SU_3 Invariance in Nucleon-Antinucleon Annihilation*

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The relations among the amplitudes for nucleon-antinucleon annihilation into two mesons, and also those for annihilation into a baryon-antibaryon pair and a decuplet-antidecuplet pair, are obtained on the basis of SU_3 invariance. The dynamical assumptions made are that for the two-meson case the **8** representations dominate the direct channel and that for the baryon-antibaryon pair case, the **8a** and **27** representations dominate the crossed channel. The relations are compared with experiments, and some resulting tests of SU_3 invariance are suggested.

1. INTRODUCTION

VARIOUS relations among reaction amplitudes in strong interactions have been discussed on the basis of the octet model[1,2] of unitary symmetry SU_3. A set of states that transform into one another under the unitary transformations will form multiplets that are labeled by two quantum numbers (λ,μ). The octet model assigns the baryons B, pseudoscalar mesons P, and vector mesons V to the (1,1) representation of the group SU_3. The process $A+B\rightarrow C+D$ can be described as two octets transforming into two other octets. Two octets (1,1) can couple together to form the product representations (2,2), $(1,1)_s$, (0,0), (0,3), (3,0) and $(1,1)_a$. Under an R transformation[1] that is independent of SU_3, the representation $(1,1)_s$ transforms with a positive phase whereas the representation $(1,1)_a$ transforms with a negative phase.

There are thus six channel amplitudes A_{27}, A_{8s}, A_1, $A_{\overline{10}}$, A_{10}, and A_{8a} which are diagonal elements of the S matrix for the representations (2,2), $(1,1)_s$, (0,0), (0,3), (3,0), and $(1,1)_a$, respectively. There are also two nondiagonal matrix elements A_{as} and A_{sa} that couple the representations $(1,1)_s$ and $(1,1)_a$.

A group-theoretical method[3] is used to obtain consequences of the SU_3 invariance in the interactions $\bar{N}+N\rightarrow P(V)+P(V)$ in Sec. 2, and $\bar{N}+N\rightarrow\bar{B}+B$ in Sec. 3, where N represents the nucleons. Finally, consequences in the interactions $\bar{N}+N\rightarrow\bar{D}+D$ are considered in Sec. 4, where D represents the decuplet of baryon-pion resonances.

2. $\bar{N}+N\rightarrow P(V)+P(V)$

With the aid of the fact[4] that $A_{10}=A_{\overline{10}}$ for $(\bar{N}N|PP)$ amplitudes, the nucleon-antinucleon annihilation amplitudes have been expressed in terms of channel amplitudes and are listed in Table I. When the channel amplitudes are eliminated, one obtains the relations

$$(\bar{p}p|K^-K^+)+(\bar{p}p|\bar{K}^0\bar{K}^0)=(\bar{p}n|K^-K^0)\,,$$
$$(\bar{p}p|\pi^-\pi^+)+(\bar{p}p|\pi^0\pi^0)=(\bar{p}n|\pi^-\pi^0)/\sqrt{2}\,, \quad (1)$$
$$\sqrt{2}(\bar{p}p|\eta\pi^0)=(\bar{p}n|\eta\pi^-)\,.$$

These are the relations that exist among the nine amplitudes of Table I; they are consequences of SU_2 or charge independence. If we make the dynamical assumption that the **8** representations dominate the intermediate states, Eqs. (1) are replaced by stronger relations that are consequences of SU_3 and this assumption. Then the initial and final states are in the **8**

TABLE I. Nucleon-antinucleon annihilation amplitudes in terms of channel amplitudes. The amplitudes have been multiplied by 4.

	A_{27}	A_{8s}	A_1	A_{10}	A_{8a}	A_{as}	A_{sa}
$(\bar{p}p\|\pi^-\pi^+)$	$-\frac{1}{10}$	$-\frac{2}{5}$	$\frac{1}{2}$	$-\frac{2}{3}$	$\frac{2}{3}$	$\frac{2\sqrt{5}}{5}$	$\frac{2\sqrt{5}}{5}$
$(\bar{p}p\|K^-K^+)$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{3}$	...	...
$(\bar{p}p\|\bar{K}^0K^0)$	$\frac{1}{10}$	$\frac{2}{5}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{2\sqrt{5}}{5}$	$\frac{2\sqrt{5}}{5}$
$(\bar{p}p\|\pi^0\pi^0)$	$\frac{1}{10}$	$\frac{2}{5}$	$-\frac{1}{2}$	...	...	$-\frac{2\sqrt{5}}{5}$	...
$(\bar{p}p\|\eta\eta)$	$\frac{9}{10}$	$-\frac{2}{5}$	$-\frac{1}{2}$	...	...	$\frac{2\sqrt{5}}{5}$	...
$(\bar{p}p\|\eta\pi^0)/\sqrt{3}$	$\frac{2}{5}$	$-\frac{2}{5}$	...	...	...	$-\frac{2\sqrt{5}}{15}$	...
$(\bar{p}n\|\pi^-\pi^0)/\sqrt{2}$	...	...	...	$-\frac{2}{3}$	$\frac{2}{3}$	...	$\frac{2\sqrt{5}}{5}$
$(\bar{p}n\|\eta\pi^-)/\sqrt{6}$	$\frac{2}{5}$	$-\frac{2}{5}$	...	...	...	$-\frac{2\sqrt{5}}{15}$	...
$(\bar{p}n\|K^-K^0)$	$\frac{4}{5}$	$\frac{6}{5}$	...	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{2\sqrt{5}}{5}$	$\frac{2\sqrt{5}}{5}$

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); California Institute of Technology Report CTSL-20, 1961 (unpublished).
[2] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] P. Tarjanne, Ann. Acad. Sci. Fennicae, Series A VI, 105 (1962).
[4] K. Itabashi and K. Tanaka (to be published).

representations so that from Table I one finds, for example, that

$$(\bar{p}p|\pi^0\pi^0)=(\bar{p}p|\eta\eta)=(\bar{p}p|\eta\pi^0)/\sqrt{3}=(\bar{p}n|\eta\pi^-)/\sqrt{6}=0\,,$$
$$(\bar{p}p|\pi^-\pi^+)=(\bar{p}n|\pi^-\pi^0)/\sqrt{2}=(\bar{p}n|K^-K^0)=(\bar{p}p|\bar{K}^0K^0)+(\bar{p}p|K^-K^+)\,,\quad(2)$$

when the two mesons are in the antisymmetric state $(1,1)_a$; and

$$(\bar{p}p|\pi^-\pi^+)=-(\bar{p}p|\pi^0\pi^0)=(\bar{p}p|\eta\eta)=(\bar{p}p|\bar{K}^0K^0)-(\bar{p}p|K^-K^+)\,,\quad(3)$$
$$-\sqrt{3}(\bar{p}p|\eta\pi^0)=(\bar{p}n|K^-K^0)=-(3/2)^{1/2}(\bar{p}n|\eta\pi^-)=(\bar{p}p|\bar{K}^0K^0)+(\bar{p}p|K^-K^+)\,,$$

when the two mesons are in the symmetric state $(1,1)_s$. In Eq. (2), J is odd and $I=1$ for the two-pion states, and J is odd and $I=0$, 1 for the two-kaon states; but in Eq. (3), J is even and $I=0$ for the two-pion states, and J is even and $I=0$, 1 for the two-kaon states.

The experimental result that $(\bar{p}p)$ annihilation takes place at rest from $^3S_1$ states[5] (for which $C=-1$) suggests that $(\bar{p}p)$ proceeds through the antisymmetric state for P+P. If this is the case, then Eq. (2) is valid and one finds

$$\sigma(\pi^0\pi^0)=\sigma(\eta\eta)=\sigma(\eta\pi^0)=0\,,\quad(4)$$
$$\sigma^{1/2}(K_1^0K_2^0)+\sigma^{1/2}(K^-K^+)\geqslant[\sigma(\pi^-\pi^+)/\rho]^{1/2}\geqslant|\sigma^{1/2}(K_1^0K_2^0)-\sigma^{1/2}(K^-K^+)|\,,\quad(5)$$

where $\rho=1.57$ is the ratio of the momentum dependences of the cross sections for two pions and two kaons.[6,7] There are two other inequalities in which the three terms in Eq. (5) are permuted. When the experimental result[5] $\sigma(\pi^+\pi^-):\sigma(K^-K^+):\sigma(K_1^0K_2^0)=3:1:0.43$ is substituted into inequality (5) and the inequalities similar to Eq. (5), they are satisfied. Equation (4) can also be obtained from invariance under charge conjugation. From Table I, the cross sections for antiprotons on neutrons are $\sigma(\eta\pi^-)=0$ and $\sigma(\pi^-\pi^0)=2\sigma(K^-K^0)=2\sigma\times(\pi^-\pi^+)$, provided the $(\bar{p}n)$ proceeds through the antisymmetric state for $P+P$.

It is evident that Eqs. (2) and (3) also hold when both pseudoscalar mesons are replaced by their corresponding vector mesons in all the amplitudes of the relations, and also when one pseudoscalar meson is replaced by its corresponding vector meson. It is noted here that Table I is not valid for the case of $V+P$, since $A_{10}=A_{\overline{10}}$ was used.

For the case of $V+V$, the total spin of the VV system is 0 or 2, as shown in Ref. 4, when the spin of the initial state is 1. In particular, the $(\rho^-\rho^+)$ is in a state with $C=-1$, $I=1$. Then the orbital angular momentum is odd so that the two vector mesons are in the antisymmetric state $(1,1)_a$. The equations corresponding to (2) are valid and the relations corresponding to Eqs. (4) and (5) are[8]

$$\sigma(\rho^0\rho^0)=\sigma(\phi^0\rho^0)=0\,,\quad(6)$$
$$\sigma^{1/2}(K_1{}^{*0}K_2{}^{*0})+\sigma^{1/2}(K^{*-}K^{*+})\geqslant\sigma^{1/2}(\rho^-\rho^+)\geqslant|\sigma^{1/2}(K_1{}^{0*}K_2{}^{0*})-\sigma^{1/2}(K^{*-}K^{*+})|\,.\quad(7)$$

The prediction that $(\rho^0\rho^0)$ should not be observed agrees with experiment.[9] The cross sections for antiprotons on neutrons are similarly $\sigma(\phi^0\rho^-)=0$ and $\sigma(\rho^-\rho^0)=2\sigma\times(K^{*-}K^{*0})=2\sigma(\rho^-\rho^+)$.

For the case of $V+P$, annihilation in the $^3S_1$ state $(C=-1)$ proceeds via the symmetric state for which $C=-1$. Then, the relations corresponding to Eq. (3) are valid so that

$$\sigma(\rho^-\pi^+)=\sigma(\rho^+\pi^-)=\sigma(\rho^0\pi^0)=\sigma(\phi^0\eta)\,,\quad(8)$$
$$\sigma^{1/2}(K_1{}^{*0}K_2{}^0)+\sigma^{1/2}(K^{*-}K^+)\geqslant\sqrt{3}\sigma^{1/2}(\phi^0\pi^0)\geqslant|\sigma^{1/2}(K_1{}^{*0}K_2{}^0)-\sigma^{1/2}(K^{*-}K^+)|\,,\quad(9)$$
$$\sigma^{1/2}(K_1{}^{*0}K_2{}^0)+\sigma^{1/2}(K^{*-}K^+)\geqslant\sigma^{1/2}(\rho^-\pi^+)\geqslant|\sigma^{1/2}(K_1{}^{*0}K_2{}^0)-\sigma^{1/2}(K^{*-}K^+)|\,.$$

The first two equalities of Eq. (8) agree with the experimental data[10]; the remaining relations as well as relations similar to Eqs. (9) are based on SU$_3$ invariance, dominance of the 8 representation, and the fact that annihilation takes place from $^3S_1$ states. The cross sections for antiprotons on neutrons are $\frac{1}{2}\sigma(\phi^0\pi^-)=\frac{1}{2}\sigma(\rho^-\eta)=\sigma(K^{*-}K^0)/3=\frac{1}{3}\sigma(K^{*0}K^-)=\sigma(\phi^0\pi^0)=\sigma(\rho^0\eta)$, provided the $(\bar{p}n)$ proceeds through the symmetric state for $V+P$.

3. $\bar{N}+N\rightarrow\bar{B}+B$

The amplitudes for $\bar{N}+N\rightarrow\bar{B}+B$ can be expressed in terms of channel amplitudes by substituting $(\Lambda,\Sigma,N,\Xi)\rightarrow(\eta,\pi,K,\bar{K})$ and $(\bar{\Lambda},\bar{\Sigma},\bar{N},\bar{\Xi})\rightarrow(\eta,\bar{\pi},\bar{K},K)$ in Table I. The resulting expressions from Table I and

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[5] R. Armenteros, L. Montanet, D. R. O. Morrison, S. Nilsson, A. Shapiro, *et al.*, *Proceedings of the International Conference on High Energy Nuclear Physics*, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 351; G. B. Chadwick, W. T. Davies, M. Derrick, J. H. Mulvey, D. Radojicic, *et al.*, *Proceedings of the Aix-en-Provence International Conference on Elementary Particles* (Centre d'Etudes Nucleaires de Saclay, Seine et Oise, 1961), p. 269.

[6] It is assumed that the decay rate is proportional to p^3, the cube of the momentum of the meson in the barycentric system, because the initial state of $(\bar{p}p)$ appears to be $^3S_1$ and the relative angular momentum of the two final mesons is 1.

[7] Y. Dothan, H. Goldberg, H. Harari, and Y. Ne'eman, Phys. Letters **1**, 308 (1962). The authors discuss these cross sections on the basis of a stronger assumption. C. A. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Rev. Letters **1**, 307 (1962), discuss them on the basis of R invariance.

[8] We define

$$K_1{}^*=(K^{*0}+CK^{*0})/\sqrt{2}=(K^{*0}-\bar{K}^{*0})/\sqrt{2}\,,$$
$$K_2{}^*=-i(K^{*0}-CK^{*0})/\sqrt{2}=-i(K^{*0}+\bar{K}^{*0})/\sqrt{2}\,,$$

so that the $(K^{*0}\bar{K}^0)$ in the antisymmetric (symmetric) state is $K_1{}^{*0}K_1{}^0$ or $K_2{}^{*0}K_2{}^0$ ($K_1{}^{*0}K_2{}^0$ or $K_2{}^{*0}K_1{}^0$). See S. M. Berman and R. J. Oakes, Nuovo Cimento **29**, 1329 (1963); H. Goldberg and Y. Ne'eman, Nucl. Phys. **42**, 638 (1963).

[9] M. Cresti, A. Grigoletto, S. Limentani, A. Loria, L. Peruzzo, *et al.*, (to be published).

[10] G. B. Chadwick, W. T. Davies, M. Derrick, C. J. B. Hawkins, J. H. Mulvey, *et al.*, Phys. Rev. Letters **10**, 62 (1963).

the fact that $A_{as}=A_{sa}$ for the amplitudes $\bar{N}+N \rightarrow \bar{B}+B$ (in addition to $A_{10}=A_{\overline{10}}$) are[4]

$$
\begin{aligned}
(\bar{p}p|\bar{\Sigma}^-\Sigma^-)&=-(\bar{p}p|\bar{\Xi}^0\Xi^0),\\
\sqrt{2}(\bar{p}p|\bar{\Lambda}\Sigma^0)&=\sqrt{2}(\bar{p}p|\bar{\Sigma}^0\Lambda)=(\bar{p}n|\bar{\Lambda}\Sigma^-),\\
(\bar{p}p|\bar{\Sigma}^+\Sigma^+)+(\bar{p}p|\bar{\Sigma}^-\Sigma^-)&=-2(\bar{p}p|\bar{\Sigma}^0\Sigma^0),\\
(\bar{p}p|\bar{p}p)+(\bar{p}p|\bar{n}n)&=(\bar{p}n|\bar{p}n),\\
(\bar{p}p|\bar{\Sigma}^+\Sigma^+)+(\bar{p}p|\bar{\Sigma}^0\Sigma^0)&=(\bar{p}n|\bar{\Sigma}^+\Sigma^0)/\sqrt{2},\\
(\bar{p}p|\bar{\Xi}^-\Xi^-)+(\bar{p}p|\bar{\Xi}^0\Xi^0)&=(\bar{p}n|\bar{\Xi}^0\Xi^-).
\end{aligned}
\tag{10}
$$

Equations (10) lead to various triangular inequalities; all except the first relation are consequences of SU_2. The rigorous equalities of cross sections are $\sigma(\bar{\Sigma}^-\Sigma^-)=\sigma(\bar{\Xi}^0\Xi^0)$, and $2\sigma(\bar{\Lambda}\Sigma^0)=2\sigma(\bar{\Sigma}^0\Lambda)=\sigma(\bar{\Lambda}\Sigma^-)$.

The production of hyperon-antihyperon pairs $(\bar{Y}Y)$ are known to be consistent with a peripheral model based on exchange of a vector meson between the proton and antiproton.[11] In order to consider the exchanged system further, we obtain the channel amplitudes in the crossed channel[3,4] $(\bar{p}n|\bar{\Lambda}\Sigma^-)\rightarrow(\Lambda\bar{p}|\Sigma^-\bar{n})$. The resulting expressions listed in Table II were derived by use of the fact that in the crossed channel $B_{\overline{10}}=B_{10}$ and $B_{as}=B_{sa}$.[4] The amplitudes for processes that are not listed can be found with the aid of Eqs. (10).

TABLE II. Proton-antiproton annihilation amplitudes in terms of channel amplitudes in the crossed channel. The amplitudes have been multiplied by 4.

	B_{27}	B_{8s}	B_1	B_{10}	B_{8a}	B_{as}
$(\bar{p}p)$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{3}$	⋯
$(\bar{n}n)$	$\frac{4}{5}$	$\frac{6}{5}$	⋯	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{4}{\sqrt{5}}$
$(\bar{\Lambda}\Lambda)$	$-\frac{9}{5}$	$-\frac{1}{5}$	⋯	-1	-1	$\frac{2}{\sqrt{5}}$
$(\bar{\Sigma}^0\Sigma^0)$	$-\frac{7}{5}$	$-\frac{3}{5}$	⋯	$-\frac{5}{3}$	$-\frac{1}{3}$	$-\frac{2}{\sqrt{5}}$
$(\bar{\Sigma}^+\Sigma^+)$	$\frac{4}{5}$	$\frac{6}{5}$	⋯	$\frac{4}{3}$	$\frac{2}{3}$	$\frac{4}{\sqrt{5}}$
$(\bar{\Sigma}^-\Sigma^-)$	2	⋯	⋯	2	⋯	⋯
$\sqrt{3}(\bar{\Lambda}\Sigma^0)$	$-\frac{3}{5}$	$\frac{3}{5}$	⋯	1	-1	$-\frac{2}{\sqrt{5}}$
$(\bar{\Xi}^-\Xi^-)$	4	⋯	⋯	⋯	⋯	⋯

First, we note that a vector-meson exchange is important in these processes because it explains the fact that the antihyperons are emitted in the forward direction with respect to the incident antiproton. If this vector meson emerges from the demand of local gauge invariance,[12] then the Lagrangian between the vector meson and the baryons must be F type so that the B_{8a} amplitude dominates over the B_{8s} and B_{as} amplitudes which vanish.

If the $\bar{\Sigma}^-$ in the $(\bar{\Sigma}^-\Sigma^-)$ events are emitted in the forward direction as they appear to be,[13] then the proton and antiproton need to exchange a system that has $Q=2$, $Y=1$, $I=\frac{3}{2}$ and belongs to B_{27} and B_{10}. One such system is $K^+\pi^+$. If a similar mechanism is valid for the $(\bar{\Xi}^-\Xi^-)$ events, then a system with $Q=2$, $Y=2$, $I=1$ (e.g., K^+K^+) is exchanged; i.e., the exchanged system belongs to B_{27}. To account for these events, we assume that the B_{27} amplitude dominates over the B_{10} amplitude. The channel amplitude B_1 does not appear in $(\bar{Y}Y)$ production but presumably is responsible for the very large elastic cross sections.

On the basis of our dynamical assumption that the B_{8a} and B_{27} amplitudes dominate, we attempt to explain the relative production cross sections of those processes in Table II. In terms of $a\equiv\int|B_{8a}|^2d\Omega$, $a\xi^2\equiv\int|B_{27}|^2d\Omega$, and $a\xi\cos\phi\equiv\int\mathrm{Re}(B_{8a}{}^*B_{27})d\Omega$, the cross sections are

$$
\begin{aligned}
\sigma(\bar{\Lambda}\Lambda)&=a\left\{1+\frac{18}{5}\xi\cos\phi+\frac{81}{25}\xi^2\right\},\\
\sigma(\bar{\Sigma}^0\Sigma^0)&=a\left\{\frac{1}{9}+\frac{14}{15}\xi\cos\phi+\frac{49}{25}\xi^2\right\},\\
\sigma(\bar{\Sigma}^+\Sigma^+)&=a\left\{\frac{4}{9}+\frac{16}{15}\xi\cos\phi+\frac{16}{25}\xi^2\right\},\\
\sigma(\bar{\Sigma}^-\Sigma^-)&=a\left\{4\xi^2\right\},\\
\sigma(\bar{\Lambda}\Sigma^0)&=a\left\{\frac{1}{3}+\frac{2}{5}\xi\cos\phi+\frac{3}{25}\xi^2\right\},\\
\sigma(\bar{\Xi}^-\Xi^-)&=a\left\{16\xi^2\right\}.
\end{aligned}
\tag{11}
$$

The masses of all the hyperons are assumed to be the same. The parameters depend on the momentum variables.

At this stage, it does not appear fruitful to carry out a detailed analysis. It is found that for $1\geqslant\cos\phi\geqslant0$, all

[11] H. D. D. Watson, Nuovo Cimento **29**, 1338 (1963); H. Goldberg, S. Nussinov, and G. Yekutieli, Nuovo Cimento **28**, 446 (1963).

[12] R. Utiyama, Phys. Rev. **101**, 1597 (1956). See, also, Dothan *et al.*, Ref. 7.

[13] R. Armenteros, E. Fett, B. French, L. Montanet, V. Nikita, *et al.*, *Proceedings of the International Conference on High Energy Nuclear Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 236.

TABLE III. Cross sections (μb) for hyperon-antihyperon pair production in $p\bar{p}$.

	Armenteros *et al.*[a] 3 BeV/*c*	Sandweiss *et al.*[b] 3.25 BeV/*c*	Theory 3 BeV/*c*
$\bar{\Lambda}\Lambda$	78.5 ±23	87±13	100
$\bar{\Lambda}\Sigma^0$	49 ±13	56±11	31
$\bar{\Sigma}^0\Lambda$	42 ±12		31
$\bar{\Sigma}^0\Sigma^0$	21 ±6		15
$\bar{\Sigma}^+\Sigma^+$	38 ±7	38±12	43
$\bar{\Sigma}^-\Sigma^-$	8 ±3		9
$\bar{\Xi}^-\Xi^-$	4 ±2.5	4±3	36

[a] Reference 13.
[b] Reference 15.

the experimental data[13–15] at 3 or 3.25 BeV/*c* (Table III), except for $\sigma(\bar{\Xi}^-\Xi^-)$, may be fitted within one standard deviation by adjusting a and ξ. For example, a fit is obtained for $\cos\phi=1$, $\xi=0.15$, $a=63$; $\cos\phi=0$, $\xi=0.16$, $a=93$. There is no fit for $\cos\phi=-1$. The last case ($\cos\phi=0$) is included in the theory column. An analysis for the momenta[14] 3.6 BeV/*c* and[15] 3.65 BeV/*c* gives rise to a situation similar to that at 3 BeV/*c*.

The cross sections for antiprotons on neutrons [denoted by $\sigma(\bar{\Xi}^0\Xi^-)$, $\sigma(\bar{\Lambda}\Sigma^-)$, and $\sigma(\bar{\Sigma}^+\Sigma^0)$] are, from Table II, Eqs. (10), and the dominance of B_{8a} and B_{27}, given by

$$\begin{aligned} \sigma(\bar{\Xi}^0\Xi^-)&=\sigma(\bar{\Sigma}^-\Sigma^-)\,,\\ \sigma(\bar{\Lambda}\Sigma^-)&=2\sigma(\bar{\Sigma}^0\Lambda)\,,\\ \sigma(\bar{\Sigma}^+\Sigma^0)&=a\left\{\frac{2}{9}-\frac{4}{5}\xi\cos\phi+\frac{18}{25}\xi^2\right\}. \end{aligned} \tag{12}$$

For the case $\cos\phi=0$, $\xi=0.16$, $a=93$, the cross sections in units of μb are $\sigma(\bar{\Xi}^0\Xi^-)=9$, $\sigma(\bar{\Lambda}\Sigma^-)=62$, and $\sigma(\bar{\Sigma}^+\Sigma^0)=22$.

A crucial test of the present model is the ratio $\sigma(\bar{\Xi}^-\Xi^-)/\sigma(\bar{\Sigma}^-\Sigma^-)=4$, which is reduced to 2.44 at 3 BeV/*c* because of the phase-space ratio. Further, this ratio is based on the symmetric amplitude B_{27} in the crossed channel.[16] Consequently, a disagreement between this ratio and experiment would necessitate complicating the analysis by introducing the B_{10} amplitude in addition to the B_{27} and B_{8a} amplitudes.[17]

The present peripheral model of the proton-antiproton annihilation, in which the amplitude B_{8a} with about a 15% mixture of the amplitude B_{27} dominates in the crossed channel, can explain the present data on hyperon-antihyperon pair production. It would be interesting to reduce the errors in the data so that a more precise check of this model can be attempted.

Note added in proof: It is noted that the present model cannot explain the largeness of the 6 $(\bar{N}N)$ cross section (Table II). The author thanks Dr. H. Harari and Dr. H. J. Lipkin for this remark.

TABLE IV. The $(\bar{p}p)$ and $(\bar{p}n)$ collision amplitudes $(\bar{N}+N\to\bar{D}+D)$ in terms of channel amplitudes in the crossed channel. The amplitudes have been multiplied by 4.

	B_{35}	B_{27}	B_{10}	B_8
$(\bar{N}^{++}N^{++})$	$\frac{1}{2}$	$\frac{9}{10}$	1	$\frac{8}{5}$
$(\bar{N}^{+}N^{+})$	$\frac{2}{3}$	$\frac{8}{5}$	$\frac{2}{3}$	$\frac{16}{15}$
$(\bar{N}^{0}N^{0})$	$\frac{5}{6}$	$\frac{23}{10}$	$\frac{1}{3}$	$\frac{8}{15}$
$(\bar{N}^{-}N^{-})$	1	3	⋯	⋯
$(\bar{Y}_1^{+}Y_1^{+})$	$\frac{4}{3}$	$\frac{4}{5}$	$\frac{4}{3}$	$\frac{8}{15}$
$(\bar{Y}_1^{-}Y_1^{-})$	2	2	⋯	⋯
$(\bar{Y}_1^{0}Y_1^{0})$	$\frac{5}{3}$	$\frac{7}{5}$	$\frac{2}{3}$	$\frac{4}{15}$
$(\bar{Z}_0Z_0)$	$\frac{5}{2}$	$\frac{1}{2}$	1	⋯
$(\bar{Z}^{-}Z^{-})$	3	1	⋯	⋯
$(\bar{\Omega}^{-}\bar{\Omega}^{-})$	4	⋯	⋯	⋯
$(\bar{N}^{++}N^{+})/\sqrt{3}$	$\frac{1}{6}$	$\frac{7}{10}$	$-\frac{1}{3}$	$-\frac{8}{15}$
$(\bar{N}^{+}N^{0})$	$\frac{1}{3}$	$\frac{7}{5}$	$-\frac{2}{3}$	$-\frac{16}{15}$
$(\bar{N}^{0}N^{-})/\sqrt{3}$	$\frac{1}{6}$	$\frac{7}{10}$	$-\frac{1}{3}$	$-\frac{8}{15}$
$(\bar{Y}_1^{+}Y_1^{0})/\sqrt{2}$	$\frac{1}{3}$	$\frac{3}{5}$	$-\frac{2}{3}$	$-\frac{4}{15}$
$(\bar{Y}_1Y_1^{-})/\sqrt{2}$	$\frac{1}{3}$	$\frac{3}{5}$	$-\frac{2}{3}$	$-\frac{4}{15}$
$(\bar{Z}^{0}Z^{-})$	$\frac{1}{2}$	$\frac{1}{2}$	−1	⋯

[14] C. Baltay, E. C. Fowler, J. Sandweiss, J. R. Sanford, H. D. Taft, *et al.*, *Proceedings of the International Conference on High Energy Nuclear Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 233.

[15] J. Sandweiss (private communication).

[16] The lowest mass state for the amplitude B_{27} is $K^+\pi^+$, which suggests a two-meson exchange for $(\bar{\Sigma}^-\Sigma^-)$ events. As a result, the mechanism for the possible forward peaking of $\bar{\Sigma}^-$ in the $(\bar{\Sigma}^-\Sigma^-)$ events is not explained here.

[17] It is noted that the local gauge invariance assumption leading to B_{8a} is probably "renormalized" by B_{8s}, B_{as} to the same extent that B_{27} is effective.

4. $\bar{N}+N\rightarrow\bar{D}+D$

The amplitudes for $\bar{N}+N\rightarrow\bar{D}+D$ can be expressed in terms of the channel amplitudes A_{27}, A_{8s}, A_{8a}, and A_1, which are the matrix elements for the representations $\mathbf{27}\leftrightarrow\mathbf{27}$, $8s\leftrightarrow\mathbf{8}$, $8a\leftrightarrow\mathbf{8}$, and $\mathbf{1}\leftrightarrow\mathbf{1}$, respectively. The decuplet D represents the baryon-pion resonances; the quartet N^{++}, N^{+}, N^{0}, N^{-}, the triplet Y_1^{+}, Y_1^{0}, Y_1^{-}, the doublet Z^0, Z^-, and the singlet Ω^-. In order to consider the exchanged system, the channel amplitudes B_{35}, B_{27}, B_{10}, and B_8 in the crossed channel $(\bar{N}N|\bar{D}D)\rightarrow(D\bar{N}|D\bar{N})$are listed in Table IV.

The experimental results[18,19] with 3.25-BeV/c incident $\bar{p}$ indicate that $\sigma(\bar{Y}_1^-Y_1^-)/\sigma(\bar{Y}_1^+Y_1^+)=7$ and $\sigma(\bar{Y}_1^-Y_1^-+\bar{Y}_1^+Y_1^+)/\sigma(\bar{N}^{++}N^{++})=10^{-2}$. Also, a necessary condition for the validity of the single-pion-exchange model for $\bar{p}+p\rightarrow\bar{N}^{++}N^{++}$ is satisfied, which suggests that the B_8 amplitude dominates. It is attractive to suggest that the ratio $\sigma(\bar{Y}_1^-Y_1^-)/\sigma(\bar{Y}_1^+Y_1^+)=7$ is determined by the B_{27} amplitude, which leads to the theoretical ratio 6.25 (Table IV), but this is not possible as the B_8 amplitude ($\bar{K}^0$ exchange), rather than the B_{27} amplitude, should dominate the reaction $\bar{p}+p\rightarrow\bar{Y}_1^+ +Y_1^+$. The preceding attempt shows that a simple analysis is not possible within the framework of SU_3.

The following equalities among cross sections in $(\bar{p}n)$ collision are noted from Table IV:

$$\sigma(\bar{N}^{++}N^{+})=\sigma(\bar{N}^{0}N^{-})=\tfrac{3}{4}\sigma(\bar{N}^{+}N^{0})$$

$$\sigma(\bar{Y}_1^{+}Y_1^{0})=\sigma(\bar{Y}_1^{0}Y_1^{-}).$$

These are consequences of SU_2. A test of SU_3 would require additional dynamical assumptions for which Table IV would be useful.

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S-Matrix Method for Calculation of Electromagnetic Corrections to Strong Interactions*

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We develop an *S*-matrix method for calculating the effect of small perturbations on a partial-wave amplitude, and in particular, on the positions and residues of bound states. The method is applicable to both nonrelativistic and relativistic problems. It has, as a particular virtue, rapid convergence of the dispersion integrals. Electromagnetic corrections to strong interactions are the main application we have in mind, and modifications useful for handling the infrared divergence that occurs in this case are described in detail.

I. INTRODUCTION

TRADITIONALLY, electromagnetic corrections to strong interactions have been formulated in terms of off-mass-shell propagators and vertex functions. For example, the usual method for calculating the neutron-proton mass difference consists of finding the electromagnetic corrections to the nucleon propagator.

On the other hand, much recent progress in strong interaction dynamics has come from a study of the two-body scattering amplitude on the mass shell. In this paper we use the on-mass-shell, *S*-matrix formalism to study electromagnetic effects, including corrections to masses and coupling constants.

We feel that this approach has several advantages: (i) According to the "bootstrap" hypothesis, all strongly interacting particles are bound states or resonances. From this point of view, the mass differences among the members of an isotopic multiplet result from electromagnetic corrections to the interactions which hold the particles together. Now, in *S*-matrix studies, closely related methods apply to both nonrelativistic and relativistic problems; one can therefore use the understanding of bound states, resonances, and perturbations on the interaction that one has in nonrelativistic quantum mechanics as a guide in relativistic problems which, according to the "bootstrap" hypothesis, possess these same features. (ii) The customary approximation scheme in strong interactions emphasizes the long-range parts of the

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