lambda muonic events for which the muon stops in the chamber, and making the necessary corrections for detection efficiency. We are aware of three such events having been observed in hydrogen.[8] For a weighted average of R_μ we obtain[9]

$$R_\mu = (1.3 \pm 0.7) \times 10^{-4}.$$

Muon-electron universality predicts $R_e/R_\mu = 6.2$ and hence $R_\mu = 1.7 \times 10^{-4}$, which is consistant with both of the above R_μ determinations.

Comparing our results with those of Cabibbo,[10] we find a remarkable agreement. Using $R_e = (1.07 \pm 0.13) \times 10^{-3}$, $\sin\theta = 0.206$ [11] (Sakurai's correction to $\theta = 0.26$ reported by Cabibbo) we calculate $|k| = 1.09 \pm 0.09$.

$$\Gamma(\Lambda \to p + e^- + \bar{\nu}) = 2.15 \times 10^7 \text{ sec}^{-1} \sin^2\theta (1 + 3|k|^2).$$

This is certainly consistent within errors with k as measured above. At $e^{-1/2}$ times the maximum in the likelihood function we obtain as the errors on k, -0.70, and $+0.34$.

Finally, this experiment does not exclude the possibility of a mixture of S and T instead of V and A as the interaction currents. If S and T were the correct currents, a likelihood calculation favors $C_T = -0.50 C_S$.

Our results are consistant with the conclusions of the experiment of C. Baglin *et al.*[12] They rule out pure V but do not decide between pure A and $|C_V| = |C_A|$.

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[8] The detection efficiency for stopping muons from lambda muonic decays depends on the liquid used in the chamber, the size of the chamber, and the momentum of the lambdas. Monte Carlo calculations are used for estimating these efficiencies. For the reported lambda muonic decays we used the curves prepared by W. E. Humphrey, J. Kirz, A. H. Rosenfeld, and J. Leitner, *Proceedings of the 1962 International Conference on High Energy Physics, CERN* (CERN, Geneva, 1962), p. 442. The reported events, their respective detection efficiency, and sample size of observed lambda decays are: M. H. Alston, J. Kirz, J. Neufeld, F. T. Solmitz, and P. G. Wohlmut, UCRL-10926, 1963 (unpublished), 23%, 30 000; M. L. Good and V. G. Lind, Phys. Rev. Letters **9**, 518 (1962), 28%, 11 500; F. Eisler, J. M. Gaillard, J. Keren, M. Schwartz, and S. Wolf, *ibid.* **7**, 136 (1961), 24%, 900.

[9] Two (not completely unambiguous) Λ_μ events in freon in an effective sample of 19 700 lambdas have been observed. The experimenters deduce that $R_\mu \leqslant 4.5 \times 10^{-4}$ at the 5% significance level. A. Kernan, W. M. Powell, C. L. Sandler, W. L. Knight, and F. R. Stannard, Phys. Rev. **133**, B1271 (1964).

[10] Nicola Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[11] J. J. Sakurai, Phys. Rev. Letters **12**, 79 (1964).

[12] C. Baglin, V. Brisson, A. Rousset, J. Six, H. H. Bingham *et al.*, Phys. Letters **6**, 186 (1963).

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Macroscopic Bodies in Quantum Theory*

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It is shown that if the wave function of a massive body is $\psi = \Sigma c_n \psi_n$, where the ψ_n are macroscopically distinguishable states, then the observation of interferences between the various ψ_n requires inconceivable laboratory conditions (e.g., the experiment may last longer than the lifetime of the universe). It is therefore proposed to interpret $\Sigma c_n \psi_n$ as a mixture of states, and not as a superposition. This new interpretation of wave functions is consistent with experience and is free from the paradoxical features of the "orthodox" measurement theory.

It may seem strange that more than thirty years after von Neumann's classic work, [1] the problem of measurement in quantum theory is not yet considered as settled.[2] The difficulty can easily be illustrated as follows. Suppose we have an instrument designed so as to measure a dynamical variable A belonging to a quantum system S. If S is initially in an eigenstate ϕ_i of A, then the pointer of the instrument will show the corresponding eigenvalue a_i. If S is initially in an eigenstate ϕ_j, the pointer will show the eigenvalue a_j (we suppose $a_j \neq a_i$). Now, if S is initially in the state $\phi = 2^{-1/2}(\phi_i + \phi_j)$, then the pointer will finally indicate either a_i or a_j, with equal probabilities. It will *not* be partly at a_i and partly at a_j, even though the initial state of S was a superposition of ϕ_i and ϕ_j. In other words, *the superposition principle is violated in a measurement process.*

Quite generally, von Neumann has shown that interactions can be constructed such that, if the initial state of S is $\phi = \sum c_n \phi_n$, and if the initial state of the instru-

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[1] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer-Verlag, Berlin, 1932).

[2] We quote only a few recent papers in which many further references may be found: S. Amai, Progr. Theoret. Phys. (Kyoto) **30**, 550 (1963); H. Margenau, Ann. Phys. (N. Y.) **23**, 469 (1963); A. Shimony, Am. J. Phys. **31**, 755 (1963); E. P. Wigner, *ibid.* **31**, 6 (1963); M. M. Yanase, *ibid.* **32**, 208 (1964).

ment is Ψ_0, then the Schrödinger equation for the compound system leads to

$$\sum c_n\phi_n\Psi_0 \rightarrow \sum c_n\phi_n\Psi_n\,, \qquad (1)$$

where the Ψ_n are orthogonal to each other. The process (1) is called by von Neumann a "measurement," but this definition agrees with the usual meaning of the word "measurement" only if the states Ψ_n are macroscopically distinguishable (e.g., different positions of a pointer on a scale) and, moreover, if the final state of the compound system is *any one* of the $\phi_n\Psi_n$ (with respective probabilities $|c_n|^2$) and *not* a superposition of the $\phi_n\Psi_n$ (with respective amplitudes c_n). In other words, the sum on right-hand side of (1) should be interpreted as a *mixture* rather than a *superposition* of states. This phenomenon has been given the name "reduction of the wave packet," and it can be shown formally that it cannot be explained within the frame of conventional quantum theory.[3]

The purpose of this paper is to investigate how quantum theory must be modified so as to encompass the measurement process. Let us emphasize, once and for all, that we do *not* intend here to bring any modification in the mathematical framework of the theory, the consistency and accuracy of which are beyond any doubt. What we challenge is the usual *physical interpretation* of this mathematical formalism, i.e., the set of correspondence rules which relate the abstract elements of the mathematical theory (which is left untouched) with the concrete elements of experience.[4] We intend to show here that these correspondence rules can and must be modified if macroscopic bodies are involved, in precisely such a way that the final state in (1) is to be considered as a mixture, and not as a superposition.

Our plan is quite simple. First we note that in orthodox quantum theory we are *compelled* to interpret $\phi=\sum c_n\phi_n$ as a superposition of states, and not as a mixture, because interference effects between the various components ϕ_n can be demonstrated *experimentally*. We now intend to show that if we are dealing with macroscopically distinguishable states Ψ_n of a particle of sufficiently large mass, it is *experimentally* impossible to get them to interfere with one another. It follows that, in this case, we are not compelled to interpret the right-hand side of (1) as a superposition and can therefore interpret it as a mixture while maintaining agreement with experiment. Our problem then (as already recognized by earlier authors[2]) is to show that macroscopically distinguishable states of massive bodies cannot be brought to interfere.

Let us start with a mental experiment: we have a macroscopic body of size a, density ρ and mass $m\sim\rho a^3$, passing through a screen in which two slits have been cut a distance b apart (of course, $b>a$) and impinging on another screen, a distance L beyond. If this experiment is repeated several times, interference fringes may be expected on the second screen, a distance $d\simeq\lambda L/b$ apart, where $\lambda=h/p$ is the de Broglie wavelength. Thus,

$$L\simeq bd/\lambda > pad/h\,,$$

and, since $p=mv/[1-(v^2/c^2)]^{1/2}\gtrsim mv$, it follows that the duration of each experiment is

$$T=L/v>mad/h\sim\rho a^4d/h\,.$$

Now, we should be reasonable and admit that $T\lesssim 10^{18}$ sec (the estimated total lifetime of our universe).[5,6] Moreover, in order to have observable fringes, d cannot be smaller than 10^{-8} cm, the interatomic distance in solid bodies. We thus obtain, with $\rho\sim1$ g/cm^3 (this is a universal constant: a few nucleon masses per cubic Bohr orbit),

$$a<1\text{ cm}\,,\qquad m<1\text{ g}\,. \qquad (2)$$

Macroscopic objects which do not satisfy (2) cannot display interference effects in our experiment. Note that the above result was obtained on grounds which transcend ordinary quantum mechanics (we had to invoke cosmology). However, it respects the spirit of quantum theory, according to which, effects that cannot be observed are nonexistent in principle.

At this point it may be argued that it is not necessary to go to a distance $L\simeq bd/\lambda$ to observe interference effects. A rigorous solution of the wave equation shows that such effects are present at distances much shorter than L. However, the amplitudes of the interference terms are then extremely small, so that a large number of observations would be required to detect them. In this case, the increase in the number of observations needed will more than make up for the decrease in the time required to carry out an observation.

The above is just one example of an experiment to attempt to observe interference in the case of a macroscopic body. To be sure, other experiments are conceivable, but in every case one is bound to encounter similar limitations arising from the shortness of the wavelength.

We thus see that a free macroscopic particle cannot exhibit interference effects.[7] However, to complete our proof, we must also show that the same holds for a macroscopic system coupled to a microscopic one. In particular, we must show that the process (1), which

[3] A. Komar, Phys. Rev. **126**, 365 (1962).

[4] We quote from Einstein's autobiography, in *Albert Einstein, Philosopher-Scientist*, edited by P. A. Schilpp (Harper & Brothers, New York, 1959), pp. 11–13: "I see on one side the totality of sense-experiences, and, on the other, the totality of concepts and propositions which are laid down in books. The relations between concepts and propositions are of a logical nature . . . [but they] get meaning, viz., content, only through their connection with sense-experiences. The connection of the latter with the former is *purely intuitive, not itself of a logical nature*."

[5] A. Sandage, Astrophys. J. **133**, 355 (1961); D. Layzer, *ibid.* **136**, 138 (1962).

[6] R. H. Dicke, Nature **192**, 440 (1961).

[7] Although one is familiar with the existence of large-scale quantum behavior, e.g., in the case of superconductivity or in the Mössbauer effect, the states involved are not what we referred to as "macroscopically distinguishable states" that could serve as the final states of a measuring instrument.

was obtained from the Schrödinger equation, is *irreversible*. At this point, it is customary[2] to invoke the many degrees of freedom of the macroscopic system as the cause of irreversibility. While it is indeed true that most measuring instruments (such as bubble chambers, Geiger counters, etc.) involve an irreversible amplification and registering mechanism with many degrees of freedom, it seems to us that this kind of irreversibility is not fundamental and perhaps may even be eliminated. It may also be argued that the many degrees of freedom of the macroscopic body are not relevant, because only a single degree of freedom is actually used for the measurement, e.g., the position of the center of mass of the pointer. Thus, to avoid a possible controversy, we shall not take issue on this problem, and instead, point out a much simpler cause of irreversiblity.

In order to reverse the arrow in (1) in such a way that the left-hand side will again be interpretable as a superposition, we need a mechanism which brings back the macroscopic body from its possible final positions *with the correct phase*. This implies, in the WKB approximation, that we need an accurate control of the phase factor $\exp[i\int p dq/\hbar]$ for the whole process. Roughly speaking, we must have

$$p\delta q < \hbar\,,$$

where p is the mean momentum of the macroscopic body and δq is the uncertainty in its *total* path q (i.e., the limit of reproducibility of the experimental setup).

Let us proceed to some estimates. The total duration of the experiment will be

$$T\sim mq/p > mq\delta q/\hbar\,.$$

For a macroscopic setup, we cannot achieve anything better than $\delta q\sim 10^{-8}$ cm (the interatomic distance) and we obtain, with, e.g., $m=1$ g and $q=1$ cm, that the experiment must last longer than the estimated total life-time of the universe.

The above discussion also shows that *not every Hermitian operator is observable*; for instance, an operator which rigidly displaces the state of a macroscopic body through a macroscopic distance, is not. Let us indeed consider the Hermitian operator

$$K=\cos(pR/\hbar)\,,$$

the effect of which is

$$Kf(q)=\tfrac{1}{2}[f(q+R)+f(q-R)]\,, \tag{3}$$

where q is the position of a macroscopic body and R a macroscopic distance.

If the process (3) were realizable, it would correlate wave packets a distance R apart, for instance macroscopically distinguishable states Ψ_1 and Ψ_2. We would therefore be compelled to consider the expression $c_1\Psi_1+c_2\Psi_2$ as a superposition, because its components Ψ_1 and Ψ_2 could be brought to interfere. We have seen however, that it is inconceivable (i.e., practically impossible) to displace a massive body through a macroscopic distance without appreciably perturbing its phase. It follows that the dynamical variable K is physically meaningless, even though it is a function of p.

(There is no inconsistency in this apparently paradoxical result, because if one tries to compute K from a measurement of p, one needs the knowledge of p with an accuracy better than $\hbar/R$—otherwise K would be completely uncertain. As a result of such a measurement, q becomes uncertain by more than R, so that our previous Ψ_1 and Ψ_2 are no longer distinguishable, and the whole measurement problem does not arise.)

Finally, one may raise the question of the status of an expression $\sum c_n\Psi_n$, when the mass of the body is not 1 g, but only 0.1 g, or 10^{-10} g, etc. Where should we place the limit between the superposition interpretation of wave functions (which is certainly valid for elementary systems)[8] and the mixture interpretation, which we propose for large bodies? For instance, if the state of an electron is measured by means of a heavy atom, which is itself measured by a macromolecule, etc., and the result of the measurement is finally recorded on a punched card, at which stage of this chain is the wave packet reduced and does the superposition yield place to a mixture?

The answer we propose is that as long as experiments can be performed in which interference effects may show up, then $\sum c_n\Psi_n$ is a superposition. It becomes a mixture beginning from the stage at which such experiments become inconceivable. The striking feature of this approach is that the determination of the nature of $\sum c_n\Psi_n$ (superposition, viz., mixture) has a certain subjective aspect: A poorly equipped physicist may interpret it as a mixture, while a better endowed one might still be able to display interference effects. This subjective aspect, however, is no new feature in physics. One might likewise ask, e.g., where should the limit be placed between the irreversible behavior of a gas and the reversible mechanical laws of single molecules? Does irreversibility start when we have 10 molecules, or 10^{10} molecules, etc? It is obvious that a wealthy laboratory, equipped with fast computers, will be able to push the reversibility limit farther than a poorer group of searchers. However, irreversibility will always appear when the number of molecules is sufficiently large—no laboratory is rich enough to hire a Maxwell demon!

The limit between superpositions and mixtures likewise depends on the technical means which are available. Future inventions and discoveries may displace it towards larger and larger masses, but we may believe that there will always be a size beyond which interference effects will not be observable, and therefore the traditional interpretation of a sum of wave functions as a superposition can be replaced by its interpretation as a mixture.

We are grateful to Professor W. H. Furry for clarifying discussions.

[8] D. Bohm and Y. Aharonov, Phys. Rev. **108**, 1070 (1957).