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Test of Normal Statistics for K Mesons

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A direct experimental test of the statement that K mesons obey Bose statistics has not been carried out. For a certain class of reactions, we point out a purely geometrical experimental consequence of the statement that the wave function for a system of two spin zero K mesons is either completely symmetric or completely antisymmetric under simultaneous interchange of the isotopic spin and spatial variables of the two particles. The test can be used to investigate the possibility of a more general kind of statistics (parastatistics) for K mesons, in which case, the above experimental manifestation of complete symmetry or complete antisymmetry would not, in general, hold.

IN this note we point out a simply testable experimental consequence of the statement that the wave function for a system of two spin zero K mesons (K^+-K^+, K^+-K^0, K^0-K^0) is either completely symmetric or completely antisymmetric under simultaneous interchange of the isotopic spin and spatial variables of the two particles. These are the two cases of normal statistics, Bose-Einstein and Fermi-Dirac, respectively. It is, of course, assumed that K mesons obey Bose statistics. In the absence of a direct experimental test of the normal statistics there have been attempts to construct theories in which the strange particles obey a more general kind of statistics.[1,2] We propose a test for normal statistics that can be carried out on high-energy reactions that will be under intensive experimental study for other strong reasons. Three such reactions are

$$\Omega^- + \mathrm{D} \rightarrow \bar{K}^- + \bar{K}^0 + {}_{\Lambda}\mathrm{H}^3 \qquad (820\ \mathrm{MeV}), \tag{1}$$

$$\Omega^- + \mathrm{He}^4 \rightarrow \bar{K}^- + \bar{K}^0 + {}_{\Lambda}\mathrm{He}^5 \qquad (640\ \mathrm{MeV}), \tag{2}$$

$$\bar{\Lambda} + \mathrm{D} \rightarrow K^+ + K^0 + \Lambda \qquad (890\ \mathrm{MeV}). \tag{3}$$

The numbers to the right of reactions (1) and (2) are the laboratory kinetic energies of the Ω^- at threshold; reaction (3) is exothermic with the indicated energy release. Reactions (1) and (2) involve the study of the nuclear interactions of the newly discovered hypercharge -2 baryon.[3] Production of two of the lighter hyperfragments with the possibility of observing their subsequent interactions and decay in the chamber provides further usefulness to a study of these reactions. In addition, the possibility exists of observing hyperfragments with two and possibly three bound lambdas, produced in the following reactions:

$$\Omega^- + \mathrm{D} \rightarrow K^- + {}_{\Lambda\Lambda}\mathrm{H}^3 \qquad (\sim 150\ \mathrm{MeV}), \tag{4}$$

$$\Omega^- + \mathrm{He}^4 \rightarrow K^- + {}_{\Lambda\Lambda}\mathrm{He}^5 \qquad (\sim 150\ \mathrm{MeV}), \tag{5}$$

$$\Omega^- + \mathrm{He}^4 \rightarrow \pi^0 + {}_{\Lambda\Lambda\Lambda}\mathrm{H}^5 \qquad (\mathrm{exothermic}). \tag{6}$$

A search for reaction (6) provides a rather direct test of the statistics of the lambda hyperon. Fermi statistics and the Pauli principle forbid a third lambda from occupying the s shell and hence greatly inhibit the reaction. Parastatistics of multiplicity two[2] would allow up to four lambdas in the s shell. A study of reaction (3) would be useful in a search for resonant structure in mesonic states of hypercharge ± 2, a search of particular interest now in view of the existence of the Ω^-. Such possible resonant structure could also be looked for in reactions (1) and (2).

To illustrate the test of normal statistics for K mesons consider, for definiteness, its application to reaction (3). It is necessary to assume that isotopic spin is conserved in the reaction, to within small electromagnetic corrections. The initial state has total isotopic spin zero, since both projectile and target have isotopic spin zero. Since the final lambda has isotopic spin zero, the K^+-K^0 system must have isotopic spin zero. In the rest system of the two K mesons call $\mathbf{k}$ the incident antilambda momentum, $\mathbf{p}$ the recoil lambda momentum, and $\mathbf{Q}$ the relative momentum of the two K mesons, as measured, say, from K^+ to K^0. Define ϕ as the azimuthal angle between the planes whose normals are $\hat{p}\times\hat{k}$ and $\hat{Q}\times\hat{k}$, respectively, and define a polar angle of $\mathbf{Q}$ by $\cos\theta=\hat{Q}\cdot\hat{k}$. Call the distribution in these variables for reaction (3), $F(\theta,\phi)$. Conservation of parity requires

$$F(\theta,\phi)=F(\theta,2\pi-\phi). \tag{7}$$

Consider the isotopic spin of each K meson to be $\frac{1}{2}$.[4] Now the statement of Bose (Fermi) statistics is that in the matrix element the isotopic spin zero component of the K^+-K^0 system occurs in conjunction with a spatial function that is purely odd (even) under $\hat{Q}\rightarrow-\hat{Q}$. The direct consequence of normal statistics

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[1] O. W. Greenberg and A. Messiah (private communication).

[2] H. Feshbach, Phys. Letters 3, 317 (1963).

[3] V. E. Barnes, P. L. Connolly, D. J. Crenell, B. B. Culwick, W. C. Delaney, W. B. Fowler, P. E. Hagerty, E. L. Hart, N. Horwitz, P. V. C. Hough, J. E. Jensen, J. K. Kopp, K. W. Lai, J. Leitner, J. L. Lloyd, G. W. London, T. W. Morris, Y. Oren, R. B. Palmer, A. G. Prodell, D. Radojicic, D. C. Rahm, C. R. Richardson, N. P. Samios, J. R. Sanford, R. P. Shutt, J. R. Smith, D. L. Stonehill, R. C. Strand, A. M. Thorndike, M. S. Webster, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters 12, 204 (1964).

[4] The argument is independent of this statement. That is the common isotopic spin of each K meson can be taken to be any half-integer or integer. In the latter case, the words odd and (even) in the following sentence are interchanged.

(either Bose *or* Fermi), taken together with Eq. (7), is therefore

$$F(\theta,\phi)=F(\pi-\theta,\pi+\phi)=F(\pi-\theta,\pi-\phi)\,. \tag{8}$$

Thus normal statistics *requires* the density of events at (θ,ϕ) to be equal to the density of events at $(\pi-\theta,\pi-\phi)$. To achieve greater statistics in an experiment one would integrate the distribution F over the azimuthal angle ϕ from 0 to π. Calling this distribution $P(\theta)$, we have

$$P(\theta)=P(\pi-\theta)\,. \tag{9}$$

We note that: (a) if K mesons do not obey normal statistics, Eq. (9) need not be satisfied, since a given isotopic spin component of the two K meson system could enter in conjunction with terms both even and odd under $\hat{Q}\rightarrow-\hat{Q}$; (b) if the K^+ and K^0 were nonidentical isotopic singlets[5] Eq. (9) need not be satisfied. Also, we note that the restriction in Eq. (9) is more general than certain simple production mechanisms for reaction (3) would imply. For example, if the mechanism were antilambda annihilation on the proton into a forward produced virtual K^+ which then interacted with the neutron via K^0 exchange to produce the final K^+-K^0 system, the distribution F would simply tend to be independent of ϕ, (neglecting internal deuteron motion) by the argument of Treiman and Yang.[6] However, the dominance of K^0 exchange in producing a single resonant K^+-K^0 system with angular momentum L would lead to $F\propto|P_L(\cos\theta)|^2$ and hence Eq. (9) would be satisfied.

The argument for reactions (1) and (2) parallels that given above and the test of normal statistics is given, for the corresponding distributions, by Eqs. (8) and (9).

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[5] A. Pais, Phys. Rev. **112**, 624 (1958).

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Monte Carlo Calculations Applied to a Determination of Four-Momentum Transfer in Ultra-High Energy Interactions[*]

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Four-momentum transfer between two groups of particles produced in ultra-high energy interactions is studied employing 3000-BeV Monte Carlo jets generated for an earlier paper from information based largely on experimentally determined average properties of 10^{12}-eV jets. First, a parameter introduced by Hasegawa and Yokoi for measuring a deliberate underestimate of this four-momentum transfer is calculated using the Monte Carlo jets. In general, the results agree with the experimental findings of Fujioka *et al.*, in that the values of this parameter are concentrated above one nucleon mass. Detailed differences between the Monte Carlo and the experimental distributions may be explained by differences between properties of the Monte Carlo and the experimental jets after clarification of the properties of the parameter. Next, after assuming values of the masses and the transverse momenta of the baryons surviving the collision, it was possible to calculate the actual value of the four-momentum transfer between two groups of the produced particles for the Monte Carlo jets. The Hasegawa and Yokoi parameter is found to underestimate this actual value by a factor of about 0.6 ± 0.2, the limits defining the approximate 68% confidence interval for statistical fluctuations of individual measurements about the mean factor 0.6. Finally, the distribution of this actual value itself shows a mean around two nucleon masses, indicating that jets having average properties like the Monte Carlo jets belong to a different class of events from those encompassed by "linked-peripheral" models for which virtual π-meson transfer predominates.

I. INTRODUCTION

In recent years, the trend that the meson shower (jet) particles produced in an ultra-high energy interaction often exhibit spatial groupings has led to the proposal of a model[1–4] in which this generation of particles in groups or "fireballs" is the fundamental production mechanism. A basic quantity for characterizing these interactions is the four-momentum transfer between the groups; this four-momentum transfer is studied here. The importance of this quantity goes beyond its relation to the fireball model because it is a Lorentz invariant quantity, and such quantities are valuable for jet studies where there is difficulty in determining the incident particle's energy.

A parameter for measuring the four-momentum transfer between fireballs was first introduced by Niu in his version of the fireball model. He called this

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