Calculation of the Rate of $\Omega^- \to \Xi^0 + e^- + \bar{\nu}$*

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(Received 15 April 1964)

The rate of electronic decay of the Ω^- hyperon is calculated, making the assumptions of Cabibbo and using a Goldberger-Treiman relation for the axial vector part. The final result contains one free parameter.

WE consider here the electronic decay mode of the Ω^- hyperon.[1]

$$\Omega^- \to \Xi^0 + e^- + \bar{\nu}. \tag{1}$$

It is assumed that this particle has spin-parity $3/2^+$ in conformity with the predictions of unitary symmetry.[2] Further, we assume $V-A$ leptonic coupling. The most general relativistic interaction is then

$$\bar{\psi}_\Xi\{(f_1+g_1\gamma_5)\delta_{\mu\lambda}+\gamma_\mu(f_2+g_2\gamma_5)q_\lambda+(f_3+g_3\gamma_5)q_\mu q_\lambda + (f_4+g_4\gamma_5)\sigma_{\mu\nu}q_\nu q_\lambda\}\psi_\Omega{}^\lambda\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_\nu = M, \tag{2}$$

where time reversal invariance implies that (f_i,g_i) are real functions of the momentum transfer squared, and we use the Rarita-Schwinger formalism[3] to describe the spin 3/2 hyperon. The expression for the rate, neglecting the lepton mass, and performing the angular integration, is

$$\frac{\Gamma}{2} = \Big\{ 2(r-1)^2 J(f_1^2;1/2) - 4(r-1/3)J(f_1^2;3/2) + (r-1)^2 J(g_1^2;3/2) - 2(r-1/3)J(g_1^2;5/2) + 4/3(r-1)\left[J(f_1f_2;3/2)\left(1+\frac{5r-1}{4}\right) + J(g_1g_2;3/2)\left(1-\frac{5r-1}{4}\right)\right] + 4J(f_2^2;5/2)+8J(g_2^2;3/2)\Big\}\sigma + \text{terms in } g_4 \text{ and } f_4 \cdots, \tag{3}$$

where

$$J(h;n)=\int_0^{\epsilon_0}\epsilon^n d\epsilon h(\epsilon); \quad \epsilon=\frac{E_\Xi - M_\Xi}{M_\Xi}, \quad \epsilon_0=\epsilon^{\max}\cong 0.03$$

$$r=M_\Omega/M_\Xi, \quad \sigma=\frac{\sqrt{2}M_\Xi{}^5G^2}{24(2\pi)^3}=1.58\times10^{11}, \quad \text{sec}^{-1} \tag{4}$$

and higher order terms in ϵ_0—$\cong 3\%$—have been neglected. The expression (3) is obtained using the projection operators given by Behrends and Fronsdal.[4] The form factors (f_3,g_3) do not appear as a result of neglecting the electron mass and using the Dirac equation. There are no interference terms of the form f_jg_i as a result of the symmetry properties of the lepton trace.[5]

Now we make the hypothesis, following Cabibbo,[6] that the vector part of the weak current—in this case the g_i terms in (3)—is in the same octet as the electromagnetic current. Further we adopt the definition of universality given in Ref. 6. Then we can relate $g_{1,2}(q^2=0)$ to the coupling constants in the photoproduction of N^* as follows:

$$g_{1,2,4}(0)=\sqrt{3}V_{1,2,4}\sin\theta, \tag{5}$$

where the V's are defined[7] by the coupling to the photon:

$$[eV_1\bar{\psi}_N\gamma_5\psi_{N^*}{}^\mu + eV_2\bar{\psi}_N\gamma_\mu\gamma_5 q_\nu\psi_{N^*}{}^\nu + eV_4\bar{\psi}_N\sigma_{\mu\nu}\gamma_5 q_\nu q_\lambda\psi_{N^*}{}^\lambda]A_\mu \tag{6}$$

and $\tan\theta$ is, as defined by Cabibbo,[6] just the factor that relates the $\Delta S=0$, and $\Delta S=1$ weak currents while the factor $\sqrt{3}$ comes from the Clebsch-Gordan coefficients for SU_3.[8] From Refs. 6 and 7, respectively, we get

$$\sin\theta=0.26 \quad V_1=(M_{N^*}+M_N)V_2; \quad V_2=0.37/M_\pi; \quad V_4=0. \tag{7}$$

We now evaluate the contribution of the axial current. Assuming the matrix element of the axial current to be dominated by the K^- pole, we obtain a relation of the Goldberger-Treiman type[9]:

$$[f_1+(M_\Omega - M_\Xi)f_2]_{q^2=0}=\gamma_{\Omega\Xi K}F/M_K, \tag{8}$$

where the f_4 piece does not appear since its divergence is zero. Again using SU_3 invariance, we get

$$\gamma_{\Omega\Xi K}=\sqrt{3}\gamma_{N^*N\pi} \tag{9}$$

* Work supported in part by the U. S. Atomic Energy Commission.

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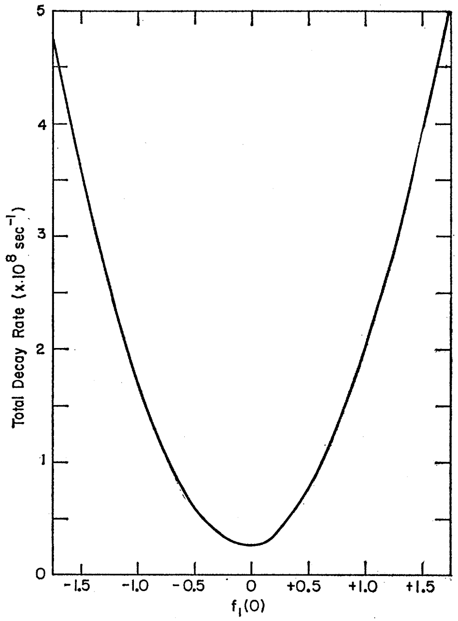

FIG. 1. The total decay rate is shown as a function of the dimensionless quantity $f_1(0)$.

F is related to the rate of the decay $K^- \to \mu^- + \bar{\nu}$ by

$$\Gamma_{K^- \to \mu^- + \bar{\nu}} = \left(\frac{m_\mu}{m_K}\right)^2 \frac{F^2}{M_K} \frac{G^2(M_K{}^2 - M_\mu{}^2)^2 \sin^2\theta}{8\pi}. \qquad (10)$$

Using the experimental value[10] $\tau_{K^-} = 1.22 \times 10^{-8}$ sec, and the branching ratio[10] 64%, we get

$$\sin\theta F / M_K = 5.88 \times 10^{-2}. \qquad (11)$$

From Ref. 7 we get $\gamma_{N^* N\pi} = 2.07$. Substituting this in (8) and using (11),

$$f_1(0) + (M_\Omega - M_\Xi) f_2(0) = 0.208. \qquad (12)$$

Assuming the form factors to be constant over the physical range of q^2 and setting them equal to their values at $q^2 = 0$, we get for the vector and axial vector contributions to the rate:

$$\Gamma_V = 2.45 \times 10^7 \text{ sec}^{-1};$$

$$\Gamma_A = [1.53 f_1{}^2(0) + 0.15 f_1(0) + 0.075] \times 10^8 \text{ sec}^{-1}. \qquad (13)$$

In Fig. 1 the total decay rate, $\Gamma = \Gamma_A + \Gamma_V$, is plotted as a function of $f_1(0)$.

If we use $\tau_{\Omega \to \Xi^0 \pi^-} = 0.7 \times 10^{-10}$ sec and $\tau_{\Omega \to K\Lambda} = 1.3 \times 10^{-10}$ sec,[11] we get an approximate lower limit to the branching ratio:

$$\frac{\Gamma_{\Omega \to \Xi^0 e^- \bar{\nu}}}{\Gamma_{\text{total}}} \gtrsim 0.54\%. \qquad (15)$$

Note added in proof: Recently, a preprint was received on this subject from Professor J. Mathews. His calculation gives $V_2 = 0.26/m_\pi$ as opposed to $0.37/m_\pi$ as in (7). The vector rate then becomes $\Gamma_V = 1.25 \times 10^7$ sec^{-1}, according to (3). Mathews' calculation gives $\Gamma_V = 1.07 \times 10^7$ sec^{-1}, making no approximation to the integrals and including the lepton mass.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Dr. P. G. O. Freund for a great number of helpful discussions, to R. Torgeson for several informative comments, and also to Professor Y. Nambu for valuable suggestions. I would also like to acknowledge the help of Professor Jon Mathews, who kindly read the original version of this report and pointed out several errors which would otherwise have gone unnoticed.

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