gamma + p \rightarrow \pi^+ + n$ , the differential cross section in the 90° plane does not contain the  $\pi^+$ -exchange amplitude, but does contain the vector  $\rho^+$ -exchange amplitude.

#### I. INTRODUCTION

HE matrix element for the photoproduction on protons of a pion<sup>1</sup> or a kaon<sup>2</sup> tends to be a rather complicated entity, even under simplifying kinematical and dynamical circumstances. In the differential cross section, the contributions from the charge and anomalous magnetic moments of the baryons involved, from the charge of the boson, and from the exchange of vector bosons, give rise, squared and in interference, to a rather complex expression even before less welldefined dynamical effects, such as nearby resonances, are taken into account. As a consequence of this initial complexity, it is difficult to even partially isolate the effect on the differential cross section of a particular contribution to the matrix element, in the physical region of the reaction.<sup>3</sup> Consider, for example, the simplest strange particle photoproduction reaction.

$$\gamma + p \to K^+ + \Lambda. \tag{1}$$

In the expression for the differential cross section for this reaction induced by unpolarized photons on unpolarized protons the contributions from the exchange between  $(p,\Lambda)$  and  $(\gamma,K^+)$ , of a  $K^+$  and of a vector  $(K^*)^+$ , give rise to very similar characteristic features away from threshold. *Both* such exchange amplitudes, squared and in interference with each other and with of a  $\sin^2\theta$  term, where  $\theta$  is the center-of-mass scattering angle. Each exchange amplitude is inversely proportional to a propagator which is the square of the fourmomentum transfer for the reaction minus the square of the mass of the exchanged particle. In each case this dependence alone tends to produce a forward peaking of the produced  $K^+$  in the center-of-mass system, this effect being more marked from the  $K^+$ -exchange amplitude. If one knew the renormalized coupling of  $K^+$  to p and A, one could, in principle, fix the  $K^+$ exchange amplitude and attempt to extract from the experimental differential cross section information about the  $(K^*)^+$ -exchange amplitude.<sup>2</sup> However, it would clearly be a useful circumstance, if an experimental quantity could be studied in which the  $K^+$  and  $(K^*)^+$  amplitudes do not simultaneously occur. One purpose of this note is to point out the existence of such a quantity, namely the differential cross section for reaction (1) induced by linearly polarized photons on unpolarized protons, when the scattering plane (the plane of the initial and final momenta) lies at 90° to the plane of polarization (the plane containing the photon polarization vector and the beam momentum). This quantity does not contain the  $K^+$ -exchange amplitude, but does contain the  $(K^*)^+$  exchange amplitude. This is, of course, a general result; for example, in the reaction

charge and magnetic amplitudes, produce the growth

$$\gamma + p \to \pi^+ + n \,, \tag{2}$$

this experimental quantity does not contain the  $\pi^+$ exchange amplitude, but does contain the amplitude generated by the exchange of a vector  $\rho^+$ . In Sec. II we note the origin of this result. In Sec. III we examine reaction (1) in detail under the conditions of a model<sup>2</sup> recently used to correlate the Cornell data on this reaction in the region within 200 MeV of threshold. Under the conditions of this model, we show that, to a

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<sup>&</sup>lt;sup>1</sup>G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

<sup>&</sup>lt;sup>2</sup> T. K. Kuo, Phys. Rev. 129, 2264 (1963).

<sup>&</sup>lt;sup>3</sup> Extrapolation from physical measurements to the sometimes distant singularity in a single-particle exchange amplitude is, of course, a well-known "isolation" technique. The difficulties are exemplified in M. Moravscik, Phys. Rev. **125**, 734 (1962), where the effects of neutral vector meson exchange are sought in the data on the reaction  $\gamma + \rho \rightarrow \pi^0 + \rho$ . This is a favorable reaction because the  $\pi^0$ -exchange amplitude is absent from the start.

good approximation in the low-energy region, one can define an experimental quantity which is essentially determined by the  $(K^*)^+$ -exchange amplitude in the sense that as the latter goes to zero, the quantity in question goes to zero.

### **II. GENERAL RESULT**

The conventional general form<sup>1</sup> for the transition operator for reaction (1) in the center-of-mass system is given by

## $F = F_1 i \boldsymbol{\sigma} \cdot \hat{\boldsymbol{e}} + F_2 \boldsymbol{\sigma} \cdot \hat{\boldsymbol{q}} \boldsymbol{\sigma} \cdot \hat{\boldsymbol{k}} \times \hat{\boldsymbol{e}} + F_3 i \boldsymbol{\sigma} \cdot \hat{\boldsymbol{k}} \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{e}} + F_4 i \boldsymbol{\sigma} \cdot \hat{\boldsymbol{q}} \hat{\boldsymbol{q}} \cdot \hat{\boldsymbol{e}}.$ (3)

Here  $\hat{k}$  and  $\hat{q}$  are unit vectors in the direction of the incident photon and the final  $K^+$ , respectively;  $\hat{e}$  is a unit polarization vector for the photon; and  $\sigma$  is the baryon spin operator. The  $F_1 \cdots F_4$  are functions of the energy and scattering angle. The expression for the center-of-mass differential cross section for linearly polarized photons is

$$\frac{d\sigma}{d\Omega} = \left(\frac{q}{k}\right) \left[|F_1|^2 + |F_2|^2 - 2x \operatorname{Re}(F_1^*F_2) + (1 - x^2) \cos^2\varphi \{|F_3|^2 + |F_4|^2 + 2 \operatorname{Re}(F_1^*F_4) + 2 \operatorname{Re}(F_2^*F_3) + 2x \operatorname{Re}(F_3^*F_4)\}\right].$$
(4)

Here  $x = \cos \theta$  and  $\varphi$  is the azimuthal angle between the plane of polarization and the plane of scattering; k and q are the magnitudes of the initial and final three-momenta, respectively. Now consider the contribution to the transition operator from exchange of a vector  $(K^*)^+$  between  $(p, \Lambda)$  and  $(\gamma, K^+)$ 

$$F(K^*) = \frac{(eg_{\Lambda})}{t - (m^*)^2} \left(\frac{\lambda}{m_x}\right) \left[\hat{q} \cdot \hat{k} \times \hat{e}(kq) \left\{ a \left[1 + \frac{W - m}{W + m} \left(1 + \frac{qx}{E_{\Lambda} + m_{\Lambda}}\right) + \frac{\omega}{E_{\Lambda} + m_{\Lambda}}\right] \right. \\ \left. + 2Wg_T \left(1 - \frac{(W - m)qx}{(W + m)(E_{\Lambda} + m_{\Lambda})}\right) \right\} + i\boldsymbol{\sigma} \cdot \hat{e} \left\{ (k\omega)a \left[\left(\frac{W - m}{W + m}\right) \left(1 - \frac{qx}{\omega}\right) + \left(\frac{q}{E_{\Lambda} + m_{\Lambda}}\right) \left(-x + \frac{q}{\omega}\right)\right] \right. \\ \left. + (kq)b \frac{(W - m)q(1 - x^2)}{(W + m)(E_{\Lambda} + m_{\Lambda})} \right\} + i\boldsymbol{\sigma} \cdot \hat{k}\hat{q} \cdot \hat{e} \left\{ (kq) \left[\frac{a\omega}{E_{\Lambda} + m_{\Lambda}} + \frac{b(W - m)qx}{(W + m)(E_{\Lambda} + m_{\Lambda})}\right] \right\} \\ \left. + i\boldsymbol{\sigma} \cdot \hat{q}\hat{q} \cdot \hat{e} \left\{ - (kq) \left[\frac{aq}{E_{\Lambda} + m_{\Lambda}} + \frac{b(W - m)q}{(W + m)(E_{\Lambda} + m_{\Lambda})}\right] \right\} \right], \quad (5)$$

with  $t=m_K^2-2\omega k+2qkx$ . In Eq. (5),  $m^*$  is the  $K^*$  mass,  $m_K$  is the  $K^+$  mass, m is the proton mass,  $m_A$  is the  $\Lambda$  mass; in the center-of-mass system, W is the total energy;  $E_A$  is the total  $\Lambda$  energy, and  $\omega$  is the total  $K^+$  energy; e is the charge unit and  $g_A$  is the renormalized coupling constant for  $p \to K^+ + \Lambda$  (these factors have been factored out for later convenience). The other quantities in Eq. (5) are defined as follows:

$$a = -g_V - (m + m_\Lambda)g_T,$$
  

$$b = -g_V - (2W + m + m_\Lambda)g_T,$$
(6)

with the quantities  $g_V$ ,  $g_T$ , and  $\lambda/m_x$  defined by the following matrix elements for  $(K^*)^+ \to K^+ + \gamma$  and  $p \to \Lambda + (K^*)^{+4}$ :

$$(4\omega k)^{1/2} \langle K^+, \gamma | (j_K^*)_\nu | 0 \rangle = - (\lambda e/m_x) \epsilon_{\nu\mu\sigma\rho} q_\mu e_\sigma k_\rho, \qquad (7a)$$

$$(EE_{\Lambda}/mm_{\Lambda})^{1/2} \langle \Lambda | (j_{K^{*}})_{\mu} | p \rangle$$
  
=  $-g_{\Lambda} \overline{u}_{\Lambda} \{ g_{V} \gamma_{\mu} - g_{T} \sigma_{\mu\nu} (p_{\Lambda} - p)_{\nu} \} u$ , (7b)

with u and  $\bar{u}_{\Lambda}$  initial and final spinors, respectively,  $\sigma_{\mu\nu} = \frac{1}{2} (\gamma_{\mu} \gamma_{\nu} - \gamma_{\nu} \gamma_{\mu})$ , E the total proton energy, and

 $q_{\mu}, k_{\mu}, (p_{\Lambda})_{\mu}$  and  $p_{\mu}$  the four-momenta of the  $K^+$ , the photon, the  $\Lambda$ , and the proton, respectively. In (7a)  $m_x$  is a mass parameter,  $\lambda$  is then dimensionless. The quantities  $\lambda$ , and  $g_V$  and  $g_T$  are, in general, functions of  $(q+k)_{\mu^2}$  and  $(p_{\Lambda}-p)_{\mu^2}$ , respectively. In Eq. (5) and in all subsequent expressions, these functions will be understood to be evaluated at  $(q+k)_{\mu}^2 = (p_{\Lambda}-p)_{\mu}^{\lambda}$  $=(m^*)^2$ ;  $\lambda e/m_x$  is thus the renormalized coupling constant defined by the physical decay  $(K^*)^+ \rightarrow K^+ + \gamma$ and  $g_{\Lambda}g_{V}$  and  $g_{\Lambda}g_{T}$  are the renormalized vector and tensor couplings for  $p \to \Lambda + (K^*)^+$ . The expression in Eq. (5) is identical with that given by  $Kuo^2$ , but is put in a rather different form which makes manifest the momentum and angular dependence of the various terms, as well as their dependence on two particular combinations of the coupling constants a and b. The expression in Eq. (5) can readily be put into the form of Eq. (3) by use of the identity

$$\hat{q} \cdot \hat{k} \times \hat{e} = \boldsymbol{\sigma} \cdot \hat{q} \boldsymbol{\sigma} \cdot \hat{k} \times \hat{e} - i \boldsymbol{\sigma} \cdot \hat{k} \hat{q} \cdot \hat{e} + i \boldsymbol{\sigma} \cdot \hat{e} \hat{q} \cdot \hat{k}.$$
(8)

However, it is convenient to retain the form of Eq. (5), for now consider that all contributions other than  $(K^*)^+$  exchange to the transition operator have been written in the form of Eq. (3). To these contributions

<sup>&</sup>lt;sup>4</sup> In Eq. (7b) we have omitted a third form factor in the general expression for the matrix element. This form factor is related to  $g_V$  by gauge invariance and does not contribute to reaction (1).

add the last three terms on the right-hand side of Eq. (5) and call this F'. Call the first term on the righthand side of Eq. (5)  $F'(K^*)$ . Then, upon computing

the differential cross section for linearly polarized incident photons from the total transition operator  $F' + F'(K^*)$ , we have

$$d\sigma/d\Omega = (q/k) [(1-x^2) \sin^2\varphi \{ |F'(K^*)|^2 + 2 \operatorname{Re}F'(K^*)(F_2')^* \} + \{ |F_1'|^2 + |F_2'|^2 - 2x \operatorname{Re}(F_1')^*F_2' \} + (1-x^2) \cos^2\varphi \{ |F_3'|^2 + |F_4'|^2 + 2 \operatorname{Re}(F_1')^*F_4' + 2 \operatorname{Re}(F_2')^*F_3' + 2x \operatorname{Re}(F_3')^*F_4' \} ], \quad (9)$$

where the  $F_1' \cdots F_4'$  are defined by the decomposition of F' according to Eq. (3). The statement that the differential cross section at  $\varphi = \pi/2$  depends on the  $(K^*)^+$ -exchange amplitude but not on the  $K^+$ -exchange amplitude follows by noting that the latter amplitude contributes only to  $F_3'$  and  $F_4'$ ; i.e.,  $F_1'(K) = F_2'(K) = 0$ . The contributions  $F_{3}'(K)$  and  $F_{4}'(K)$  are given explicitly by the first terms on the right-hand sides of Eqs. (10c) and (10d) of Sec. III; they are those given by Kuo.<sup>2</sup>

Thus, experiments with linearly polarized photons in which the differential cross section at  $\varphi = \pi/2$  is measured, are analyzable in terms of the effects of the  $(K^*)^+$ -exchange amplitude without the need of having to separate these effects from the similar ones contributed by the  $K^+$ -exchange amplitude. Of course, even at  $\varphi = \pi/2$  the (K\*)+-exchange effects must still, in general, be disentangled from a morass of other contributing amplitudes; in particular, the effects of charge-generated amplitudes and amplitudes generated by perhaps poorly known magnetic moments. In the next section we show that, if there is some validity to a special model<sup>2</sup> recently used to correlate the Cornell data on reaction (1) with unpolarized photons, then an experimental quantity can be found that is essentially determined by the  $(K^*)^+$ -exchange amplitude, utilizing data for the reaction induced by unpolarized and by linearly polarized photons.

### III. APPLICATION TO A "LOW-ENERGY" MODEL

The presently available measurements<sup>5</sup> on the differential cross sections for reaction (1) at several energies up to about 170 MeV above threshold are adequately described by distributions of the form,  $a+b\cos\theta+c\cos^2\theta$ . The center-of-mass momentum of the  $K^+$  at the higher energies is still moderate, of the order ot 250 MeV/c. Therefore, an initial attempt to describe the situation might well be made by summing the real amplitudes generated by the charge, by the anomalous magnetic moments of the baryons, and by the  $K^+$ - and  $(K^*)^+$ -exchange mechanisms, and then adding the complex amplitudes describing the contributions of nearby resonances in the direct (or s) channel, that is, resonances with total energy near that of the total  $K^+\Lambda$  energy, and with spins  $J \leq \frac{3}{2}$ . This is essentially the starting point of the model of Kuo,<sup>2</sup> who, however, makes two further restrictions: (a) Only nearby resonances which lead to the  $K^+\Lambda$  system in S or P states are considered, and (b) the effects of the anomalous magnetic moment of the  $\Lambda$ ,  $\mu_{\Lambda}$ , and of the transition moment between  $\Sigma^0$  and  $\Lambda$ ,  $\mu_T$ , are neglected. A reasonable correlation of the differential cross sections at several energies is then achieved with a  $J=\frac{1}{2}$ , Pwave  $K^+\Lambda$  resonant state at about 1700 MeV induced by an  $M_1^-$  multipole.<sup>6</sup> Let us write down the amplitudes which appear in Eq. (9) without the above restrictions (a) and (b), including the  $T=\frac{1}{2}, J=\frac{3}{2}, D$ wave resonant state at 1510 MeV induced by an  $E_2^$ multipole.  $F'(K^*)$  is given by the coefficient of the quantity  $\hat{q} \cdot \hat{k} \times \hat{e}$  in Eq. (5). The other amplitudes are as follows:

$$F_{1}'(x) = \frac{(W^{2} - m^{2})}{8\pi W} \left(\frac{E_{\Lambda} + m_{\Lambda}}{2W}\right)^{1/2} (eg_{\Lambda}) \left[\frac{(1 - (W + m)(\mu/e))}{W^{2} - m^{2}} + \left(\frac{\lambda}{m_{x}}\right) a \left(\frac{W - m}{2W}\right) \frac{\omega}{t - (m^{*})^{2}} \right] \\ \times \left\{ \left(\frac{W - m}{W + m}\right) \left(1 - \frac{qx}{\omega}\right) + \left(\frac{q}{E_{\Lambda} + m_{\Lambda}}\right) \left(-x + \frac{q}{\omega}\right) \right\} + \left(\frac{\lambda}{m_{x}}\right) b \left(\frac{W - m}{W + m}\right) \left(\frac{q}{E_{\Lambda} + m_{\Lambda}}\right) \frac{q(1 - x^{2})}{t - (m^{*})^{2}} \right] \\ - \frac{(m_{\Lambda} - m_{\Sigma})(\mu_{T}/e)(g_{\Sigma}/g_{\Lambda})}{u - m_{\Sigma}^{2}} + \left\{-(W - m) + \frac{(W + m)(\omega - qx)}{W} - 2(m_{\Lambda} - m_{\Sigma})\right\} \\ \times \left\{\frac{(\mu_{\Lambda}/e)}{u - m_{\Lambda}^{2}} + \frac{(\mu_{T}/e)(g_{\Sigma}/g_{\Lambda})}{u - m_{\Sigma}^{2}}\right\} + E_{2}^{-}, \quad (10a)$$

<sup>&</sup>lt;sup>5</sup> R. L. Anderson, E. Gabathuler, D. Jones, B. D. McDaniel, and A. J. Sadoff, Phys. Rev. Letters 9, 131 (1962). <sup>6</sup> The original suggestion concerning this state was made by A. Kanazawa, Phys. Rev. 123, 997 (1961) based on an analysis of  $\pi^-+p \rightarrow \Lambda + K^0$ . It is, of course, possible that the effect in this region is due to the well-established  $T = \frac{1}{2}$ ,  $F_{5/2}$  state at 1680 MeV [see S. Hatsukade and H. Schnitzer, Phys. Rev. 132, 1301 (1963)]. However the experimental differential cross sections do not reflect the important cos<sup>3</sup>  $\theta$  and cos<sup>4</sup>  $\theta$  terms that such a state would, *in general*, give rise to. It is likely that the possible influence of this state would be more significant on the  $\Lambda$  polarization than on the differential cross section.

$$F_{2}'(x) = \frac{(W^{2} - m^{2})}{8\pi W} \left(\frac{E_{\Lambda} - m_{\Lambda}}{2W}\right)^{1/2} (eg_{\Lambda}) \left[ -\frac{(1 + (W + m)(\mu/e))}{W^{2} - m^{2}} + \frac{(m_{\Lambda} - m_{\Sigma})(\mu_{T}/e)(g_{\Sigma}/g_{\Lambda})}{u - m_{\Sigma}^{2}} + \left\{ -(W - m) + \frac{(W + m)(\omega - qx)}{2W} + 2(m_{\Lambda} - m_{\Sigma}) \right\} \left\{ \frac{(\mu_{\Lambda}/e)}{u - m_{\Lambda}^{2}} + \frac{(\mu_{T}/e)(g_{\Sigma}/g_{\Lambda})}{u - m_{\Sigma}^{2}} \right\} \right] + M_{1}^{-}, \quad (10b)$$

$$F_{3}'(x) = \frac{(W^{2} - m^{2})}{8\pi W} \left(\frac{E_{\Lambda} + m_{\Lambda}}{2W}\right)^{1/2} (eg_{\Lambda}) \left[ -\frac{2q}{(W + m)(t - m_{K}^{2})} + \left(\frac{\lambda}{m_{x}}\right) \left(\frac{1}{t - (m^{*})^{2}}\right) \left(\frac{q}{E_{\Lambda} + m_{\Lambda}}\right) \right] \times \left\{ a\omega \left(\frac{W + m}{2W}\right) + bqx \left(\frac{W - m}{2W}\right) \right\} + 2q \left\{ \frac{(\mu_{\Lambda}/e)}{u - m_{\Lambda}^{2}} + \frac{(\mu_{T}/e)(g_{\Sigma}/g_{\Lambda})}{u - m_{\Sigma}^{2}} \right\} \right], \quad (10c)$$

$$F_{4}'(x) = \frac{(W^{2} - m^{2})}{8\pi W} \left(\frac{E_{\Lambda} - m_{\Lambda}}{2W}\right)^{1/2} (eg_{\Lambda}) \left[ \frac{2q}{(W + m)(t - m_{K}^{2})} + \left(\frac{\lambda}{m_{x}}\right) \left(\frac{q}{t - (m^{*})^{2}}\right) \left(\frac{q}{E_{\Lambda} + m_{\Lambda}}\right) \times \left\{ a\left(\frac{W + m}{2W}\right) + b\left(\frac{W - m}{2W}\right) \right\} + 2q \left\{ \frac{(\mu_{\Lambda}/e)}{u - m_{\Lambda}^{2}} + \frac{(\mu_{T}/e)(g_{\Sigma}/g_{\Lambda})}{u - m_{\Sigma}^{2}} \right\} - 3E_{2}^{-}. \quad (10d)$$

In Eqs. (10),  $\mu$  is the proton anomalous moment, 1.8(e/2m),  $m_{\Sigma}$  is the  $\Sigma^0$  mass, and  $g_{\Sigma}$  is the renormalized coupling constant for  $p \to K^+ + \Sigma^0$ ;  $u = m_{\Lambda^2} - (W^2 - m^2)(E_{\Lambda} + qx)/W$ . The respective resonant amplitudes are denoted by  $M_1^-$  and  $E_2^-$  in Eqs. (10). Now if we study reaction (1) induced by linearly polarized photons and look at  $\varphi = \theta = \pi/2$ , we have, from Eq. (9),

$$\frac{d\sigma}{d\Omega} \left( \varphi = \frac{\pi}{2}, x = 0 \right) = \left( \frac{q}{k} \right) \left[ \left\{ |F'(K^*)|^2 + 2\operatorname{Re}F'(K^*)(F_2')^* \right\} + \left\{ |F_1'|^2 + |F_2'|^2 \right\} \right] = \sigma_0(x = 0) + \sigma_1(x = 0).$$
(11)

The terms in the first curly bracket,  $\sigma_0$  (x=0), on the right-hand side of Eq. (11) depend directly on the ( $K^*$ )<sup>+</sup>exchange amplitude,  $F'(K^*)$ . It would be nice if one could measure the terms in the second curly bracket,  $\sigma_1$  (x=0), and subtract them away. One cannot quite do this, but one can get close in a fortuitous circumstance. The differential cross section for reaction (1) with unpolarized photons is given by Eq. (9) integrated over  $\varphi$  and divided by  $2\pi$ , call this  $d\bar{\sigma}/d\Omega$ . Assume, as indicated by experiment, that  $d\bar{\sigma}/d\Omega = a + b \cos\theta + c \cos^2\theta$ . Consider one-half the sum of this differential cross section at  $\theta = 0$  and at  $\theta = \pi$ 

$$\frac{1}{2} \left[ d\bar{\sigma}/d\Omega(x=1) + d\bar{\sigma}/d\Omega(x=-1) \right] = \frac{1}{2} \left[ |F_1'(x=1)|^2 + |F_2'(x=1)|^2 - 2\operatorname{Re}(F_1'(x=1))^* F_2'(x=1) + |F_1'(x=-1)|^2 + |F_2'(x=-1)|^2 + 2\operatorname{Re}(F_1'(x=-1))^* F_2'(x=-1) \right] = \sigma_1(x=0) + C, \quad (12)$$

where C is simply defined by the expansion

$$|F_{1}'(x)|^{2} + |F_{2}'(x)|^{2} - 2x \operatorname{Re}(F_{1}'(x))^{*}F_{2}'(x) = A + Bx + Cx^{2}$$
(13)

and is explicitly computable from Eqs. (10) expanded to second order in x. The left-hand sides of Eqs. (12) and (11) are straightforward experimental cross sections measured with unpolarized and linearly polarized photons, respectively. Subtracting Eq. (12) from Eq. (11), we obtain

$$\frac{d\sigma}{d\Omega} \left(\varphi = \frac{\pi}{2}, x = 0\right) - \frac{1}{2} \left[\frac{d\bar{\sigma}}{d\Omega} (x = 1) + \frac{d\bar{\sigma}}{d\Omega} (x = -1)\right] = \left(\frac{q}{k}\right) \{|F'(K^*)|^2 + 2\operatorname{Re}F'(K^*)(F_2')^*\} - C.$$
(14)

The last step is to note that by direct computation from Eqs. (10a) and (10b), provided that the terms in  $\mu_{\Lambda}$  and  $\mu_{T}$  can be neglected, the quantity C depends directly on terms arising from the  $(K^*)^+$ -exchange amplitude, that is, just as the term in curly brackets on the right-hand side of Eq. (14), C depends no less than linearly (as well as quadratically) on terms generated by  $(K^*)^+$  exchange. We give the explicit expressions

$$|F'(K^*)|^2 + 2 \operatorname{Re} F'(K^*)(F_2')^* = \frac{(W+m)^2(W^2 - m^2)(E_\Lambda + m_\Lambda)q}{256\pi^2 W^4(y - (m^*)^2)} \left(\frac{\lambda}{m_x}\right) (eg_\Lambda)^2 \\ \times \left[a\left(1 + \frac{W-m}{W+m} + \frac{\omega}{E_\Lambda + m_\Lambda}\right) + 2Wg_T\right] \left\{\left(\frac{\lambda}{m_x}\right) \left[a\left(1 + \frac{W-m}{W+m} + \frac{\omega}{E_\Lambda + m_\Lambda}\right) + 2Wg_T\right](y - (m^*)^2)^{-1}(kq) - \frac{2(W-m)q(1 + (W+m)(\mu/e))}{(E_\Lambda + m_\Lambda)(W^2 - m^2)} + \frac{16\sqrt{2}\pi W^{3/2} \operatorname{Re} M_1^{-1}}{(W+m)(E_\Lambda + m_\Lambda)^{1/2}(eg_\Lambda)}\right\}, \quad (15)$$

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$$C = \frac{(W^2 - m^2)^2 (eg_{\Lambda})^2}{256\pi^2 W^4} \left(\frac{\lambda}{m_x}\right) \frac{\omega}{(m^*)^2 - y} \left\{ \left[\frac{\omega(E_{\Lambda} + m_{\Lambda})(W + m)^2(\lambda/m_x)a}{((m^*)^2 - y)(2W)} \right] \right. \\ \left. \times \left[ -\frac{q}{E_{\Lambda} + m_{\Lambda}} - \frac{(W - m)q}{(W + m)\omega} + \frac{q(W^2 - m^2)\left(\frac{W - m}{W + m} + \frac{q^2}{\omega(E_{\Lambda} + m_{\Lambda})}\left(1 + \frac{b}{a}\frac{W - m}{W + m}\right)\right)}{W((m^*)^2 - y)} \right] \\ \left. + 2q(W - m)\left\{ -\frac{(1 + (W + m)(\mu/e))}{W^2 - m^2} + \frac{8\sqrt{2}\pi W^{3/2}(E_{\Lambda} + m_{\Lambda})^{1/2}\operatorname{ReM}_1^{--}}{(W^2 - m^2)q(eg_{\Lambda})} \right\} \right] \right] \\ \left. \times \left[ -\frac{q}{E_{\Lambda} + m_{\Lambda}} - \frac{(W - m)q}{(W + m)\omega} + \frac{q(W^2 - m^2)\left(\frac{W - m}{W + m} + \frac{q^2}{\omega(E_{\Lambda} + m_{\Lambda})}\left(1 + \frac{b}{a}\frac{W - m}{W + m}\right)\right)}{W((m^*)^2 - y)} \right] a \\ \left. + 2bq^2 \frac{(W + m)}{\omega} \left[ \frac{(1 - (W + m)(\mu/e))}{W^2 - m^2} - \left(\frac{\lambda}{m_x}\right) \frac{\omega}{(m^*)^2 - y} \left\{ a\left(\frac{W + m}{2W}\right) \left(\frac{W - m}{W + m} + \frac{q^2}{\omega(E_{\Lambda} + m_{\Lambda})}\right) \right. \\ \left. + b\left(\frac{W - m}{2W}\right) \left(\frac{q^2}{\omega(E_{\Lambda} + m_{\Lambda})}\right) \right\} + \frac{8\sqrt{2}\pi W^{3/2}\operatorname{ReE}_2^{--}}{(eg_{\Lambda})(W^2 - m^2)(E_{\Lambda} + m_{\Lambda})^{1/2}} \right] \right\}, \quad (16)$$

where

$$y=m_K^2-\frac{\omega(W^2-m^2)}{W}.$$

Concerning the neglect of  $\mu_{\Lambda}$  and  $\mu_{T}$ , it must be said, that although this approximation was made in the Kuo model, it is not possible to justify it by the usual talk about these singularities in the crossed graphs (u channel) being distant. These contributions are not negligible in the physical region at the photon energies involved here for anomalous moments of the order of a nuclear magneton. However, one circumstance could diminish the contributions of these terms substantially,<sup>7</sup> that is, a partial cancellation between  $\mu_{\Lambda}$  and  $\mu_{T}$ . To within small terms of order of the  $\Sigma$ - $\Lambda$  mass difference divided by the photon energy, the quantity that enters is  $\mu_{\Lambda} + \mu_T (g_{\Sigma}/g_{\Lambda})$ . Early measurements<sup>8</sup> suggest  $\mu_{\Lambda} \sim -1$ in units of nuclear magnetons;  $\mu_T$  which controls the  $\Sigma^0$  lifetime is unknown experimentally. A theoretical estimate<sup>9</sup> gives  $\mu_T \sim +1$ . For  $g_2 \sim g_A$ , a substantial cancellation is possible. The results of this section, Eqs. (14), (15), and (16), could then be used as a basis for an experimental effort to determine the parameters associated with the K\*, namely,  $\lambda g_V/m_x$  and  $g_T/g_V$ , through

experiments with unpolarized and linearly polarized photons at several energies within 200 MeV of threshold. Experiments at photon laboratory energies around 1 BeV might allow neglect of the relatively distant resonant contributions from  $E_2^-$  and  $M_1^-$  (the existence of this resonance is, of course, not well established at this time).

#### IV. SUMMARY

We have pointed out that for reaction (1) induced by linearly polarized photons, the differential cross section is independent of the  $K^+$ -exchange amplitude when the plane of scattering is at  $90^{\circ}$  to the plane of polarization. Measurement and analysis of this cross section thus provides a relatively direct means of studying the unknown effects of the exchange of the vector  $(K^*)^+$ and of extracting the parameters associated with this process. Experiments at photon energies around 1 BeV may be analyzable in terms of the  $(K^*)^+$ -exchange amplitude and amplitudes generated by charge and anomalous moments, the latter amplitudes being calculable from a knowledge of the moments. Such experiments would be accessible to a large number of electron accelerators, not only the largest ones at CEA and Hamburg, but also the lower energy machines at the California Institute of Technology, Cornell, Frascati, Tokyo, and Lund.

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<sup>&</sup>lt;sup>7</sup> This has been noted in Ref. 2 (see footnote 13). <sup>8</sup> R. L. Cool, E. Jenkins, T. F. Kycia, D. A. Hill, L. Marshall, and R. A. Schluter, Phys. Rev. **127**, 2223 (1962). <sup>9</sup> J. Dreitlein and B. W. Lee, Phys. Rev. **124**, 1274 (1961).