upon isotropic scattering of most particles off magnetic inhomogeneities. As a result, no such reflection is assumed in this paper.

3. Conclusion

It is not yet possible to choose confidently between various theories of cosmic-ray origins. Current cosmic ray data can be reconciled with many. Even the prediction of a break in the energy and mass spectra between 10^{15}–10^{17} eV can be obtained from other theories. It comes from the assumption of rigidity dependence in cosmic-ray propagation. Any model which allows primaries of rigidity 10^{15} V to begin escaping prematurely will predict a significant steepening of the energy spectrum and a shift in abundances to the heavies. (This process could occur in the sources themselves or in the diffusion region.) If this model allows a second less intense population of particles extending to 10^{20} eV, current data can be fairly well fitted.

However, many other approaches can be used, the eclectic diffusion model seems to be worth exploring further. In recent years persuasive arguments against an eclective model have questioned the smooth fit of different fluxes into a single straight energy spectrum. It is now becoming increasingly clear that fine structure in the energy and mass spectra can be found. These weaken the arguments against an eclectic approach and suggest the advantages of carefully reexamining it.

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Birefringence of the Vacuum

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It is known that Maxwell's equations become nonlinear if the effect of virtual electron-positron pair creation is included. The vacuum, thus behaving like a polarizable continuum, is shown to exhibit the phenomenon of birefringence.

THE fact that in quantum electrodynamics two photons can interact through the creation of virtual electron-positron pairs leads to additional terms in the Lagrangian density which are biquadratic in field intensities. It is shown in this note that this causes birefringence of the vacuum. An estimate of the Kerr constant of the vacuum gives a value of $(7/90\pi)\times(e^2/\hbar c)^2(1/mc^2)(\hbar/mc)^3\Lambda^{-1}$, where Λ is the wavelength of the light.

The interaction between two photons has been treated[1–5] in quantum electrodynamics by considering the production of virtual pairs in the vacuum. It is now well known that this phenomenon of scattering of one photon by another (Fig. 1) leads to a nonlinear interaction between electromagnetic fields in vacuum. The S-matrix element $(k_3,k_4|S|k_1,k_2)$ for the scattering of two photons of 4-momenta k_1 and k_2 into k_3 and k_4 can be written in the following manner:

$$(k_3,k_4|S|k_1,k_2)=\left(k_3,k_4\left|-i\int \mathfrak{L}_{I\mathrm{eff}}d^4x\right|k_1,k_2\right), \quad (1)$$

where the effective Lagrangian density $\mathfrak{L}_{I\mathrm{eff}}$ must satisfy the requirements of relativistic invariance and gauge invariance and hence must involve the invariants $(\frac{1}{2}F_{\mu\nu}F^{\mu\nu})^2=(\mathbf{B}^2-\mathbf{E})^2$ and $(\frac{1}{8}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma})^2=(\mathbf{B}\cdot\mathbf{E})^2$ of the electromagnetic field, where $F_{\mu\nu}=\partial_\nu A_\mu-\partial_\mu A_\nu$ and the indices μ and ν take the values 0, 1, 2, 3. According to Schwinger,[5] and Karplus and Neuman,[4] the result for Fig. 1 is given by

$$\mathfrak{L}_{I\mathrm{eff}}=\frac{2\alpha^2}{45(4\pi)^2m^4}[(\mathbf{B}^2-\mathbf{E}^2)^2+7(\mathbf{B}\cdot\mathbf{E})^2], \quad (2)$$

where the fine structure constant $\alpha=e^2$ (in naturalized Gaussian units, $\hbar=c=1$) and m is the mass of the electron. The complete Lagrangian for the electromagnetic field, including the effect of virtual pair

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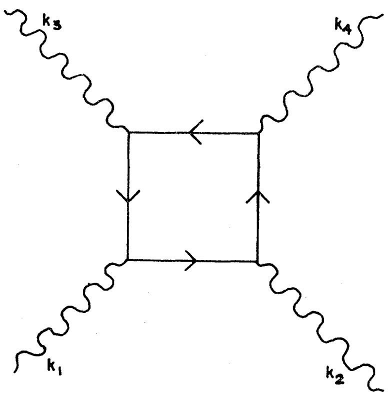

FIG. 1. Feynman diagram for photon-photon scattering.

creation to order α^2 is now given by

$$L=\int d^4x\left[\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}+\frac{2\alpha^2}{45(4\pi)^2m^4}\{(\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu})^2+7(\tfrac{1}{8}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma})^2\}\right]. \quad (3)$$

The modified Maxwell's equations can be obtained by varying the Lagrangian with respect to the four potentials A_μ. This results in the usual set of Maxwell's equation with the following redefinitions[6]

$$D_i=\sum_{j=1}^{3}\epsilon_{ij}E_j=\sum_j(\delta_{ij}+\epsilon_{ij}')E_j, \quad (4)$$

$$H_i=\sum_{j=1}^{3}\mu_{ij}B_j=\sum_j(\delta_{ij}+\mu_{ij}')B_j, \quad (5)$$

where

$$\epsilon_{ij}'=\frac{1}{45\pi}\frac{\alpha^2}{m^4}[2(E^2-B^2)\delta_{ij}+7B_iB_j], \quad (6)$$

$$\mu_{ij}'=\frac{1}{45\pi}\frac{\alpha^2}{m^4}[2(E^2-B^2)\delta_{ij}-7E_iE_j]. \quad (7)$$

From Eqs. (4)–(7), it is clear that ϵ_{ij}' and μ_{ij}' are interpretable as the electric and magnetic susceptibilities of the vacuum.

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BIREFRINGENCE OF THE VACUUM

Let us consider a plane wave traveling in the z direction between two plates of a capacitor having a uniform static field E_s along the x direction ($E_x=E_s$, $E_y=E_z=0$). Since the fields associated with the wave are small compared with the static field E_s, one may approximate $\epsilon_{ij}'\simeq(2/45\pi)(\alpha^2/m^4)E_s^2\delta_{ij}$, so that

$$\mathbf{D}=(1+2\lambda E_s^2)\mathbf{E}, \quad (8)$$

where

$$\lambda=(1/45\pi)(\alpha^2/m^4). \quad (9)$$

The principal components of μ_{ij}' are given by

$$\mu_{xx}'\simeq-5\lambda E_s^2 \quad \text{(parallel component)}$$
$$\left.\begin{matrix}\mu_{yy}'\simeq2\lambda E_s^2\\ \mu_{zz}'\simeq2\lambda E_s^2\end{matrix}\right\} \quad \text{(perpendicular components)}. \quad (10)$$

Therefore

$$H_{\parallel}\simeq(1-5\lambda E_s^2)B_{\parallel},$$
$$H_{\perp}\simeq(1+2\lambda E_s^2)B_{\perp}, \quad (11)$$

where $H_{\parallel}$, $B_{\parallel}$ correspond to the parallel components of the field and $H_\perp$, $B_\perp$ to the perpendicular components. The phase velocities $c_{\parallel}$ and $c_\perp$ of the wave in the parallel (extraordinary ray) and perpendicular (ordinary ray) directions, respectively, are given by

$$c_{\parallel}=1/[(1+2\lambda E_s^2)(1-5\lambda E_s^2)]^{1/2}\simeq1/(1-\tfrac{3}{2}\lambda E_s^2)>1, \quad (12)$$

$$c_\perp=1/[(1+2\lambda E_s^2)^2]^{1/2}=1/(1+2\lambda E_s^2)<1. \quad (13)$$

Thus, the wave travels as if passing through a birefrigent medium, the component of the **B** vector at right angles to E_s traveling more slowly, and the component parallel to E_s traveling more rapidly than in the absence of the strong static field.

In order to compute the Kerr constant for the vacuum we make use of the following equation:

$$n_0-n_e=B\Lambda E_s^2, \quad (14)$$

where n_0 and n_e are the ordinary and extraordinary refractive indices, Λ the wavelength of light, and B is the Kerr constant. From Eqs. (12) and (13), for the vacuum $n_0-n_e\simeq\frac{7}{2}\lambda E_s^2$ so that $B\simeq\frac{7}{2}\lambda/\Lambda$. For sodium light, $\Lambda\sim5\times10^{-5}$ cm and therefore the estimated value of $B\sim2\times10^{-27}$ cm^2/erg ($\sim2\times10^{-34}$ m/V^2). This may be compared with the Kerr constant for water, viz., B(water)$\sim5\times10^{-14}$ m/V^2, which is a factor 10^{20} larger than for vacuum.