at 1.248 MeV with relative intensities 5 and 9 (±15%), respectively. The 0.722-MeV transition from the spin-2 state to the spin-2 first excited state is 94% quadrupole and 6% dipole ($\delta=4.1\pm0.6$).

The introduction of a β condition in the study of γ-γ angular correlations has proved to be not too difficult and it should be very useful in reducing ambiguities in the interpretation of the γ rays involved in γ-γ cascades in complex beta decays.

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Expectation Values of Various Operators in the Triton

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The trial functions used in a calculation of the triton binding energy with realistic forces have been used to derive the expectation values of a number of operators. These include the Coulomb energy for point and for finite protons; various products of powers of the interparticle distances; and the charge and magnetic moment form factors as given by Schiff.

INTRODUCTION

As a by-product of our calculations of the triton binding energy[1] we have calculated the expectation values of a number of simple operators over the variational ground-state functions which we have used.

These operators are of three types:

(1). Coulomb energy operators. We have calculated the expectation value of the Coulomb energy for both point protons and for protons of finite size, with parameters as given by Pappademos.[2]

(2). Products of powers of the three interparticle distances, $r_{12}{}^{\alpha}r_{23}{}^{\beta}r_{31}{}^{\gamma}$.

(3). Charge and magnetic moment form factors. We have calculated the form factors F_1 and F_2 defined by Schiff.[3]

We give here the results for two distinct wave functions ψ_1 and ψ_2.

The first of these is the wave function used in the calculations of Blatt, Derrick, and Lyness[4]; the second is derived from this by adding a component representing a neutron bound to a deuteron, and is the function (A) of a previous note.[1] In each case, results are given for three potentials, which we label Hamada,[5] Yale,[6] and Gammel-Brueckner (GB).[7] The matrix elements have not been calculated for the (better) wave functions (B) and (C) of Ref. 2.

(1) Coulomb energy for point and for finite protons. The Coulomb energy operator for point protons is

$$C(r_{12})=e^2/r_{12}, \tag{1}$$

while for protons of finite size the appropriate expression has been given by Pappademos,[2] assuming the protons to be undistorted within the triton. Using the same parameters for the proton charge distribution as does Pappademos, we obtain the results given in Table I. In this table we include for reference the variational

TABLE I. The Coulomb energy of He^3 for point and finite proton (MeV).

Potential	GB		Hamada		Yale	
Wave functions	ψ_1	ψ_2	ψ_1	ψ_2	ψ_1	ψ_2
Point protons	0.692	0.616	0.717	0.549	0.691	0.520
Finite protons	0.661	0.593	0.685	0.532	0.662	0.505
$E(H^3)$	−5.72	−6.186	−2.57	−4.35	−2.54	−4.24

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TABLE II. Expected values of powers of the interparticle distances.[a,b]

Potential	GB		Hamada		Yale	
Wave function α,β,γ	ψ_1	ψ_2	ψ_1	ψ_2	ψ_1	ψ_2
0,0,0	3.0	3.0	3.0	3.0	3.0	3.0
1,0,0	7.85	8.89	7.49	10.3	8.08	10.8
2,0,0	26.6	34.7	23.1	48.6	27.8	53.7
3,0,0	1.16×10^2	1.73×10^2	86.1	3.02×10^2	1.20×10^2	3.50×10^2
4,0,0	6.45×10^2	1.07×10^2	3.82×10^2	2.36×10^3	6.29×10^2	2.86×10^3
1,1,0	23.2	30.1	20.4	41.1	24.5	45.4
2,1,0	92.4	1.37×10^2	69.7	2.30×10^2	97.0	2.66×10^2
3,1,0	4.9×10^2	8.0×10^2	2.9×10^2	1.70×10^3	4.8×10^2	$2.0(5)\times10^3$
2,2,0	4.30×10^2	7.1×10^2	2.0×10^2	1.5×10^3	4.3×10^2	1.8×10^3
1,1,1	76.5	1.13×10^2	59.0	1.7×10^2	82.1	2.05×10^2
2,1,1	3.4×10^2	5.75×10^2	2.1×10^2	1.1×10^3	3.5×10^2	1.0×10^3
−1,0,0	1.48	1.35	1.51	1.21	1.44	1.17
−2,0,0	0.944	0.809	0.945	0.667	0.875	0.625
−3,0,0	0.757	0.626	0.730	0.478	0.658	0.439
−4,0,0	0.741	0.601	0.671	0.422	0.589	0.378
−5,0,0	0.85	0.69	0.71	0.445	0.60	0.380
−6,0,0	1.1	0.890	0.830	0.50	0.680	0.43
$R_{\rm rms}$	1.72	1.96	1.60	2.32	1.76	2.44

[a] $\langle\alpha,\beta,\gamma\rangle\equiv\langle r_{12}{}^{\alpha}r_{23}{}^{\beta}r_{31}{}^{\gamma}\rangle$+cyclic terms.
[b] Lengths are in $F=10^{-13}$ cm.

estimates of the triton binding energy given by these wave functions.

(2) *The rms radius and other measures of the size of the nucleus.* A measure of the matter distribution is given by the expectation values of operators of the form

$$\langle\alpha,\beta,\gamma\rangle\equiv\langle\psi|W(\alpha,\beta,\gamma)|\psi\rangle,$$
$$W(\alpha,\beta,\gamma)\equiv r_{12}{}^{\alpha}r_{23}{}^{\beta}r_{31}{}^{\gamma}+r_{23}{}^{\alpha}r_{31}{}^{\beta}r_{12}{}^{\gamma}+r_{31}{}^{\alpha}r_{12}{}^{\beta}r_{23}{}^{\gamma}. \quad (2)$$

We have calculated these expectation values for a range of α, β, γ; the results are given in Table II.

(3) *Charge and magnetic moment form factors.* Schiff has given[3] formulas for the charge and magnetic moment form factors, including only S-state contributions. In this approximation the results are expressible in terms of two form factors F_1 and F_2 defined as follows: If we expand the trial function in terms of the angular functions Y_i of Ref. 8,

$$\psi=\sum_{i=1}^{10,2} f_i\mathcal{Y}i,$$

then

$$F_1(q)=\int e^{i\mathbf{q}\cdot\mathbf{r}_1}f_1{}^2d\tau,$$

$$F_2(q)=\int\{[\exp(i\mathbf{q}\cdot\mathbf{r}_1)-\exp(i\mathbf{q}\cdot\mathbf{r}_2)]f_1f_{3,2}-\sqrt{3}\exp(i\mathbf{q}\cdot\mathbf{r}_2)f_1f_{3,1}\}d\tau.$$

In these formulas, f_1 is the radial function of the principal (space symmetric) S state, while $f_{3,1}$ and $f_{3,2}$ are the two components of the S state of mixed symmetry.

Other states contribute in principle to F_1 and F_2, but their contribution is neglected by Schiff. The expectation values of F_1 and F_2 are given in Table III; the accuracy of the numerical integration is such that for $q\lesssim2$, the results are expected to be accurate to better than 1%, while for $q>3$, the accuracy falls off rapidly.

[8] G. Derrick and J. M. Blatt, Nucl. Phys. 8, 310 (1958).

DISCUSSION

Any comparison of these results with experiment must be preceded by the obvious caution that they represent expectation values over a variational wave function; they are not in themselves variational estimates, nor is there any way of estimating how they differ from the exact results for the potentials quoted. One estimate of their significance is given by comparing

TABLE III. Values of the form factor $F_1(q)$.[a]

Potential	GB		Hamada		Yale	
Wave function q	ψ_1	ψ_2	ψ_1	ψ_2	ψ_1	ψ_2
0	0.916	0.923	0.916	0.915	0.924	0.929
0.2	0.898	0.899	0.911	0.897	0.905	0.894
0.4	0.848	0.834	0.868	0.818	0.850	0.801
0.6	0.773	0.741	0.801	0.707	0.768	0.677
0.8	0.683	0.635	0.716	0.588	0.671	0.547
1.0	0.588	0.528	0.623	0.473	0.569	0.429
1.2	0.495	0.430	0.527	0.373	0.471	0.329
1.4	0.408	0.343	0.436	0.288	0.381	0.248
1.6	0.330	0.269	0.353	0.220	0.303	0.185
1.8	0.262	0.207	0.280	0.165	0.237	0.136
2.0	0.205	0.157	0.218	0.123	0.182	0.099
2.2	0.16	0.12	0.17	0.090	0.14	0.071
2.4	0.12	0.086	0.12	0.066	0.10	0.050
2.6	0.087	0.062	0.092	0.048	0.074	0.035
2.8	0.062	0.043	0.067	0.034	0.052	0.023
3.0	0.04	0.03	0.05	0.02(5)	0.03(5)	0.01(5)
3.2	0.03	0.02	0.03	0.02	0.02	0.01
3.4	0.02	0.01	0.02	0.01	0.01	0.005

[a] q is measured in F^{-1}.

TABLE IV. Values of the form factor $F_2(q)$.[a]

Potential	GB		Hamada		Yale	
Wave function	ψ_1	ψ_2	ψ_1	ψ_2	ψ_1	ψ_2
q						
0	0	0	0	0	0	0
0.2	-2.09×10^{-4}	-6.34×10^{-4}	2.73×10^{-5}	-2.73×10^{-3}	2.69×10^{-4}	-2.84×10^{-3}
0.4	-8.03×10^{-4}	-2.37×10^{-3}	1.16×10^{-4}	-9.19×10^{-3}	9.78×10^{-4}	-8.74×10^{-3}
0.6	-1.69×10^{-3}	-4.72×10^{-3}	2.45×10^{-4}	-1.48×10^{-2}	1.89×10^{-3}	-1.33×10^{-2}
0.8	-2.74×10^{-3}	-6.98×10^{-3}	3.93×10^{-4}	-1.71×10^{-2}	2.72×10^{-3}	-1.47×10^{-2}
1.0	-3.82×10^{-3}	-8.56×10^{-3}	5.34×10^{-4}	-1.65×10^{-2}	3.30×10^{-3}	-1.34×10^{-2}
1.2	-4.79×10^{-3}	-9.24×10^{-3}	6.44×10^{-4}	-1.42×10^{-2}	3.51×10^{-3}	-1.10×10^{-2}
1.4	-5.56×10^{-3}	-9.17×10^{-3}	7.07×10^{-4}	-1.13×10^{-2}	3.39×10^{-3}	-8.49×10^{-3}
1.6	-6.07×10^{-3}	-8.61×10^{-3}	7.23×10^{-4}	-8.60×10^{-3}	3.01×10^{-3}	-6.32×10^{-3}
1.8	-6.30×10^{-3}	-7.81×10^{-3}	6.87×10^{-4}	-6.29×10^{-3}	2.47×10^{-3}	-4.68×10^{-3}
2.0	-6.26×10^{-3}	-6.90×10^{-3}	6.17×10^{-4}	-4.47×10^{-3}	1.88×10^{-3}	-3.52×10^{-3}
2.2	-6.0×10^{-3}	-6.0×10^{-3}	5.2×10^{-4}	-3.1×10^{-3}	1.3×10^{-3}	-2.7×10^{-3}
2.4	-5.6×10^{-3}	-5.1×10^{-3}	4.0×10^{-4}	-2.2×10^{-3}	7.7×10^{-4}	-2.2×10^{-3}
2.6	-5.0×10^{-3}	-4.3×10^{-3}	2.8×10^{-4}	-1.6×10^{-3}	3.5×10^{-4}	-1.8×10^{-3}
2.8	-4.4×10^{-3}	-3.6×10^{-3}	1.7×10^{-4}	-1.2×10^{-3}	2.8×10^{-5}	-1.5×10^{-3}
3.0	-4×10^{-3}	-3×10^{-3}	7×10^{-5}	-1×10^{-3}	-2×10^{-4}	-1×10^{-3}
3.2	-3×10^{-3}	-2×10^{-3}	-8×10^{-6}	-8×10^{-4}	-3×10^{-4}	-1×10^{-3}
3.4	-3×10^{-3}	-2×10^{-3}	-7×10^{-5}	-7×10^{-4}	-4×10^{-4}	-8×10^{-4}

[a] q is measured in F^{-1}.

the results for the two variational wave functions. The differences are in some cases quite large; they reflect chiefly the feature that ψ_2 extends out rather further than ψ_1, which is almost certainly too compact for the Hamada and Yale potentials. The sensitivity of each type of operator to various details of the wave function is discussed below.

For a given trial function, the results obtained are limited in accuracy by the numerical integration method used. The accuracy attained is estimated to be better than 1% except in special cases where there was severe cancellation, or where contributions from large interparticle distances were dominant. The accuracy estimated in each case is indicated in the tables; we have kept one more figure than can be guaranteed.

(*1*) *Coulomb energy and rms radius.* The Coulomb energy and rms radius of He^3 give two ways of estimating the size of the system; they are insensitive to the fine details of the wave function. Experimentally, we have

$$E_{\text{Coul}}=0.764 \text{ MeV},$$
$$R_{\text{rms}}=1.78\pm0.09 \text{ F}.$$

The "experimental" R_{rms} has been calculated by subtracting from the measured He^3 charge radius that of the free proton.[9] Compared with our results, we see that our calculated Coulomb energies are too low for all potentials and both wave functions, while ψ_1 gives reasonable rms radii, those for ψ_2 being too large. The difference in R_{rms} between ψ_1 and ψ_2 is easily accounted for. For the Hamada and Yale potentials, ψ_1 was restrained from spreading too far by its manner of selection.[1] This constraint was removed for ψ_2, and these wave functions spread in a manner consistent with their low binding energies. The GB potential gives higher binding energies, and for this potential ψ_1 and ψ_2 differ much less.

The consistently low Coulomb energies cannot be explained in this way. The values we obtain are very similar to those found by Pappademos[2b] using wave functions which fit the rms radius well and are adjusted to represent the two-body correlations and asymptotic form closely. These results are consistent with a deviation of about 0.3%–0.5% from charge symmetry for the S-wave nn and pp interactions.

(*2*) *Correction for finite size of the proton.* The differences between the Coulomb energy for "point" and "finite" protons are about 5% in all cases. This agrees with the estimate of Pappademos, and of Ohmura.[10]

(*3*) *Form factors.* The electron scattering data have been analyzed by Schiff[3] to give "experimental" values of F_1 and F_2. These values are reproduced in Table V; they agree poorly with all of the "theoretical" form factors of Tables III and IV.

TABLE V. "Experimental" form factors from Schiff (Ref. 7).[a]

q^2	q	F_L	F_0	F_1	F_2
1	1	0.649	0.731	0.676	0.082
1.5	1.225	0.481	0.479	0.480	−0.002
2	1.414	0.396	0.422	0.405	0.026
2.5	1.581	0.343	0.422	0.369	0.079
3	1.732	0.293	0.380	0.322	0.087
3.5	1.871	0.245	0.331	0.274	0.086
4	2	0.219	0.314	0.251	0.095
4.5	2.121	0.154	0.207	0.172	0.053
5	2.236	0.134	0.145	0.138	0.012

[a] $F_1=\frac{2}{3}F_L+\frac{1}{3}F_0$: $F_2=F_0-F_L$.

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(*a*) F_1. The values of F_1 in Table III are consistently lower than the experimental values, over the region of momentum transfer measured. Since the normalization is such that $F_1(0)=1$,[11] our wave functions are *too smooth*. With our present trial functions, this discrepancy is not significant.

(*b*) F_2. The values of F_2 given by ψ_1 are much too low; those given by ψ_2 are also much too low and have the wrong sign. Neither of these results is significant. The low magnitudes reflect the percentages of state 3 which we find; these are rather lower than the 4% required by Schiff to fit F_2. It is not clear whether a better trial function would increase the percentage of state 3; but, in any event, the discrepancies are of the general order of magnitude of the contributions expected from the D states. No significance can be attached to the sign of F_2 given by ψ_2. This wave function has the form $\psi_2=\alpha\psi_1+\beta\psi_{n+d}$, where ψ_{n+d} is constructed to represent loosely a neutron bound to a deuteron. The major contribution to state 3 in ψ_2 comes from ψ_{n+d}, and from the manner of construction of ψ_{n+d} its amplitude is *fixed* to be equal (and of opposite sign) to that of the state 1 part of ψ_{n+d}. Hence, the sign of F_2 has been essentially fixed in advance in this way.

[11] In Schiff's approximation. Actually $F_1(0)$ gives the probability of the principal S state.

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Be^9 States from the Reaction $Li^6(Li^7,\alpha)Be^9$†

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Li^6 and Li^7 ions accelerated to energies between 2.15 and 3.0 MeV were used to study the reaction $Li^6(Li^7,\alpha)Be^9$. Alpha particles were distinguished from other reaction products by a dE/dX-E system based upon a general-purpose digital computer. States were observed at 0.00, 1.75, 2.43, 3.04, and 11.9 MeV excitation in Be^9 in the presence of a very prominent continuum. The width of the 11.9±0.2-MeV state was measured at 500±100 keV. No other states up to an upper limit of 13.0-MeV excitation could be observed above the continuum. The yield of the 2.43-MeV state at 0° was found to have an energy dependence different from the ground and 3.04-MeV states. Angular distributions of the ground-state alpha particles obtained at Li^6 bombarding energies of 2.2 and 3.0 MeV were observed to be slightly energy-dependent.

INTRODUCTION

RECENT papers by Garin *et al.*[1,2] and Karadeny *et al.*[3] at Saclay, France, studying the alpha particles and neutrons from the reaction Li^6 on Li^7 at 1.8 MeV have concluded that the predominant reaction sequence contributing to the alpha-particle continuum is

$$Li^6+Li^7\rightarrow\alpha_1+Be^{9*},$$
$$Be^{9*}\rightarrow\alpha_{11}+He^5,$$
$$He^5\rightarrow\alpha_{111}+n.$$

It is postulated that α_1 comes from a state in Be^9 having a Li^7+d character and an excitation of between 11 and 15 MeV. Garin *et al.*[1] have reported that they have been unable to observe this state directly due to the low energy of α_1, lack of particle identification, and the interference of the contamination reaction:

$$Li^6+p\rightarrow He^3+He^4+4.02\text{ MeV}.$$

In the present experiment the target and projectile have been interchanged to remove the contamination reaction products. A dE/dX and E detection system was employed to identify low-energy alpha particles.

In addition, energy spectra have been taken at 0° of alpha particles from the ground and first three excited states of Be^9 at Li^6 energies of 2.15, 2.6, and 3.0 MeV. Angular distributions of the ground state were obtained at 2.2 and 3.0 MeV.

EXPERIMENTAL

Lithium ions were accelerated by the State University of Iowa Van de Graaff. The momentum of the ions was determined by measuring the current applied to a bending magnet which deflected the beam through 25°. The beam was defined by a series of apertures which assured homogeneity of energy to 1%.

The relative angular distributions were measured with the target chamber shown in Fig. 1. The axis of the chamber is inclined at 20°30′ to the vertical, which allows rotation of the movable counter from 0° to 139°. A fixed monitor consisting of a solid-state detector is located at 90° to the beam path.

The particle identification system consisted of a

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