final state involving the same particles as in the initial state. The effect due to this process is not included in the iC term. This is the reason why the C term may interfere with the term $\exp\frac{1}{2}(A_0+A_1t)$.

Foley *et al.*[2,4] have adopted $d\sigma/dt=\exp(a+bt+ct^2)$ as an expression of $d\sigma/dt$ in the small $|t|$ region. This empirical formula gives better fits for the diffraction peaks than $d\sigma/dt=\exp(A_0+A_1t)$ because of the existence of an additional parameter. If we adopt, in the expression (6), $\exp\frac{1}{2}(a+bt+ct^2)$ instead of $\exp\frac{1}{2}(A_0+A_1t)$, it is difficult to obtain the empirical formula which can fit the experimental data for scattering over all angles, particularly the large angles.[17] In order to perform partial-wave analysis, it is important to take into account the character of large-angle scattering, as was emphasized previously.[9]

[17] Recently Orear tried to express $d\sigma/d\Omega$ in terms of transverse momentum $p_\perp$ [J. Orear, Phys. Rev. Letters **12**, 112 (1964)]. However, his expression is applicable in a limited $|t|$ region. Krisch has expressed $d\sigma/d\Omega$ by a sum of three exponentials [A. D. Krisch, Phys. Rev. Letters **11**, 217 (1963), and private communication].

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Measurement of the Form Factor Ratio in $K_{\mu_3}^-$ Decay

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150 000 photographs from a propane-Freon bubble chamber have been scanned for examples of in-flight $K_{\mu3}$-mode decay of 440-MeV/c K^- mesons. 150 candidates have been found from which 138 events have been selected for final analysis. These events have been analyzed assuming $K_{\mu3}$ decay to be dominated by vector coupling with form factors that may be considered energy-independent. The experimentally observed muon total-energy spectrum has been used in a likelihood calculation to determine the ratio of the form factors. The data strongly favor the ratio $\xi=0$, which yields a muon-energy spectrum favoring low-energy decay muons.

PREVIOUS investigations of form factor behavior in the three-body leptonic decays of the long-lived K mesons have utilized the decays $K^+\rightarrow \ell^+ +\pi^0+\nu$ and $K_2{}^0\rightarrow \ell^\pm+\pi^\mp+\nu(\bar{\nu})$. In this paper we report a measurement of the form factor ratio in the hitherto ignored decay, $K^-\rightarrow\mu^-+\pi^0+\bar{\nu}$.

The usual theoretical description[1] of the generalized decay process, $K\rightarrow\mu+\pi+\nu$, assumes a universal $V-A$ interaction. For the decay $K^-\rightarrow\mu^-+\pi^0+\bar{\nu}$, this leads to a matrix element of the form:

$$M=\tfrac{1}{2}[f_+(q^2)Q_\lambda+f_-(q^2)q_\lambda][\bar{u}_\mu\gamma_\lambda(1+\gamma_5)v_\nu], \quad (1)$$

where

$$Q_\lambda\equiv P_{K;\lambda}+P_{\pi;\lambda};\ q_\lambda\equiv P_{K;\lambda}-P_{\pi;\lambda};\ q^2\equiv q_\lambda q_\lambda, \quad (1a)$$

and f_+ and f_-, the form factors, are scalar functions of the invariant q^2. If time-reversal invariance holds, they may be taken to be real. Their dependence on q^2 is expected to be mild and to a first approximation they may be assumed to be constants. Their ratio, $\xi\equiv f_-/f_+$, is reasonably accessible to experimental determination in several ways. The shape of the muon-energy spectrum depends on the value of ξ as does the muon longitudinal polarization. In this experiment we utilize the first approach.

In an effort to evaluate the parameter ξ from the shape of the muon-energy spectrum, we have analyzed 138 examples of the decay process: $K^-\rightarrow\mu^-+\pi^0+\bar{\nu}$. These events were identified in a scan of 150 000 bubble chamber photographs taken using the 30-in. Lawrence Radiation Laboratory bubble chamber. The chamber was filled with a propane-Freon mixture (24% C_3H_8–76% CF_3Br by weight) and operated in a 13-kG magnetic field. The beam particles were K^- mesons from the 800-MeV/c separated beam of Murray *et al.*,[2] degraded first to 550 MeV/c by a copper absorber upstream from the chamber, and finally to 440 MeV/c by a 1-in. copper plate placed inside the chamber 5 in. from the beam entrance. From the range distribution of stopping beam tracks, we estimate the beam momentum to be 440±25 MeV/c at the downstream edge of the copper plate.

At the scanning level we have accepted 419 examples of beam particle decay satisfying the following three criteria: (1) the decay has only one charged secondary; (2) the charged secondary comes to rest in the chamber,

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[1] See, for example, J. Bernstein and S. Weinberg, Phys. Rev. Letters **5**, 481 (1960); N. Brene, L. Egardt, and B. Qvist, Nucl. Phys. **22**, 553 (1961). The relation $R\equiv W(K_{\mu3})/W(K_{e3})=0.651+0.126\xi+0.0189\xi^2$ is taken from this reference. See also P. Dennery and H. Primakoff, Phys. Rev. **131**, 1334 (1963).

[2] P. Bastien, O. Dahl, J. Murray, M. Watson, R. G. Ammar, and P. Schlein, in *Proceedings of an International Conference on Instrumentation for High Energy Physics, Lawrence Radiation Laboratory, University of California at Berkeley, 1960* (Interscience Publishers, Inc., New York, 1961).

then decays into an electron; (3) the K^- decay point is within 27.5 cm of the copper plate. Assuming the nominal value of 440 MeV/c at the plate, this places a lower bound of 300 MeV/c on the K^- momentum at the time of decay. The second condition affords a good separation between π^- and μ^- secondaries and provides the most accurate determination of the muon momentum. A lower bound on the K^- momentum is necessary to avoid mistaking for decays, slow K^- nuclear interactions. The lower bound also limits the size of the uncertainty in our determination of the K^- momentum. We determine this momentum from the length of the K^- track between the copper plate and the point of decay, the known rate of energy loss for K mesons in the propane-Freon mixture, and the nominal beam momentum at the plate (440 MeV/c). Because the beam track trajectories tend to be distorted in the heavy liquid due to multiple scattering and energy loss, we have found this method to be considerably more reliable than calculating beam track momenta from track curvature. On the other hand, our procedure suffers in that the propagated error in the beam momentum rises nonlinearly with track length and a cutoff is correspondingly dictated. At 27.5 cm from the plate our estimate of beam momentum is 300 ± 50 MeV/c.

A histogram of the muon total energy in the K^- rest frame is shown in Fig. 1(a). All 419 events are included and no corrections for losses or biases have been made. The large peak centered around 260 MeV comes from $K_{\mu 2}^-$ events. $K_{\mu 2}^-$ and $K_{\mu 3}^-$ decays have the same topology on the scanning board; no attempt was made to distinguish between these two decays at the scanning level.

To determine the value of ξ from these data, we calculate a likelihood function L, and from it ascertain the relative likelihoods for various values of ξ. The muon total energy spectrum in the K^- rest frame has the form[1]:

$$W(\xi,E_\mu)\propto[(\Delta-2m_KE_\mu-m_\pi^2)/(\Delta-2m_KE_\mu)]^2 \times[E_\mu^2-m_\mu^2]^{1/2}[m_KE_\mu(\Delta-2m_KE_\mu) +\tfrac{1}{4}m_\mu^2(m_KE_\mu-m_\mu^2)+\tfrac{1}{2}\xi m_\mu^2(2m_K^2-3m_KE_\mu+m_\mu^2) +\tfrac{1}{4}\xi^2m_\mu^2(m_KE_\mu-m_\mu^2)], \quad (2)$$

where m_K, m_π, and m_μ are the K^-, π^0, and μ^- masses, respectively, and $\Delta=m_K^2+m_\mu^2$. For the idealized case (no scanning biases or losses from finite chamber volume) the likelihood function would be simply

$$L(\xi)=\prod_i W(\xi,E_i), \quad (3)$$

where i indexes our events and E_i is the muon total energy for the ith event. In our case we use (3) in the altered form:

$$L(\xi)=\prod_i\left[W(\xi,E_i)\Big/\int_{E_{l;i}}^{E_{u;i}}W(\xi,E)dE\right]. \quad (4)$$

The normalization integral in the denominator of (4) corrects each event for decay configurations that would have been undetected due to scanning bias and/or the muon leaving the chamber, given the position of the event in the chamber and its rest-frame decay angles. Thus, the energy limits, $E_{l;i}$ and $E_{u;i}$, are those muon energies (in the K^- rest frame) that bound the portion of the energy spectrum which could have yielded recognizable decay configurations assuming the position and decay angles of the ith event. In this respect, our corrections for chamber size and scanning bias are all contained in the calculation of these energy limits.

To be consistent with this calculation, we have selected for consideration only those $K_{\mu 3}^-$ decays satisfying the following criteria: (1) the entire event lies within a given fiducial volume; (2) the μ^- track length exceeds one cm; (3) the laboratory decay angle exceeds ten degrees. These requirements are sufficient to insure a reasonably high, constant scanning efficiency.

In addition to the above three requirements we have included a fourth for rather different reasons. Our ability to separate $K_{\mu 2}^-$ events from $K_{\mu 3}^-$ events is not good enough to prevent the last 5 MeV of the $K_{\mu 3}$ spectrum from being seriously contaminated by the two body decays. Figures 1(a) and 1(b) show this very clearly. For this reason we have calculated L, always assuming an upper cutoff on the spectrum to avoid this region. We have found the calculation to be quite insensitive to the position of this cutoff as long as it is below 235 MeV. The actual value used in the calculations we report here was 220 MeV. This particular value was selected with consideration for the approximate symmetry of the $K_{\mu 2}$ distribution and its apparent upper limit of 300 MeV.

If one assumes both the universal $V-A$ interaction and constant form factors, it is possible to show that the ratio of the $K_{\mu 3}$ decay rate to the K_{e3} decay rate is given by the quadratic expression[1]:

$$R=0.651+0.126\xi+0.0189\xi^2. \quad (5)$$

It is clear that from a measurement of R, (5) will yield two possible choices for ξ. Roe *et al.*[3] have reported a value for R of 1.0 ± 0.2 for K^+ decay. If this value is used in (5), one determines $\xi=2$, -9. A very recent result[4] from a larger sample of the same events gives $R=0.63\pm0.1$. For this case one finds $\xi=0$, -6.5.

In Fig. 1(b) we have replotted that portion of the observed spectrum, Fig. 1(a), that we will consider further. The data in Fig. 1(b) have been edited for fiducial volume and the minimum acceptance criteria mentioned earlier for μ^- track length and laboratory decay angle. Figures 1(c) and 1(d) are $K_{\mu 3}$ spectra generated artificially by a Monte Carlo procedure.

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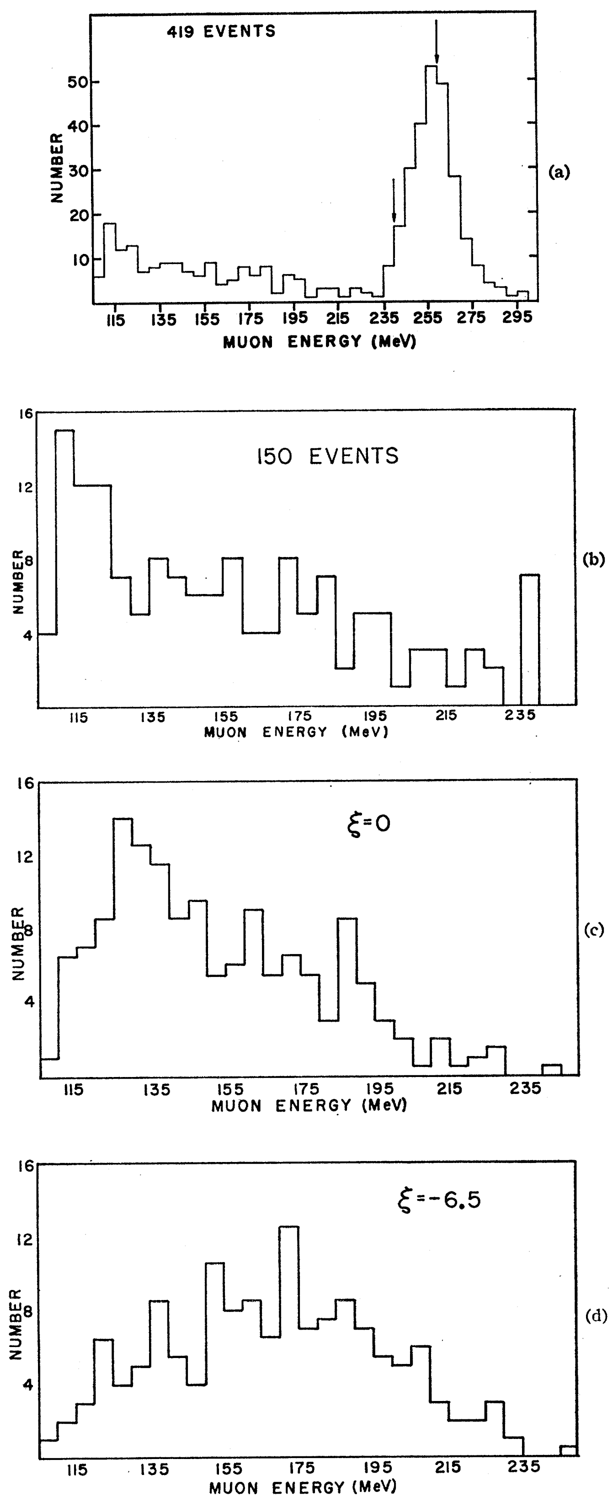

FIG. 1. (a) Observed muon total energy spectrum. Arrows indicate upper limit of $K_{\mu 3}$ spectrum ($\approx$240 MeV) and nominal $K_{\mu 2}$ energy ($\approx$260 MeV). (b) Observed $K_{\mu 3}$ spectrum edited for fiducial volume. (c), (d) Energy spectra generated by Monte Carlo calculation.

Figure 1(c) has been prepared assuming $\xi=0$; Fig. 1(d) is consistent with the assumption $\xi=-6.5$. Both include effects for finite chamber size, scanning bias, and spread in beam momentum. The histograms in Figs. 1(b), 1(c), and 1(d) are all normalized to have the same number of events over the energy interval $m_\mu \leqslant E_\mu < 235$ MeV. It is quite evident that the observed data correspond more closely with the $\xi=0$ spectrum than with the $\xi=-6.5$ choice.

In Fig. 2 we have plotted the logarithm of our likelihood as a function of ξ. It is apparent that, of the choices for ξ suggested by the decay-rate results, our data strongly favor the solution in the neighborhood of zero. (For values of $\xi>0$, the energy spectrum (2) is too insensitive for our data to resolve a definite maximum in $\log L$ corresponding to a most likely value for ξ.) The actual value of the likelihood ratio given by our data for

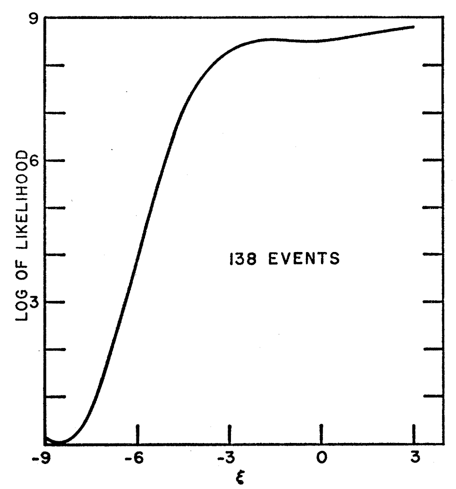

FIG. 2. Logarithm of the likelihood as a function of the form factor ratio (ξ).

the most recent decay rate results ($\xi=0, -6.5$) is $\approx 10^6$ to 1 in favor of $\xi=0$.

We have tested how strongly this result is affected by our uncertainty in the momenta of the K^- particles at the time of decay. The dependence has proven to be quite mild. In this case we modify (3) to the form

$$L(\xi)=\prod_i \int S(E_i,E) \times\left[W(\xi,E)\Big/\int_{E_{l;i}}^{E_{u;i}} W(\xi,E')dE'\right]dE. \quad (6)$$

We are assuming at this point that we can represent each observed event by a weighted sum of kinematically consistent decay configurations each having the laboratory decay angle and muon trajectory of the observed event (these being the well-determined quantities). The configurations differ from one another in beam momentum which we assume determined only up to a probability distribution. The weighted sum is then taken over this parameter. We assume the probability distribution to be of Gaussian form, centered at the nominal value, with a width consistent with our assumption of a beam momentum of 440 ± 25 MeV/c at the copper plate. The weighted sum is taken from -2σ to $+2\sigma$, where σ is the standard deviation of the Gaussian. We find that this correction reduces our odds on $\xi=0$ versus $\xi=-6.5$ by less than an order of magnitude.

References to existing measurements of ξ for the decays $K^+\to\mu^++\pi^0+\nu$ and $K_2{}^0\to\mu^\pm+\pi^\mp+\nu(\bar{\nu})$ are given in Refs. 5–11. Although the early results[5–7] seem to be in disagreement, the more recent values[8–11] all appear to be consistent with the value $\xi=0$. In conclusion we find that to the extent that we have been able to test the decay $K^-\to\mu^-+\pi^0+\bar{\nu}$, the form factor behavior is quite consistent with what has been reported recently on the decays $K^+\to\mu^++\pi^0+\nu$ and $K_2{}^0\to\mu^\pm+\pi^\mp+\nu(\bar{\nu})$.

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Solutions in Pion-Pion Scattering for Two ρ Mesons*

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The theory of pion-pion scattering with inelastic effects is applied to the experimental evidence for the B particle which is interpreted as a second resonance in the $J=1^-$ pion-pion scattering amplitude. The full width of the B particle is predicted and the positions of the ρ and B particles are found to be consistent with the assumption that the $\pi-\omega$ channel dominates the inelastic intermediate state.

SOME time ago, we discussed solutions of the low-energy pion-pion scattering problem with inelastic contributions to the intermediate states.[1,2] The most interesting feature of these solutions was the appearance of two resonances with the same quantum numbers when the inelastic pion-pion scattering was small. It is interesting to reexamine this result in the light of the recent experimental discovery of the B particle, which suggests a companion resonance to the ρ meson. This new resonance ρ' is conjectured to have the decay channels $\rho'\to\pi+\pi(f^0)$ and $\rho'\to\pi+\omega(B)$ and its quantum numbers are $J=1$, $P=-1$, and $G=+1$ as for the ρ meson.[4] In this note, we wish to relate this situation to our previous results and to find the conditions under which two resonances with these masses could be expected. Our main result is the prediction of the ratio of the total widths of the ρ and ρ'.

The basic conclusions of the two previous papers[1,2] are as follows: If there exists a solution with inelastic intermediate states set equal to zero and which exhibits a resonance in the P wave, then if small inelastic contributions are introduced with a threshold just below this resonance, a second resonance will appear at a

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