$$\cos\phi_1=\frac{\alpha_1}{\beta_1}=\frac{s^2-s(M_1^2+M_2^2+m_1^2+m_2^2-2\mu_1^2)+(M_1^2-M_2^2)(m_1^2-m_2^2)}{[\Delta(s,M_1^2,M_2^2)\Delta(s,m_1^2,m_2^2)]^{1/2}}, \tag{A1}$$

$$\cos\phi_2=\frac{\alpha_2}{\beta_2}=\frac{s^2-s(M_3^2+M_4^2+m_1^2+m_2^2-2\mu_2^2)+(M_4^2-M_3^2)(m_1^2-m_2^2)}{[\Delta(s,M_3^2,M_4^2)\Delta(s,m_1^2,m_2^2)]^{1/2}}, \tag{A2}$$

where the symbols are defined in Fig. 5 and Eq. (1). If the two parts in Fig. 8 are now put together to form the transition in Fig. 7(a), and if ϕ is defined to be the angle between M_1 and M_4, then

$$\cos\phi=\cos(\phi_1+\phi_2)=\cos\phi_1\cos\phi_2-[(1-\cos^2\phi_1)(1-\cos^2\phi_2)]^{1/2}. \tag{A3}$$

We now look at Fig. 5, and express t in terms of s, $\cos\phi$, and the M_i^2,

$$t=(1/2s)\{-s^2+s(M_1^2+M_2^2+M_3^2+M_4^3)-(M_1^2-M_2^2)(M_4^2-M_3^2)+\cos\phi[\Delta(s,M_1^2,M_2^2)\Delta(s,M_3^2,M_4^2)]^{1/2}\}. \tag{A4}$$

Substituting (A1)–(A3) into (A4) and replacing s, t and the M_i^2 by x, y, and the x_i according to Eqs. (9a), (9b), there results Eq. (11).

The remark at the end of part B, Sec. III can also be easily verified. For x outside both the intervals $L_{12}^-<x<L_{12}^+$ and $L_{23}^-<x<L_{34}^+$, then $\cos\phi_1$ and $\cos\phi_2$ defined in (A1) and (A2) are both larger than unity in magnitude. Therefore, by (A3), so is $\cos\phi$, and y_+ does not lie in the physical region $|\cos\phi|<1$.

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Depolarization of Spin-½ Particles by Electromagnetic Scatterings

C. K. IDDINGS* AND G. L. SHAW*
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California
AND
Y. S. TSAI†
Stanford Linear Accelerator Center, Stanford University, Stanford, California
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A study is made of the depolarization of polarized, relativistic fermions (spin ½) passing through matter. The final polarization of the projectile shows two features, (i) a rotation of the polarization vector so that it does not have the same direction as the initial polarization with respect to the initial or final momenta: *rotation*; (ii) an unpolarized component so that the magnitude of the polarization has diminished: *shrinkage.* We consider the scattering of the incident polarized fermion off unpolarized target electrons and nuclei to lowest order in α. Whereas to this order no polarization can be produced, i.e., the magnitude of the polarization vector cannot increase, the magnitude of the spin vector can decrease if the target has spin. General formulas are presented for the spin-½ particles scattered electromagnetically from an unpolarized target with arbitrary spin in terms of form factors. Numerical results are presented for processes (i) and (ii) in the cases of positrons and muons scattered by unpolarized electrons. Process (ii) is proportional to t^2 (for small momentum transfer t). If one expands the expressions for the polarization phenomena keeping only the linear term in t, then the shrinkage (ii) vanishes and the rotation effects (i) all reduce to those for the pure Coulomb scattering case. (As is well-known the depolarization due to Coulomb scattering is negligible for small-angle scattering.) However, if one is concerned with particles scattered into a sizeable solid angle, then (a) the rotation effects in, e.g., positron-electron scattering become enormously larger than that given by Coulomb scattering; (b) they become strongly dependent on the relative orientation of the incident polarization vector: much larger rotations occur for transversely polarized beams; (c) one cannot omit the contribution from the annihilation diagram compared to that from the direct one-photon exchange; (d) and *most important* the depolarization due to shrinkage is *comparable* to the rotational effects. In multiple scattering, the shrinkage is a cumulative effect whereas the rotational contribution to depolarization is a random walk process.

I. INTRODUCTION

DETAILED knowledge of the depolarization of polarized, relativistic fermions (spin ½) passing through matter is of current interest. Our theoretical studies (which neglect bremsstrahlung, see Ref. 21) do not explain the large depolarizations found in the experiments of Dick *et al.*[1,2] However, our results show a

* Supported in part by the U. S. Air Force through Air Force Office of Scientific Research Grant AF-AFOSR-62-452.
† Supported by U. S. Atomic Energy Commission.

[1] L. Dick, L. Feuvrais, and M. Spighel, Phys. Letters 7, 150 (1963); S. Bloom, L. A. Dick, L. Feuvrais, G. R. Henry, P. C. Macq, and M. Spighel, *ibid.* 8, 87 (1964); L. Dick, L. Feuvrais, L. DiLella, and M. Spighel, *ibid.* 10, 236 (1964).

[2] However, we do get agreement with the small μ depolarization observed experimentally. Polarized muons suffer negligible depolarization in slowing down from ~70 to ~10 MeV in any type of moderator [D. D. Yovanovitch (private communication)].

number of very interesting features which have not been explicitly discussed in previous calculations.[3–6] (In particular, keeping only the lowest order term in the momentum transfer variable,[5] t, throws away all the effects of the spin of the target electrons or nuclei.)

We will be concerned with the scattering of an incident beam of polarized, relativistic fermions off an unpolarized target (the final state of the target is not observed). The simple *lowest order* (in α) processes we calculate cannot, of course, produce polarization, i.e., *increase* the magnitude of the polarization vector of the projectile (see Appendix). We wish to emphasize strongly, that even to this order in α the final polarization state of the projectile shows *both* the following features: (i) a rotation of the polarization vector so that it does not have the same direction as the initial polarization with respect to the initial or final momenta; (ii) an unpolarized component is produced so that the magnitude of the polarization has *diminished*. The production of this unpolarized component is due to the interaction with the randomly oriented spin of the target. We will distinguish these effects by calling (i) *rotation*, and (ii) *shrinkage*.

A rotation of the polarization vector can take place in scattering from any kind of target (spinless, with spin polarized or unpolarized) but a shrinking can occur only when the target has spin. Then the randomness of phase present in the target system produces a final state of the projectile which has a random component. A well-known result in scattering theory is that the Born approximation cannot produce polarization in the final state if the initial state (of both spin-$\frac{1}{2}$ particles) is unpolarized.[7] We find the interesting result that the Born approximation can shrink the polarization vector of the projectile.

Our results follow directly from lowest order perturbation theory and illustrate rather nicely (and perhaps pedagogically) some familiar principles of quantum mechanics and relativistic kinematics. A review of the relevant spin formalism is presented in Sec. II. Presenting things as simply as possible we start in Sec. III with a discussion of the scattering of a fermion from a nonidentical unpolarized target by one-photon exchange. This example includes the scattering of muons from electrons and elastic or inelastic scattering of muons, electrons or positrons from an unpolarized (or spinless) nucleus. It demonstrates both the shrinking and rotation effects. In Sec. IV we give the analogous formulas for electron-electron (Møller) and positron-electron (Bhabha) scattering. Because of the complexity of the formulas, we present in Sec. V some numerical results obtained from the analyses of Secs. III and IV. A brief discussion of multiple scattering is given in Sec. VI.

The fractional decrease in magnitude or shrinkage of the incident polarization vector for small t (and sizeable incident laboratory energy E) is proportional to

$$t^2/E^4 \text{ for longitudinal polarization;}$$

$$t^2/E^2 \text{ for transverse polarizations.} \qquad (1)$$

If we expand all the polarization phenomena expressions keeping only the linear term in the momentum transfer variable t, then the shrinkage vanishes. In addition the rotation effects all reduce to the pure Coulomb (no energy loss) scattering case: the fraction of the incident polarization vector in the scattering plane which does not follow the scattering angle in a single collision is given by[8] [see Eq. (29)]

$$tm^2/(2E^2+\tfrac{1}{2}t)(E^2-m^2)\,, \qquad (2)$$

where m is the mass of the projectile. The particular conditions of the Dick experiments,[1] namely slowing down of 50-MeV positrons to 10 MeV in a Be absorber and observing only the positrons in the very forward direction, allow us [using (2)] trivially to make an upper estimate of the depolarization. This estimate is orders of magnitude too small to explain Dick's results. This result (that for small scattering angles the depolarization due to Coulomb scattering is negligible) is known.[3–5]

On the other hand, if one accepts particles scattered into a sizeable angle, then (a) the rotation effects in Bhabha scattering become enormously larger than that given by the Coulomb scattering; (b) they become strongly dependent on the relative orientation of the incident polarization vector with respect to the incident momentum: much larger rotation for transversely than for longitudinally polarized beam; (c) one cannot omit the contribution from the annihilation diagram compared to that from the direct one-photon exchange; (d) and *most* important the depolarization due to shrinkage is *comparable* to the rotational effects, i.e., one gets a real decrease in magnitude of the polarization vector and not just a change in direction. These results are all made evident by the numerical calculations presented in Sec. V.

We include in an Appendix, an expression of the polarization resulting from the most general possible collision of a spin-$\frac{1}{2}$ particle with an arbitrary, unpolarized, target. This shows both the limitations and the general features of the results obtained with the Born approximation.

[3] M. E. Rose and H. A. Bethe, Phys. Rev. **55**, 277 (1939). L. J. Weigert and M. E. Rose, Nucl. Phys. **51**, 529 (1964), have given a general discussion of polarization phenomena for electron-nucleus scattering, emphasizing the effects of nuclear structure.

[4] G. W. Ford and C. J. Mullin, Phys. Rev. **108**, 477 (1957).

[5] C. Bouchiat and J. M. Lévy-Leblond, Nuovo Cimento (to be published).

[6] H. Olsen and L. C. Maximom, Phys. Rev. **114**, 887 (1959).

[7] We have in mind only Hermitian interactions.

[8] We use units $\hbar=c=1$.

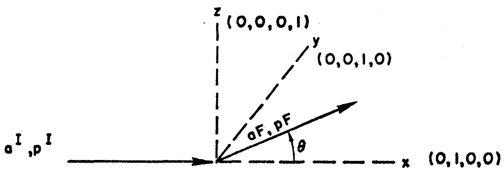

FIG. 1. Projectile with initial four-momentum p^I and polarization a^I is incident along x axis and scatters by angle θ in x-y plane to p^F and a^F.

II. REVIEW OF SPIN FORMALISM

For a spin-$\frac{1}{2}$ particle we shall describe the polarization by the usual covariant density matrix[8–10]

$$\rho=\tfrac{1}{2}(1\pm a\gamma_5). \qquad (3)$$

The polarization four-vector a satisfies the subsidiary condition

$$a\cdot p=0, \qquad (4)$$

where p is the four-momentum of the particle (with mass m). In general,

$$0\leq a^2\leq -1, \qquad (5)$$

where $a^2=-1$ corresponds to complete polarization. These relations are simple generalizations of the familiar 2×2 matrix operator $\frac{1}{2}(1+\mathbf{P}\cdot\boldsymbol{\sigma})$, $0\leq(\mathbf{P})^2\leq1$ which described an arbitrary mixture of polarized and unpolarized spin-$\frac{1}{2}$ particles. This expression holds *relativistically* but only in the rest frame of the particle where $a=(0,\mathbf{P})$.

The polarization vector a can be expressed as a linear combination of three [due to condition (4)] orthonormal four-vectors a_i:

$$a=\sum_{i=1}^{3} P_i a_i, \qquad (6)$$

where a_i reduces to a unit space vector in the rest frame of the particle. The familiar covariant spin projection operator can be written

$$\Sigma=\tfrac{1}{2}(1\pm a_i\gamma_5).$$

Consider a two-body scattering process, Fig. 1, where the incident particle with[9]

$$p^I=(E^I,p^I,0,0), \qquad (7)$$

in the laboratory system scatters to

$$p^F=(E^F,p^F\cos\theta,p^F\sin\theta,0). \qquad (8)$$

Then we choose our a_i such that $\mathbf{a}_1$ is parallel to $\mathbf{p}$ and $\mathbf{a}_2$ and $\mathbf{a}_3$ are normal[11] to $\mathbf{p}$:

$$\begin{aligned} a_1^I&=(p^I,E^I,0,0)/m,\\ a_2^I&=(0,0,1,0),\\ a_3^I&=(0,0,0,1), \end{aligned} \qquad (9)$$

and

$$\begin{aligned} a_1^F&=(p^F,\ E^F\cos\theta,\ E^F\sin\theta,\ 0)/m,\\ a_2^F&=(0,\ -\sin\theta,\ \cos\theta,\ 0),\\ a_3^F&=(0,0,0,1). \end{aligned} \qquad (10)$$

Since we will be considering scattering processes only to lowest order in α, the magnitude of a cannot *increase*. Then for this situation (see Appendix for the more general relation) we can express the transformation of a^I into a^F by the collision process in terms of

$$\begin{pmatrix} P_1^F\\ P_2^F\\ P_3^F \end{pmatrix}=\begin{pmatrix} M_{11} & M_{12} & 0\\ M_{21} & M_{22} & 0\\ 0 & 0 & M_{33} \end{pmatrix}\begin{pmatrix} P_1^I\\ P_2^I\\ P_3^I \end{pmatrix}. \qquad (11)$$

The fact that M_{12} and M_{21} are not zero represent what we have called *rotation*. *Shrinkage* is given by the expressions

$$S_i>0,\quad i=1,\,2,\,3 \qquad (12)$$

where

$$\begin{aligned} S_1&=1-(M_{11}^2+M_{21}^2)^{1/2},\\ S_2&=1-(M_{21}^2+M_{22}^2)^{1/2},\\ S_3&=1-M_{33}. \end{aligned} \qquad (13)$$

The M matrix and hence the rotation depends on the coordinate system [one could of course find a coordinate system different from (10) such that M is diagonal]. One must be careful to work in the *laboratory system* in particular for the problem of *multiple scattering* (see Sec. VI). It is possible and sometimes convenient to work in the center-of-mass system (see Sec. V); however, considerable care must be taken in defining the directions of polarization. The connection between the laboratory system and the center-of-mass system, c.m., is as follows. The laboratory and c.m. frames are related by a Lorentz transformation along the x axis with velocity u. *Denoting the c.m. quantities by primes* we have

$$p^F=Lp^{F'}=\left((E^{F'}+up^{F'}\cos\theta')\frac{1}{(1-u^2)^{1/2}},\right.$$
$$\left.(p^{F'}\cos\theta'+uE^{F'})\frac{1}{(1-u^2)^{1/2}},\ p^{F'}\sin\theta,\ 0\right), \qquad (14)$$

also

$$a_i^I=La_i^{I'}. \qquad (15)$$

However, two successive Lorentz transformations not

---

[9] Our metric is 1, −1, −1, −1, and $\gamma_5^2=+1$. We use (−) in Eq. (3) for electrons, (+) for positrons. Superscript $I(F)$ denotes initial (final) states.

[10] R. P. Feynman, *Quantum Electrodynamics* (W. A. Benjamin and Company, New York, 1961). See also: L. Michel and A. S. Wightman, Phys. Rev. **98**, 1190 (1955).

[11] The a_i are just Lorentz transforms of unit vectors in the rest frame of the particle. The functional form of Eqs. (9) and (10) is the same in the c.m. system, however, all quantities are primed, i.e., a_i', etc.

in the same direction are equivalent to a single Lorentz transformation plus a rotation so that[12] $La_1^{F\prime}$ is not parallel to $\mathbf{p}_F$ as is required by our definition (10) of a_1^F. There is an additional rotation R in the x-y plane relating a_i^F and $a_i^{F\prime}$. Let

$$\bar{a}_i^F \equiv La_i^{F\prime}; \tag{16}$$

then it is easy to show that

$$\begin{pmatrix} a_1^F \\ a_2^F \\ a_3^F \end{pmatrix} = RL \begin{pmatrix} a_1^{F\prime} \\ a_2^{F\prime} \\ a_3^{F\prime} \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{a}_1^F \\ \bar{a}_2^F \\ \bar{a}_3^F \end{pmatrix}, \tag{17}$$

where

$$\begin{aligned} D &= \left[1+u^2\cos^2\theta' + \frac{2u}{v'}\cos\theta' - u^2\left(1-\frac{1}{v'^2}\right)\right]^{1/2}, \\ \cos\delta &= [1+(u/v')\cos\theta']/D, \\ \sin\delta &= (-u/v')(1-v'^2)^{1/2}(\sin\theta')/D, \end{aligned} \tag{18}$$

and $v'=p^{F\prime}/E^{F\prime}$. The polarization M matrices are related by the same rotation R:

$$M = RM' \tag{19}$$

and thus:

$$\begin{pmatrix} P_1^F \\ P_2^F \\ P_3^F \end{pmatrix} = R \begin{pmatrix} P_1^{F\prime} \\ P_2^{F\prime} \\ P_3^{F\prime} \end{pmatrix}.$$

Note, however, that $S_i = S_i'$. Let $d\sigma/d\Omega(a_i^F, a_j^I)$ be the differential scattering cross section for initial spin in "direction" a_j^I, Eq. (9), and final spin in "direction" a_i^F, (10), then

$$M_{ij} = \frac{(d\sigma/d\Omega)(a_i^F, a_j^I) - (d\sigma/d\Omega)(-a_i^F, a_j^I)}{(d\sigma/d\Omega)(a_i^F, a_j^I) + (d\sigma/d\Omega)(-a_i^F, a_j^F)}. \tag{20}$$

[In calculating $d\sigma/d\Omega(a_i^F, a_j^I)$ one projects out the spin in direction a_i^F, e.g., by using the projection operator $(1+\mathbf{a}_i^F\gamma_5)/2$.]

III. DEPOLARIZATION PHENOMENA DUE TO GENERAL ONE-PHOTON-EXCHANGE DIAGRAM

In this section we treat the general one-photon-exchange diagram shown in Fig. 2 where p and a, Eqs. (7)–(10), represent[9] the momenta and polarization vectors of the polarized electron (positron or muon) projectile. p_T is the initial momenta of the target of mass M with arbitrary spin but unpolarized. The final state n of the target is arbitrary (elastic or inelastic) but experimentally unmeasured. Given a^I we are interested in determining a^F as a function of E^I, E^F and the scattering angle θ, or the momentum transfer variable

$$t \equiv q^2 = (p^I - p^F)^2. \tag{21}$$

Equivalently we have seen that we may calculate the cross section $d\sigma/d\Omega(a_i^F, a_j^I)$ for initial spin of projectile in "direction" a_j^I and final spin in direction a_i^F. This is given by

$$\begin{aligned} \frac{d\sigma}{d\Omega}(a_i^F, a_j^I) &\propto \frac{1}{t^2}\langle p^I a_j^I | j_\mu | p^F a_i^F\rangle^* \langle p^F a_i^F | j_\nu | p^F a_j^I\rangle \\ &\times \sum_{\substack{\text{initial spin of target}\\ \text{and all } n}} \langle p_T | J_\mu | n\rangle^* \langle n | J_\nu | p_T\rangle \\ &\equiv \frac{1}{t^2} F_{\mu\nu} T_{\mu\nu}. \end{aligned} \tag{22}$$

The second-rank tensor,

$$T_{\mu\nu} = \sum_{\substack{\text{initial spin of target}\\ \text{and all } n}} \langle p_T | J_\mu | n\rangle^* \langle n | J_\nu | p_T\rangle,$$

must be constructed from the four vectors p_T and q. Since

$$q_\mu F_{\mu\nu} = q_\nu F_{\mu\nu} = 0,$$

the most general form of $T_{\mu\nu}$ must be[13]

$$T_{\mu\nu} = g_{\mu\nu} G + p_{T\mu} p_{T\nu} F, \tag{23}$$

where G and F are functions of t and $q\cdot p_T$. Noting that the projection operator for the state $|pa_i\rangle$ is (for an electron)

$$[(\not{p}+m)/2m]\tfrac{1}{2}(1-\mathbf{a}_i\gamma_5),$$

we have for the tensor describing the projectile

$$\begin{aligned} T_{\mu\nu} = \mathrm{Tr}\left(\frac{\not{p}^I+m}{2m}\right)\left(\frac{1-\mathbf{a}_j^I\gamma_5}{2}\right)\gamma_\mu \\ \times \left(\frac{\not{p}^F+m}{2m}\right)\left(\frac{1-\mathbf{a}_i^F\gamma_5}{2}\right)\gamma_\nu. \end{aligned} \tag{24}$$

The expression for positron scattering can be obtained by the substitution rule $p^I \leftrightarrow -p^F$ and $a_i^F \leftrightarrow -a_j^I$. Contracting (23) and (24) we obtain

$$\begin{aligned} \frac{d\sigma}{d\Omega}(a_i^F, a_j^I) \propto \frac{1}{t^2}\frac{1}{4m^2}\Big[G\{2m^2+t-2m^2(a_i^F\cdot a_j^I)\} + F\{2(p^I\cdot p_T)(p^F\cdot p_T) + \tfrac{1}{2}M^2 t - (a_i^F\cdot a_j^I)(2(p^I\cdot p_T)(p^F\cdot p_T)+\tfrac{1}{2}M^2 t) \\ - M^2(p^I\cdot a_i^F)(p^F\cdot a_j^I) + t(p_T\cdot a_j^I)(p_T\cdot a_i^F) + 2(p^I\cdot p_T)(p_T\cdot a_i^F)(p^F\cdot a_j^I) + 2(p^F\cdot p_T)(p^I\cdot a_i^F)(p_T\cdot a_j^I)\}\Big]. \end{aligned} \tag{25}$$

[12] E. Wigner, Rev. Mod. Phys. **29**, 255 (1957).
[13] Y. S. Tsai, Proceedings of the International Conference on Nucleon Structure, June 1963 (Stanford University Press, to be published).

As expected, the eross section (25) for electron-nucleus scattering is the same as for positron-nucleus scattering. We see that the spin-dependent terms of one-photon-exchange cross section, (25), are bilinear in a_i^F and a_j^I. Thus, summing over $\pm a_i^F(\pm a_j^I)$ leaves (25) independent of initial (final) spin orientation. Hence, as expected, the one-photon-exchange process cannot be used as a spin polarizer or analyzer.

The polarization M matrix elements, (20), are

$$M_{ij}=[G\{-2m^2(a_i^F\cdot a_j^I)\}+F\{-(a_i^F\cdot a_j^I)2(p^I\cdot p_T)(p^F\cdot p_T)+\tfrac{1}{2}M^2t-M^2(p^I\cdot a_i^F)(p^F\cdot a_j^I)+t(p_T\cdot a_j^I)(p_T\cdot a_i^F)$$
$$+2(p^I\cdot p_T)(p_T\cdot a_i^F)(p^F\cdot a_j^I)+2(p^F\cdot p_T)(p^I\cdot a_i^F)(p_T\cdot a_j^I)\}]/[G(2m^2+t)+F(2(p^I\cdot p_T)(p^F\cdot p_T)+\tfrac{1}{2}M^2t)]. \quad (26)$$

The simplest effect to demonstrate is the shrinking for an initial polarization normal to the scattering plane: The element S_3, from (9) and (10) is given by

$$S_3=1-M_{33}=\frac{Gt}{G(2m^2+t)+F(2(p^I\cdot p_T)(p^F\cdot p_T)+\frac{1}{2}M^2t)}. \quad (27)$$

For elastic scattering from a spinless target or elastic Coulomb scattering we have

$$G=0,\quad F=F(t). \quad (28)$$

Hence $S_3=0$. It is easily shown that S_1 and S_2 are also zero, i.e., Coulomb scattering preserves the magnitude of the polarization vector. The component of the polarization vector in the scattering plane rotates, lagging the (lab) scattering angle θ by an amount β:

$$\sin\beta=M_{12}=-M_{21}=\frac{m(E^I+E^F)\sin\theta}{(2E^IE^F+\frac{1}{2}t)}, \quad (29)$$

[independent of $F(t)$]. Equation (29) shows that in the nonrelativistic limit we have $\beta=\theta$, i.e., the polarization vector $\mathbf{P}^I$ preserves its direction and magnitude; in the high-energy limit $\beta=0$, i.e., rotation of $\mathbf{P}^I$ is exactly equal to the scattering angle; for small-angle scattering

$$\beta\approx\theta(m/E^I). \quad (30)$$

When the target has spin, $G\neq 0$ and $|\mathbf{P}^I|>|\mathbf{P}^F|$ for any nonforward scattering. For the case elastic scattering off a proton target, Eq. (25) can be written as

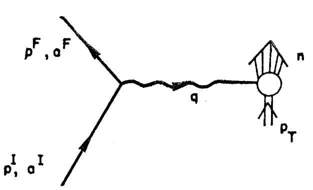

FIG. 2. General direct one-photon-exchange diagram.

$$\frac{d\sigma}{d\Omega}(a_i^F,a_j^I)=\frac{\alpha^2p^F}{2Mp^I}\frac{1}{[E^I+M-(p^IE^F/p^F)\cos\theta]}\frac{1}{t^2}\Big[\quad\Big], \quad (31)$$

where the bracket [ ] is the same as in (25) with the substitutions

$$G=\tfrac{1}{2}tG_m^2,$$
$$F=2\left(G_m^2-\frac{4M^2}{t}G_e^2\right)\Big/\left(1-\frac{4M^2}{t}\right), \quad (32)$$

where G_e and G_m are the electric and magnetic nucleon form factors. In terms of F_1 and F_2, we have

$$G_e=F_1+(Kt/4M^2)F_2,\quad G_m=F_1+KF_2, \quad (33)$$

where K is the anomalous magnetic moment. At high energy we see that the shrinkage S_3 is

$$S_3\propto t^2/(E^I)^2. \quad (34)$$

IV. DEPOLARIZATION IN MØLLER AND BHABHA SCATTERING

Møller and Bhabha scattering involve only one internal photon. However, these two processes contain diagrams, Figs. 3 and 4, which do not belong to the class of diagrams considered in Sec. III. In the exchange diagram of Møller scattering and the annihilation diagram of Bhabha scattering the incident particle p^I and the scattered particle p^F do not belong to the same charged line. Hence, one cannot write down an equation such as (23).

For positron-electron scattering, the cross section for the initial spin in direction a_i^I and final spin in direction a_j^F can be written as

$$\frac{d\sigma}{d\Omega}(a_i^F,a_j^I)=\frac{\alpha^2(p^F)^3}{2m^2p^I(E^I+m)(E^F-m)}\Bigg[\frac{1}{t^2}\left(\frac{s^2+(s+t)^2}{2}-4m^2(s-m^2)\right)+\frac{1}{ts}\left((s+t)^2-4m^4\right)$$
$$+\frac{1}{s^2}\left(\frac{t^2+(s+t)^2}{2}-4m^2(t-m^2)\right)+(a_i^F\cdot a_j^I)\left(\frac{1}{t^2}(us-4m^4)-\frac{1}{ts}\{s(s+t)+2m^2t-4m^4\}-\frac{2m^2}{s}\right)$$
$$+2(p^I\cdot a_i^F)\left(\frac{1}{t^2}\{2(p_T\cdot a_j^I)(p^F\cdot p_T)-m^2(p^F\cdot a_j^I)\}+\frac{1}{ts}\{(2(p^F\cdot p_T)+m^2)(a_j^I\cdot p_T)-m^2(a_j^I\cdot p^F)\}+\frac{m^2}{s^2}(a_j^I\cdot p_T)\right)$$
$$+2(p_T\cdot a_i^F)\left(\frac{1}{t^2}\{2(p^F\cdot a_j^I)(p^I\cdot p_T)+t(p_T\cdot a_j^I)\}+\frac{1}{ts}\{t(p_T\cdot a_j^I)+(s-m^2)(p^F\cdot a_j^I)\}+\frac{m^2}{s^2}(a_j^I\cdot p_T)\right)\Bigg], \quad (35)$$

TABLE I. S_i, r_i, and $d\sigma/d\Omega$ for Bhabha scattering. Boldface quantities represent contributions from Fig. 3(a) alone. Incident positron energy E^I and energy loss ΔE are in units of m_e, whereas the cross section has units α^2/m_e^2. All quantities refer to the laboratory system. The integers after the commas denote the powers of 10.

E^I	ΔE	S_1	$\mathbf{S}_1$	r_1	$\mathbf{r}_1$	S_2	$\mathbf{S}_2$	r_2	$\mathbf{r}_2$	S_3	$\mathbf{S}_3$	$d\sigma/d\Omega$	$\mathbf{d\sigma/d\Omega}$
200	190	0.90, 0	0.29, −3	0.91, 0	0.82, −1	0.10, 1	0.91, 0	0.10, 1	0.91, 0	0.10, 1	0.91, 0	0.27, −2	0.15, −2
	130	0.30, 0	0.41, −3	0.31, 0	0.35, −2	0.79, 0	0.38, 0	0.80, 0	0.38, 0	0.80, 0	0.38, 0	0.17, 0	0.16, 0
	50	0.58, −2	0.26, −4	0.63, −2	0.65, −4	0.15, 0	0.40, −1	0.15, 0	0.40, −1	0.15, 0	0.40, −1	0.59, 1	0.70, 1
	10	0.69, −5	0.20, −6	0.95, −5	0.32, −6	0.52, −2	0.13, −2	0.52, −2	0.13, −2	0.52, −2	0.13, −2	0.33, 3	0.34, 3
	1	0.22, −8	0.47, −9	0.28, −8	0.63, −9	0.50, −4	0.13, −4	0.50, −4	0.13, −4	0.50, −4	0.13, −4	0.39, 5	0.39, 5
	0.1	0.16, −10	0.33, −11	0.56, −10	0.60, −10	0.50, −6	0.13, −6	0.50, −6	0.13, −6	0.50, −6	0.13, −6	0.40, −7	0.40, 7
100	80	0.58, 0	0.12, −2	0.60, 0	0.24, −1	0.94, 0	0.62, 0	0.95, 0	0.62, 0	0.95, 0	0.63, 0	0.46, −1	0.33, −1
	40	0.44, −1	0.23, −3	0.48, −1	0.77, −3	0.38, 0	0.12, 0	0.38, 0	0.12, 0	0.38, 0	0.12, 0	0.13, 1	0.15, 1
	20	0.22, −2	0.27, −4	0.27, −2	0.58, −4	0.92, −1	0.24, −1	0.93, −1	0.24, −1	0.93, −1	0.24, −1	0.11, 2	0.13, 2
	1	0.35, −7	0.77, −8	0.45, −7	0.10, −7	0.20, −3	0.51, −4	0.20, −3	0.51, −4	0.20, −3	0.51, −4	0.96, 4	0.97, 4
	0.1	0.25, −9	0.53, −10	0.89, −9	0.96, −9	0.20, −5	0.50, −6	0.20, −5	0.50, −6	0.20, −5	0.50, −6	0.10, 7	0.10, 7
	0.01	0.25, −11	0.50, −12	0.99, −10	0.10, −9	0.20, −7	0.50, −8	0.20, −7	0.51, −8	0.20, −7	0.50, −8	0.10, 9	0.10, 9
60	50	0.65, 0	0.22, −2	0.68, 0	0.55, −1	0.96, 0	0.69, 0	0.97, 0	0.69, 0	0.97, 0	0.69, 0	0.32, −1	0.21, −1
	30	0.11, 0	0.81, −3	0.12, 0	0.32, −2	0.56, 0	0.20, 0	0.56, 0	0.20, 0	0.56, 0	0.20, 0	0.56, 0	0.63, 0
	10	0.99, −3	0.29, −4	0.14, −2	0.50, −4	0.63, −1	0.16, −1	0.63, −1	0.16, −1	0.63, −1	0.16, −1	0.18, 2	0.21, 2
	1	0.27, −6	0.60, −7	0.35, −6	0.80, −7	0.56, −3	0.14, −3	0.56, −3	0.14, −3	0.56, −3	0.14, −3	0.34, 4	0.34, 4
	0.1	0.19, −8	0.41, −9	0.69, −8	0.74, −8	0.55, −5	0.14, −5	0.55, −5	0.14, −5	0.55, −5	0.14, −5	0.36, 6	0.36, 6
	0.01	0.19, −10	0.39, −11	0.76, −9	0.77, −9	0.55, −7	0.14, −7	0.55, −7	0.17, −7	0.55, −7	0.14, −7	0.36, 8	0.36, 8
20	16	0.59, 0	0.11, −1	0.67, 0	0.12, 0	0.94, 0	0.66, 0	0.96, 0	0.66, 0	0.97, 0	0.67, 0	0.45, −1	0.35, −1
	1	0.22, −4	0.53, −5	0.29, −4	0.69, −5	0.50, −2	0.13, −2	0.50, −2	0.13, −2	0.50, −2	0.13, −2	0.33, 3	0.34, 3
	0.1	0.15, −6	0.33, −7	0.56, −6	0.60, −6	0.48, −4	0.13, −4	0.48, −4	0.13, −4	0.48, −4	0.13, −4	0.39, 5	0.39, 5
	0.001	0.15, −10	0.31, −11	0.63, −8	0.63, −8	0.48, −8	0.13, −8	0.11, −7	0.75, −8	0.48, −8	0.13, −8	0.40, 9	0.40, 9
2	0.5	0.77, −1	0.30, −1	0.81, −1	0.78, −1	0.17, 0	0.55, −1	0.18, 0	0.11, 0	0.16, 0	0.48, −1	0.41, 1	0.56, 1
	0.1	0.13, −2	0.42, −3	0.85, −2	0.91, −2	0.47, −2	0.17, −2	0.12, −1	0.10, −1	0.40, −2	0.13, −2	0.31, 3	0.33, 3
	0.01	0.10, −4	0.32, −5	0.83, −3	0.84, −3	0.44, −4	0.16, −4	0.87, −3	0.85, −3	0.37, −4	0.13, −4	0.39, 5	0.39, 5
	0.0001	0.10, −8	0.31, −9	0.83, −5	0.83, −5	0.43, −8	0.16, −8	0.83, −5	0.83, −5	0.36, −8	0.13, −8	0.40, 9	0.40, 9

where s and u are the usual scalar variables

$$s \equiv (p^I + p_T)^2, \qquad (36)$$
$$u \equiv (p^F - p_T)^2 = 4m^2 - s - t,$$

and p^I, p^F, p_T are physical four-momenta of the particles (energy component > 0). The contributions to (35) from the one-photon-exchange diagram, Fig. 3(a), from the annihilation diagram, Fig. 3(b), and the interference term can be identified by a denominator $1/t^2$, $1/s^2$, and $1/ts$, respectively.

The corresponding expression for the Møller scattering can be obtained by making the following substitutions in the [ ] of (35):

$$-p^I \leftrightarrow p^F, \quad a_i^F \leftrightarrow -a_j^I, \quad s \leftrightarrow u. \qquad (37)$$

V. NUMERICAL RESULTS AND DISCUSSION

The cross section and the polarization matrix M [of Eq. (11)] completely characterize the single scattering by one-photon exchange from an unpolarized target. Instead of numerically tabulating matrix elements of M, we computed the quantities r_i,

$$r_i = 1 - M_{ii} \quad i = 1, 2, \qquad (38)$$

and S_i [of Eq. (13)]. The three S_i give directly the shrinkage or decrease in magnitude of the incident polarization vector for a single scattering. They allow us to give a minimum for the depolarization in multiple scattering (due to the particular scattering process being considered). The r_i give us the fraction in the laboratory system of the incident polarization vector in the scattering plane which does not follow the scattering of the particle. (We stress again that M', the corresponding matrix in the c.m. frame, is related to M by a rotation. On the other hand, the shrinkage elements S_i are the same in both the laboratory and the c.m. frame; this is not true, however, for any arbitrary Lorentz frame.) Clearly it is much simpler to work in the laboratory system in treating multiple scattering. The r_i allow us to give a maximum for the depolarization in multiple scattering.[14]

The five quantities S_i and r_i were determined for polarized positrons scattering off unpolarized electrons by numerically evaluating (35). The calculations were performed using double-precision arithmetic on the Stanford 7090 computer. [We note that for large incident energy one gains more accuracy by evaluating cross sections in the center-of-mass system and then

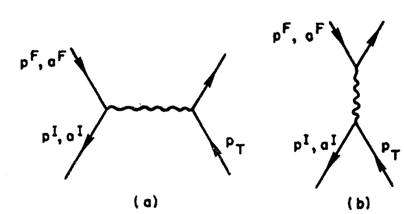

FIG. 3. Bhabha scattering of polarized incident positron off unpolarized target electron.

[14] In the literature are many calculations of depolarization effects in a single scattering; however, estimates of the depolarization in multiple scattering based on these can sometimes be misleading. For example, using estimates of depolarization in the c.m. frame of the collision [without using the rotation of Eq. (19)] or considering only the component of final polarization in the direction of the incident particle may tend to over estimate the depolarization effects. Consider small-angle Coulomb scattering where Eq. (30) holds. At high energies, the polarization direction clearly follows the direction of motion. On the other hand, at low energies, $m/E \sim 1$, and the direction of polarization remains fixed in the laboratory system (parallel to its original direction). Thus, in multiple scattering, at high energies it is more reasonable to estimate the depolarization using the r_i but at low energies it is better to consider the components of the polarization along fixed directions in the laboratory system.

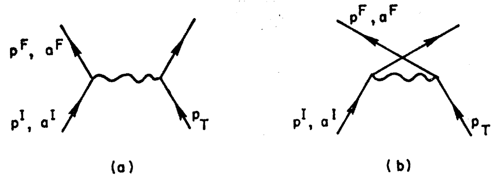

FIG. 4. Møller scattering of polarized incident electron off unpolarized target electron.

performing the transformation (17) to the laboratory system.] The results are presented in Table I. We give separately the contributions from the direct one-photon-exchange diagram Fig. 3(a) (denoted by boldface) as well as the full results from the direct, annihilation and interference terms. We have

$$s=2m_e^2+2E^Im_e$$

and

$$t=-2m_e\Delta E\,,$$

where ΔE is the energy lost by the positron.

The results of Table I for individual scatterings may be summarized as follows (again all quantities refer to the laboratory system):

For small energy loss $\Delta E/m_e\ll 1$ (and $E\gg m_e$):

(1) To lowest order in t,

$$r_i\propto t/s^4,\quad i=1,\,2$$
$$S_1\propto t^2/s^4,\qquad (39)$$
$$S_i\propto t^2/s^2,\quad i=2,\,3\,.$$

(2) In general the condition for the asymptotic expressions (39) to be valid is simply $\Delta E/m_e\ll 1$. However, in the case of r_2, it also depends on s: for sizeable $(E^I)^2\Delta E/m_e^3\sim 1$

$$r_2\propto t^2/s^2\,. \qquad (40)$$

(3) The numerical values for r_i in the asymptotic region are independent of i. Indeed they, as well as the one-photon exchange $\mathbf{r}_i$, reduce to the pure Coulomb result, Eq. (2).

(4) The S_i are a factor larger than the one-photon-exchange contribution $\mathbf{S}_i$ even in the asymptotic region.

(5) Again even in the asymptotic region the S_i (and $\mathbf{S}_i$) are a factor larger for the transverse polarizations 2 and 3 than the longitudinal polarization 1.

(6) For sizeable energy loss $\Delta E/m_e>1$

$$S_2=S_3\gg S_1\,.$$

(7) The longitudinal quantities $\mathbf{r}_1$, $\mathbf{S}_1$ are small compared to r_1, S_1.

(8) And most important, S_i is comparable in magnitude to r_i

$$S_i\sim r_i\,.$$

(9) Finally for $\Delta E\approx E^I-m_e$

$$S_i\approx 1$$

also

$$\mathbf{S}_1\approx 2(m_e/E^I)^2$$
$$\mathbf{S}_i\approx 1\quad\text{for}\quad i=2,\,3\,.$$

It is interesting to examine the above conclusions in the limit that $m\to 0$, comparing them with our notions of helicity conservation: (a) The helicity argument for a massless particle only concerns longitudinal polarization. (b) $\mathbf{S}_1$ does approach zero for $m_e/E\to 0$ regardless of ΔE. (c) On the other hand, $S_1\to 1$ for $\Delta E\to E^I-m$ even for $m_e/E\to 0$. This is due to the following: For 180° scattering in the c.m. and $m/E=0$, the direct amplitude preserves the helicity of the positron while the annihilation amplitude reverses the helicity of the positron. The relationships between the initial and final helicities are shown in Fig. 5, and can be understood simply in terms of angular momentum conservation. [Also this follows from the $m/E=0$, $-t=s$, $u=0$ limit of Eq. (35).] Since these amplitudes are equal in magnitude and do not interfere, it follows that the final state of the positron is unpolarized.

Calculations for polarized muons scattering off unpolarized electrons were performed using (26). Here

$$s=m_e^2+m_\mu^2+2m_eE^I$$

and

$$-t=2m_e\Delta E\,.$$

The maximum energy loss is

$$\Delta E_{\max}=2m_ep^{I2}/(m_e^2+m_\mu^2+2m_eE^I)\,, \qquad (41)$$

which nonrelativistically is $4m_e/m_\mu(E^I-m_\mu)$. The results are presented in Table II. For the same incident kinetic energy and same energy loss, the "maximum depolarization along the direction of motion," r_1 is much greater for μ than for e^+. Note [Eq. (41)] that the maximum energy loss for muon-electron collisions is constrained by kinematics to be small.

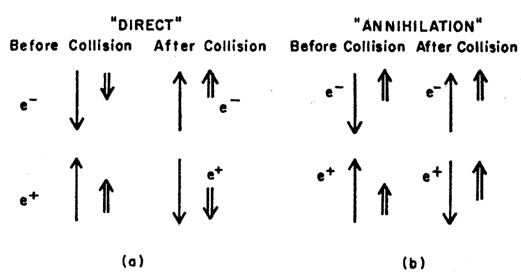

FIG. 5. Pictorial understanding of depolarization of longitudinally polarized incident e^+ scattering by 180° in c.m. system off *unpolarized* e^- in the limit $m_e/E\to 0$. The direct interaction (a) conserves helicity so by conservation of angular momentum only the (+) helicity state of the e^- interacts. The intermediate state of the annihilation interaction consists of one photon so that in (b) only the (−) helicity state of the e^- interacts. By angular momentum conservation, the helicity of the e^+ flips and amplitudes for (a) and (b), which are equal in magnitude, do not interfere. Hence the final positron is unpolarized.

TABLE II. S_i, r_i and $d\sigma/d\Omega$ for muons scattering off electrons. E^I and ΔE are in units of m_e, whereas the cross section has units α^2/m_e^2. All quantities refer to the laboratory system.

E^I	ΔE	S_1	r_1	S_2	r_2	S_3	$d\sigma/d\Omega$
400	4.0	0.15, −3	0.16, −3	0.93, −4	0.10, −3	0.11, −3	0.10, 0
	2.0	0.13, −4	0.13, −4	0.17, −4	0.17, −4	0.17, −4	0.68, 0
	1.0	0.18, −5	0.24, −5	0.41, −5	0.47, −5	0.36, −5	0.32, 1
	0.1	0.92, −8	0.22, −6	0.40, −7	0.25, −6	0.32, −7	0.37, 3
	0.01	0.84, −10	0.23, −7	0.40, −9	0.23, −7	0.31, −9	0.37, 5
	0.001	0.84, −12	0.23, −8	0.40, −11	0.23, −8	0.31, −11	0.37, 7
350	3.5	0.25, −3	0.25, −3	0.13, −3	0.13, −3	0.13, −3	0.88, −1
	2.0	0.27, −4	0.27, −4	0.25, −4	0.25, −4	0.25, −4	0.46, 0
	1.0	0.34, −5	0.46, −5	0.57, −5	0.69, −5	0.50, −5	0.24, 1
	0.1	0.16, −7	0.41, −6	0.57, −7	0.45, −6	0.42, −7	0.28, 3
	0.01	0.14, −9	0.44, −7	0.55, −9	0.44, −7	0.41, −9	0.29, 5
	0.001	0.14, −11	0.44, −8	0.55, −11	0.44, −8	0.41, −11	0.29, 7
300	2.0	0.79, −4	0.79, −4	0.43, −4	0.43, −4	0.43, −4	0.27, 0
	1.0	0.79, −5	0.10, −4	0.86, −5	0.11, −4	0.73, −5	0.16, 1
	0.1	0.30, −7	0.94, −6	0.82, −7	0.99, −6	0.57, −7	0.21, 3
	0.01	0.27, −9	0.10, −6	0.82, −9	0.10, −6	0.56, −9	0.21, 5
	0.001	0.26, −11	0.10, −7	0.82, −11	0.10, −7	0.56, −11	0.21, 7
270	1.0	0.15, −4	0.18, −4	0.11, −4	0.14, −4	0.98, −5	0.12, 1
	0.1	0.48, −7	0.18, −5	0.11, −6	0.18, −5	0.71, −7	0.17, 3
	0.001	0.40, −11	0.19, −7	0.11, −10	0.19, −7	0.69, −11	0.17, 7
230	0.4	0.33, −5	0.77, −5	0.20, −5	0.64, −5	0.18, −5	0.65, 1
	0.1	0.10, −6	0.63, −5	0.16, −6	0.63, −5	0.99, −7	0.12, 3
	0.001	0.77, −11	0.79, −7	0.17, −10	0.79, −7	0.95, −11	0.12, 7

VI. DEPOLARIZATION IN MULTIPLE SCATTERING

Let us summarize the results of our calculations (which take into account only graphs 2, 3, and 4) as they pertain to multiple scattering.[15] Although the numerical results of the previous section make it clear that for very large momentum transfers the depolarization can be appreciable, we are going to argue that this has little effect on the slowing down of high-energy beams of positrons or muons. In particular we shall conclude that under the conditions of the Dick experiment the polarization should be practically unchanged.[1,2,16]

In general, the multiple-scattering process can be treated either by Monte Carlo methods or by solving the Boltzmann equation. For elastic Coulomb scattering alone, the Boltzmann equation, including polarization effects has been solved by Toptygin.[17,18] The full Boltzmann treatment, using our Eq. (35) and the direct, one-photon exchange with the nucleus [Eq. (25)] is extremely complicated. However, such a rigorous treatment is not necessary; it is quite easy to show that there is only negligible depolarization by the one-photon processes just mentioned. We are now considering the following experimental situation[1]: Longitudinally polarized positrons of energy ~50 MeV which slow down to ~10 MeV in a Be absorber but remain within 8° of their original direction of motion.

First of all, for small energy loss ΔE, all the polarization effects in Bhabha scattering reduce to those of Coulomb scattering [just a rotation of the polarization vector, cf. remark (3) of Sec. V]. As is well known, for small-angle scattering, these effects are small and the numerical results in Table I illustrate this. Another feature which Table I shows is that for a given total energy loss, there is a much greater loss of polarization if the energy is lost in a few collisions than if the energy is lost in many small-angle collisions although the differential cross section tells us that the former process is much less likely than the later.[19] Let us consider an estimate for the polarization loss which uses a fixed, average energy loss per collision $\langle\Delta E\rangle$. In Be, $\langle\Delta E\rangle \lesssim 100$ eV.[20] The maximum longitudinal depolarization in such a collision is of order 10^{-10} and in loosing 40 MeV there are required $\sim 4\times 10^5$ such collisions. Thus, the "average" loss of polarization in passing through the medium is very small ($\sim 10^{-4}$). The one-photon-exchange process considered leads to

[15] H. Olsen and L. C. Maximon (Ref. 6) have considered the depolarization due to the emission of a single photon.

[16] The depolarization in this energy range is also small for transversely polarized positrons or muons.

[17] I. N. Toptygin, Zh. Eksperim. i Teor. Fiz. **36**, 488 (1959) [English transl.: Soviet Phys.—JETP **9**, 340 (1959)].

[18] The effect of the "Coulomb" part of the Bhabha scattering [i.e., using Eq. (30) only] on the depolarization has been considered by C. Bouchait and J. M. Lévy-LeBlond, Ref. 5.

[19] For example, if a 50-MeV positron, longitudinally polarized, loses 15 MeV by a single collision the maximum depolarization in the longitudinal direction is $r_1=0.015$ and the shrinkage is 0.013. If instead this energy loss takes place in 30 collisions averaging 0.5 MeV then we have the loss of polarization in the longitudinal direction is less than 10^{-5}.

[20] B. Rossi, *High Energy Particles* (Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1952), Chap. 2.

completely negligible depolarization for the slowing down of a 50-MeV positron to 10 MeV.[21]

On the other hand, there are cases where the depolarization effects will not be small and we shall conclude with a few remarks concerning them. As noted above, we have omitted the effect of higher order processes such as bremsstrahlung which will have to be included. The equation (35) for depolarization in the one-photon exchange will also be modified, especially at lower momentum transfers and for slower projectiles because the electrons in the absorber are initially in bound states rather than free-particle states, i.e., a Coulomb field is present.

It is not immediately evident whether the "shrinkage" or rotation effects are going to account for most of the depolarization. The rotation of the spin through an angle η in a given collision gives an rms angle $\eta N^{1/2}$ in N collisions; however, if the "shrinkage" in a single collision is ϵ then N such collisions give a shrinkage $\sim N\epsilon$. In other words, if the scattering process cannot polarize but can only rotate and shrink the polarization vector then the rotation is a random walk process but the shrinkage is a cumulative effect. (In our previous estimates, we have treated both effects as cumulative and so obtained an upper bound to the depolarization.)

For large momentum transfer collisions there is large depolarization, regardless of the energy of the incident particles. Thus if the experimental setup is such as to select particles which have undergone high momentum transfer scatterings, there will be considerable depolarization. The numbers in Table I also show the general feature that the depolarization of a longitudinally polarized beam (direction 1) is less than that of a transversely polarized (directions 2 and 3) beam. Also, for a given energy loss the depolarization is larger, the lower the initial energy.[14] When the multiple-scattering depolarization effects are large enough to be significant, a much more careful treatment, possibly the full Boltzmann equation, must be used; it is not sufficient to consider only the average momentum transfer and polarization loss. This is because only experimental conditions which select large momentum transfer scatterings will show any important depolarization effects. Such collisions are much less frequent than those having low momentum transfers so the fluctuations will be much larger. The simple approximation which considers only a succession of equal "average" collisions will not hold. We note also that the small-angle, Coulomb approximation to the cross section and polarization terms [that is, the expansion of Eq. (35) in powers of t] will not be good in the high momentum transfer region where there is large depolarization.

[21] C. Bouchait and J. M. Lévy-LeBlond, Ref. 5, have made numerical estimates of the depolarization due to bremsstrahlung using the results of Ref. 6. For the experimental conditions of L. Dick, Ref. 1, they obtain a depolarization of about 7%. However, this may be an overestimate, see Ref. 14. Thus depolarization due to bremsstrahlung, apparently the most important process, is much too small to explain the experimental result of Dick *et al.*

ACKNOWLEDGMENTS

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APPENDIX: THE POLARIZATION IN AN ARBITRARY REACTION OF A SPIN-$\frac{1}{2}$ PARTICLE

We now consider the most general scattering process whereby a spin-$\frac{1}{2}$ particle of momentum p^I and spin direction a_j^I hits an unpolarized target of momentum p_T and another spin-$\frac{1}{2}$ particle of momentum p^F and spin direction a_i^F is measured in the final state. All other details of the final state are to be ignored. For the cross section, a_i^F and a_j^I can appear at most linearly so we have:

$$(d\sigma/d\Omega)(a_i^F,a_j^I)=A+b\cdot a_j^I+c\cdot a_i^F+(a_j^I)_\mu(a_i^F)_\nu f_{\mu\nu}. \quad \text{(A1)}$$

Now a_j^I, a_i^F are axial vectors and $d\sigma$ is a scalar. In order to conserve parity (we shall always assume this) A must be a scalar, b and c pseudovectors and f a tensor. Because only one of the final particles is detected, the only independent vectors we have to construct b, c, f are p^I, p^F, p_T. We see immediately that the only possible candidates are:

$$b\cdot a_j^I=a_j^I\cdot nB\,,$$

$$c\cdot a_i^F=a_i^F\cdot nC\,,$$

$$\begin{aligned}(a_j^I)_\mu(a_i^F)_\nu f_{\mu\nu} &= (a_j^I\cdot p^F)(a_i^F\cdot p^I)F_1+(a_j^I\cdot p_T)(a_i^F\cdot p^I)F_2 \\ &+(a_j^I\cdot p^F)(a_i^F\cdot p_T)F_3+(a_j^I\cdot p_T)(a_i^F\cdot p_T)F_4 \\ &+(a_j^I\cdot n)(a_i^F\cdot n)F_5, \quad \text{(A2)}\end{aligned}$$

where we define $n_\mu=\epsilon(\mu,p^I,p^F,p_T)=\epsilon_{\mu\nu\rho\sigma}p_\nu^I p_\rho^F(p_T)_\sigma$ and A, B, C, F_k are all scalar functions of the energy $(p^I+p_T)^2$, the momentum transfer, $(p^I-p^F)^2$ and the quantity $(p^F-p_T)^2$. We have eliminated the form $(a_j^I\cdot a_i^F)$ since the four-vectors p^I, p^F, p_T and n are all independent and we can thus construct the tensor $g_{\mu\nu}$ from bilinear combinations of them.

If we choose the basis set a_i^I, a_i^F, $i=1, 2, 3$ defined in Sec. II Eq. (6) *et seq.* then this general scattering can be written in either the c.m. or laboratory frame as[22,23]:

[22] A matrix of this form was discussed by S. Chandresekhar, *Radiative Transfer* (Dover Publications, Inc., New York, 1960), p. 37, Eq. (201). In this Appendix our object is to obtain the form of this matrix for elastic or inelastic collisions, using symmetry properties alone. For spin-$\frac{1}{2}$ particle scattering this matrix was also considered by W. H. McMaster, Rev. Mod. Phys. **33**, 8 (1961). It should be noted that in this reference everything is treated only in the c.m. system. Also Eq. (48) of this reference erroneously states that polarization can be produced from an unpolarized initial state in Møller scattering.

[23] See also the second article in Ref. 3 where general features of direct one-photon-exchange processes are discussed.

$$\begin{bmatrix} \phi^F & P_1{}^F \\ \phi^F & P_2{}^F \\ \phi^F & P_3{}^F \\ & \phi^F \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{21} & M_{22} & 0 & 0 \\ 0 & 0 & (a_3{}^I\cdot n)(a_3{}^F\cdot n)F_5 & (a_3{}^F\cdot n)C \\ 0 & 0 & (a_3{}^I\cdot n)B & A \end{bmatrix} \begin{bmatrix} \phi^I & P_1{}^I \\ \phi^I & P_2{}^I \\ \phi^I & P_3{}^I \\ & \phi^I \end{bmatrix}, \tag{A3}$$

$$M_{ij}=(a_i{}^I)_\mu(a_j{}^F)_\nu f_{\mu\nu} \quad i,\, j=1,\, 2. \tag{A4}$$

The flux of particles in the beam is ϕ, and we have used $n\cdot a_1=n\cdot a_2=0$, $a_3\cdot p^I=a_3\cdot p_T=a_3\cdot p^F=0$ for $a_i=a_i{}^I$, $a_i{}^F$. Let us call the square matrix on the right-hand side of (A3) M. Striking out the last row and column of M gives a matrix closely related to the matrix M of Sec. II, Eqs. (11) and (20). All quantities appearing in Eq. (A3) are Lorentz covariant, however, there is the usual rotation of coordinate axes describing the final spin [which is defined by Eq. (18)] when one transforms from the c.m. to the lab system. This notation makes it clear that, in general, the effect of a scattering is to *induce* a polarization (C terms) in addition to the rotation and shrinkage effects discussed in Sec. II of this article. Since there can be a final-state polarization in the direction n when there is no polarization present in the initial state, the term "shrinkage" is not appropriate, but the unpolarized component can still be calculated by subtracting the magnitude of the polarization vector, a^F, from 1. Because the term B in Eq. (A3) is not zero, the differential cross section summed over final spins, will depend on the initial polarization $P_3{}^I$. In the general case it is therefore simpler to work directly with the matrix (A3) and not to define a "shrinkage" S_i.

It is a well-known result[24] which applies when time reversal is good and the initial state is the same as the final state (elastic processes) is that $B=C$. This equality states that the polarizing power of the reaction is equal to the analyzing power. We shall now establish that in the c.m. under the same conditions $-M_{12}'=M_{21}'$ in (A4) and hence M has only 6 independent elements. The method of proof is the same as that used by Bell and Mandel[25] to show that $B=C$. We consider

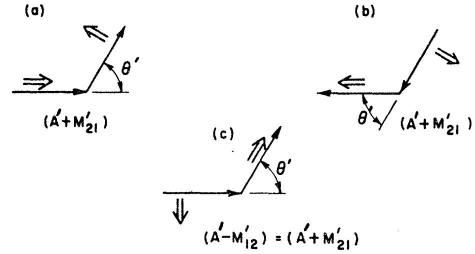

FIG. 6. Diagrams to illustrate the proof that $-M_{12}'=M_{21}'$ in the c.m. for an elastic, parity conserving, process. The cross section for (a) is proportional to $A'+M_{21}'$, etc.

a scattering through an angle θ' in the c.m. from a polarization state $a_1{}^I$ to a state $a_2{}^F$, Fig. 6(a), the time reversed reaction, Fig. 6(b) and the reaction $-a_2{}^I$, $a_1{}^F$, Fig. 6(c). Because the process is elastic, Fig. 6(b) is the same as Fig. 6(c) [rotate (b) by 180° about an axis which is in the direction of the initial momentum]. Thus we have $-M_{21}'=M_{12}'$. This proof fails in the laboratory frame because the three-momenta of the initial and final states are not equal and therefore (b) is not the same as (c). However, if there is no shrinkage [for example, Coulomb scattering, see Eq. (29)], then $M_{12}=-M_{21}$ is also true in the laboratory system because the final state is simply a rotation in the scattering plane times the initial state. One can readily obtain a condition on M in the laboratory by using the antisymmetry in the c.m. and the rotation defined in Eqs. (17) and (18). (This proof gives the relative sign between M_{12}' and M_{21}'; however, particular conventions in naming angles enter into this equation and therefore the sign will have to be checked in any specific application.) We have made use of this property of M to check the numerical results of Sec. V.

[24] L. Wolfenstein, Ann. Rev. Nucl. Sci. 6, 43 (1956).
[25] J. Bell and F. Mandel, Proc. Phys. Soc. (London) 71, 272, 867 (1958).