Fission Dynamics and the Statistical Theory*

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A posteriori justification of the statistical theory of nuclear fission is found in the shell effect on mass distribution and kinetic-energy distribution as well as in the constancy of kinetic energy at high-energy fission. *A priori* justification is found in the calculation showing that the scission time is longer than the characteristic nuclear time and the nuclear relaxation time. It is shown that the condition for the adiabatic approximation to be valid cannot be satisfied in the fission process from the saddle point to scission and therefore the validity of the adiabatic theory of fission is in doubt.

THE statistical theory of nuclear fission[1,2] has had some success, but discrepancies remain. It is appropriate to re-examine its basic assumption which states that the fission process is a slow process so that statistical equilibrium is established instantaneously throughout the process until the scission point when the two fragments separate apart. This is the more imperative in view of the existence of the adiabatic theory[3] which considers the mechanism of fission quite differently. According to this theory the ground state at the saddle point is a highly correlated state and is separated from the excited states by an energy gap. As the nucleus proceeds from the saddle point to scission, the energy gap prevents the exchange of energy with other states and fission may proceed exclusively by ground state. The distributions of fission products for low-energy fission may thus be determined by the properties of the ground state instead of by statistical considerations.

The statistical assumption may be justified *a posteriori* by the following considerations:

(1) The nuclear shell effect on mass distribution.[1] The heavy fragment peaks of the mass distribution curves of all fissioning nuclei coincide in the region where the fission fragment completes its 82-neutron and 50-proton shells. Furthermore, the position of the peaks remains the same at high-energy fission (below 50 MeV). It thus seems that the fission process is not determined by the initial condition as in any dynamical theory but is determined by the final condition as in a statistical theory. The final condition is that at the point of scission when the closed shells of the fragments are formed.

(2) The shell effect on kinetic energy distribution.[2] The determination by final condition instead of by initial condition is again evident in kinetic energy distribution.

(3) High-energy fission. The kinetic energy of fission fragments induced by high-energy particles, e.g., 90-MeV neutrons, is almost the same as that by thermal neutrons.[4] This indicates that the incident energy is rapidly dissipated into heat energy among the internal degrees of freedom and thus statistical equilibrium may soon be established. This also rules out the possibility of wave mechanism in fission. If fission is caused by a wave motion initiated by the incident particle, it is difficult to conceive that an increase in incident energy should not increase the kinetic energy of the fragments. Incidentally, the statistical theory[1] predicts that only 1 MeV out of the 90-MeV incident energy will go into the kinetic energy of the fragments. The increase in kinetic energy is too small to be noticeable, corroborating the experimental results.

Before the statistical assumption may be justified *a priori* we have to consider the dynamics of fission. The time involved in fission, of the order of 10^{-16} sec, is large compared with the characteristic nuclear time, the time required of a nucleon to cross the nuclear diameter, of the order of 2×10^{-22} sec. On this basis we may consider equilibrium to be established. Still it may be argued that most of the time is spent in reaching the saddle point, and the time from saddle point to scission may still be short. While we may believe equilibrium at the saddle point, equilibrium at the scission point, which is the basis of the statistical theory, is not obvious. Thus, we investigate the time required of a fissioning nucleus to proceed from the saddle point to the scission point; this time is defined as the scission time.

At the saddle point the potential energy is at a maximum and therefore the net force is zero—the electrostatic force which tends to produce longitudinal deformation is balanced off exactly by the surface tension. At the scission point the surface tension is reduced to zero and the force between the two fragments is just the Coulomb force of two deformed charge drops in contact, $F_s=Z_1Z_2e^2/D^2$, where Z_1 and Z_2 are the proton numbers of the two fragments and D is the distance between the two charge centers (which are very close to the mass centers and may be used to approximate the latter). The corresponding Coulomb energy $Z_1Z_2e^2/D$ cannot be greater than the experimentally observed kinetic

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1 Peter Fong, thesis, University of Chicago, 1953 (unpublished); Phys. Rev. **102**, 434 (1956).

2 Peter Fong, Phys. Rev. Letters **11**, 375 (1963).

3 Lawrence Wilets, Argonne National Laboratory Report No. ANL-6797, 1963 (unpublished), p. 406. See also Lawrence Wilets, *Theories of Nuclear Fission* (Clarendon Press, Oxford, 1964), Chap. 4.

4 J. Jungerman and S. C. Wright, Phys. Rev. **76**, 1112 (1949). The same conclusion is borne out in many later experiments.

energy of a pair of fission fragments, which is about 170 MeV. Using the latter value, we find the lower limit of D to be 1.8×10^{-12} cm and the upper limit of F_s to be 1.5×10^{8} dyn. During the scission time, the net force between the fragments increases from zero at the saddle point to F_s at scission and we may take F_s as its upper limit to calculate the lower limit of the scission time. To do so we need to know the increase of the distance between the charge centers ΔL from saddle point to scission, which may be estimated from the series of deformation shapes that Frankel and Metropolis[5] used to approximate the deformation process in spontaneous fission, the result being 8×10^{-13} cm. From the force, distance, and mass of the fragment we calculate the lower limit of the scission time to be 1.0×10^{-21} sec. This time is still 5 times longer than the characteristic nuclear time so that protons and neutrons have time to move back and forth between the two fragments and various modes of mass and charge division may be realized. To assure statistical equilibrium among all possible modes of mass and charge division we compare the lower limit of the scission time with the relaxation time of the nucleus which may be estimated by the magnitude of the imaginary part of the optical-model potential at the corresponding excitation energy, the result being 1×10^{-22} sec (thus, the nuclear mean free path is one-half of the nuclear diameter). The scission time is thus at least 10 times longer than the relaxation time and therefore any deviation from equilibrium will be given long enough time to return to equilibrium. The actual scission time may be much longer than the lower limit calculated, but the time for the part of the process just before scission, which is crucial to the statistical equilibrium at scission, should be comparable to the lower limit. The above conclusion is thus valid at the scission point. The statistical assumption as applied to mass and charge distributions is thus justified.

The above discussion does not tell us whether the statistical assumption is valid in kinetic energy distribution, i.e., whether the equilibrium between the translational and internal degrees of freedom is established. If it is, then the fragments at the scission point will have little kinetic energy (about 0.5 MeV for thermal neutron fission[1]); otherwise, this energy may be much greater. This is a point which should be investigated further.

An upper limit of the scission time may be obtained by considering the fission process to be very slow so that the kinetic energy of the fragments at the scission point is very small compared with the total amount 170 MeV, say, 1 MeV. The average force from the saddle point to scission is required to do an amount of work over the distance ΔL equal to this energy of 1 MeV; this leads to a magnitude of the average force equal to 2×10^{6} dyn. The upper limit of the scission time is calculated to be 9×10^{-21} sec; the range of the scission time is thus 1–9 times 10^{-21} sec. The lower limit of the relative velocity of the fragments at the scission point is calculated to be 1.8×10^{8} cm/sec.

Based on this information we now consider the adiabatic theory. In applying the adiabatic approximation of the time-dependent perturbation theory, we have to separate that part of the potential that is responsible for the tearing apart of the two fission fragments from the total Hamiltonian and treat it as a perturbation. In the present case the perturbation potential undergoes large changes and therefore we should use the following condition for the validity of the adiabatic approximation[6]:

$$(h/(\Delta E)^2)(\partial V/\partial t)\ll 1, \tag{1}$$

where h is the Planck constant, ΔE is the energy spacing at the ground state and $\partial V/\partial t$ is the rate of change of the perturbation potential. Near the scission point where the change of potential is fastest the perturbation potential is essentially the Coulomb potential and therefore

$$\partial V/\partial t=(Z_1Z_2e^2/D^2)(\partial D/\partial t). \tag{2}$$

Making use of the lower limit of the relative velocity of the fragments at the scission point obtained above for $\partial D/\partial t$, we find that even in the slowest scission process Eq. (1) demands that

$$\Delta E\gg 8.4\text{ MeV}. \tag{3}$$

The energy spacing at ground state for the present case of large deformation is in the neighborhood of 0.2 MeV (at most 1 MeV) and therefore the condition for the adiabatic approximation to be valid, Eq. (3), cannot be satisfied. The application of the adiabatic approximation in the fission process from the saddle point to scission is thus not valid. The change of the potential is sufficiently rapid to produce transitions of states just as in the process of Coulomb excitation.

Since the application of the statistical theory depends on many nuclear data including the nuclear masses, the deformation parameters, the level density function, etc., the remaining discrepancy of the theory may well originate from the insufficient knowledge of these data. The nuclear masses and the deformation parameters are subject to a nuclear shell effect which manifests itself in the mass and kinetic energy distributions in fission.[1,2] The level density function is also subject to a shell effect which may well explain the discrepancy of the statistical theory in the energy dependence of the mass distribution. These shell effects are not completely understood quantitatively. Only after more reliable information on them is available may the statistical theory be tested conclusively.

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[5] S. Frankel and N. Metropolis, Phys. Rev. **72**, 914 (1947).

[6] David Bohm, *Quantum Theory* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1951), p. 496.