Alpha-Particle Scattering by P^{31} at 18.2 MeV*†

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The differential cross sections for the inelastic (1.265- and 2.232-MeV states) and elastic scattering of 18.2-MeV alpha particles from vacuum-evaporated phosphorous targets have been measured using silicon surface-barrier detectors. The measurements were made at 2.5° intervals in the nominal angular range from 10 to 173°. All the angular distributions exhibit an oscillatory diffraction-like structure with rather uniformly spaced maxima for angles less than about 90°. In each angular distribution the oscillatory behavior persists to backward angles but becomes less distinct and regular. The elastic cross section increases rapidly at extreme backward angles, a behavior exhibited to a lesser degree by the inelastic cross sections. Analyses of the ground-state angular distribution were made in terms of the simple diffraction model yielding $R=6.28$ F and the sharp-cutoff Akhieser-Pomeranchuk-Blair (APB) model yielding $l_0=9$. A graphical compilation of experimental elastic alpha-particle angular distributions for a series of light nuclei corresponding to incident alpha-particle energies in the range from 18.0 to 22.5 MeV is presented, and the possible existence of a systematic difference in the backward-angle behavior of the cross sections for certain even-even and odd-A nuclei is discussed. A Blair diffraction-model analysis of the inelastic angular distribution associated with the 2.232-MeV state resulted in the following values for the collective-model parameters: $C_2/(C_2)_{\rm irrot}=7.9$ and $B_2/(B_2)_{\rm irrot}=50$ assuming a pure vibrational transition; and $|\beta_2|=0.13$ assuming a pure intra-band rotational transition with $K=\frac{1}{2}$. The inelastic angular distributions associated with the 1.265- and 2.232-MeV states were analyzed using the direct-interaction theories of Austern, Butler, and McManus; and McCarthy and Pursey. Satisfactory fits were obtained at forward angles for both angular distributions with $j_2^2(QR)$ using $R=6.52$ F, and with the expressions of McCarthy and Pursey using $R=6.62$ F, $\lambda=1.0$ F, and $\gamma=0.8$.

I. INTRODUCTION

ALTHOUGH the spins, parities, and decay properties of the low-lying levels of P^{31} are quite well known[1–3]; and these levels have been analyzed with moderate success in terms of the symmetric-core rotator model,[4] the collective vibrational model,[5] and recently the asymmetric-core rotator model[6]; the study of alpha-particle scattering provides a somewhat independent approach to the investigation of this nucleus. Alpha-particle scattering data for P^{31} is lacking at present, and typifies the more general lack of such information for odd-A nuclei in this mass region.

It is the purpose of this investigation to measure the elastic and inelastic alpha-particle scattering cross sections for P^{31}, and to analyze them in terms of various direct interaction and diffraction models, using when necessary the known spin and parity assignments of the states under consideration. Whenever possible, the information concerning the nucleus derived from the analysis of the alpha-particle scattering data will be compared with the corresponding predictions of various nuclear models.

II. EXPERIMENTAL

A description of the cyclotron experimental area; beam focusing, steering, and analyzing system; and scattering chamber has been presented elsewhere.[7] The phosphorous targets were bombarded with an 18.2-MeV alpha-particle beam having an rms energy spread of 60 keV. The beam cross section perpendicular to its direction was circular with a 5/64-in. diameter. The energy of the scattered alpha particles was measured using a silicon surface-barrier detector assembly and an electronics configuration which has also been previously described.[7] A target-counter geometry was used in which the azimuthal acceptance angle of the detector was 2.3°, and the nominal solid angle subtended by the detector was 0.001 sr. Experimental measurements were made at 2.5° intervals over an angular range from 10.0 to 172.5° in the laboratory system.

The targets were prepared by thermal vacuum-evaporation of red phosphorous powder on to thin Zaponite-lacquer and gold-leaf backings. The three targets used in this investigation had measured thicknesses of 2.06, 1.25, and 0.566 mg/cm². The 1.25-mg/cm² target had a gold backing (230 μg/cm²), and Zaponite-lacquer backings (<10 μg/cm²) were used for the other

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‡ Purdue University XR Fellow, September 1961 to September 1963.

§ National Defense Education Act Fellow, September 1960 to September 1963.

[1] Unless otherwise specified the level structure and individual level properties proposed for P^{31} in the following reference will be assumed: P. M. Endt and C. Van Der Leun, Nucl. Phys. **34**, 1 (1962). In cases where specific quantitative information and/or interpretations are under discussion, a full bibliographical reference to the original reports will be made.

[2] G. I. Harris and L. W. Seagondollar, Phys. Rev. **128**, 338 (1962).

[3] G. I. Harris and L. W. Seagondollar, Phys. Rev. **131**, 787 (1963).

[4] C. Broude, L. L. Green, and J. C. Willmott, Proc. Phys. Soc. (London) **72**, 1097 (1958).

[5] V. K. Thankappan and S. P. Pandya, Nucl. Phys. **19**, 303 (1960).

[6] B. E. Chi and J. P. Davidson, Phys. Rev. **131**, 366 (1963).

[7] B. T. Lucas, S. W. Cosper, and O. E. Johnson, Phys. Rev. **133**, B963 (1964).

two targets. The 0.566-mg/cm^2 target was used in a reflection geometry at laboratory scattering angles greater than 90°, where target thickness seriously affects the experimental resolution. The two thicker targets were used for scattering angles less than 90°, although the gold-backed target was unsatisfactory at extreme forward angles because of the gold's large elastic-scattering cross section. The total oxygen and carbon content in the phosphorous and the backings as determined from the measured elastic alpha-particle groups and the known cross sections[8] was about 4 μg/cm^2 for each element. The estimated probable systematic error in the absolute cross sections due to uncertainties in target thickness, beam integration, and experimental geometry is $\pm 15\%$.

III. RESULTS, ANALYSIS, AND DISCUSSION

A. Elastic Scattering

No previous measurements of the differential elastic-scattering cross section of alpha particles from P^{31} have been reported. The experimental elastic alpha-particle cross section for P^{31} at 18.2 MeV is presented in Fig. 1.

The broken line in Fig. 1 corresponds to the familiar theoretical (α,α) cross section derived on the basis of the black-nucleus model[9] and is given by

$$\sigma_E(\theta)=(kR^2)^2[J_1^2(x)/x^2], \quad (1)$$

where k and θ are the center-of-mass wave number and scattering angle of the incident and scattered alpha particle, respectively; R is the interaction radius; $x\equiv 2kR\sin\theta/2$; and $J_1(x)$ is the ordinary Bessel function of first order. The interaction radius is the only adjustable parameter and there is no arbitrary normalization. The broken curve corresponds to an evaluation of Eq. (1) taking $R=6.28$ F, a value resulting from a compromise fitting of the angular positions of the experimental maxima. Although there is good agreement between theory and experiment in the angular positions of the maxima, there are large discrepancies between the magnitudes.

Blair[10] presented an interpretation of the elastic scattering of low-energy alpha particles by heavy nuclei that was based on a semiclassical model which had been previously proposed by Akhieser and Pomeranchuk[11] for the description of the small-angle scattering of high-energy charged particles. The APB model (α,α) differential cross section may be written in the form[12]

$$\sigma_E(\theta)=\tfrac{1}{4}\lambda\!\!\!^{-2}\,|[-i\eta/\sin^2(\theta/2)]\times\exp[-i\eta\ln\sin^2(\theta/2)]-\sum_{l=0}^{\infty}(2l+1)\times\exp[2i(\sigma_l-\sigma_0)][1-|A_l|]P_l(\cos\theta)|^2, \quad (2)$$

[8] J. C. Corelli, E. Bleuler, and D. J. Tendam, Phys. Rev. **116**, 1184 (1959).

[9] J. S. Blair, Phys. Rev. **108**, 827 (1957).

[10] J. S. Blair, Phys. Rev. **95**, 1218 (1954).

[11] A. Akhieser and I. Pomeranchuk, J. Phys. (USSR) **9**, 471 (1945).

[12] R. M. Eisberg and C. E. Porter, Rev. Mod. Phys. **33**, 190 (1961).

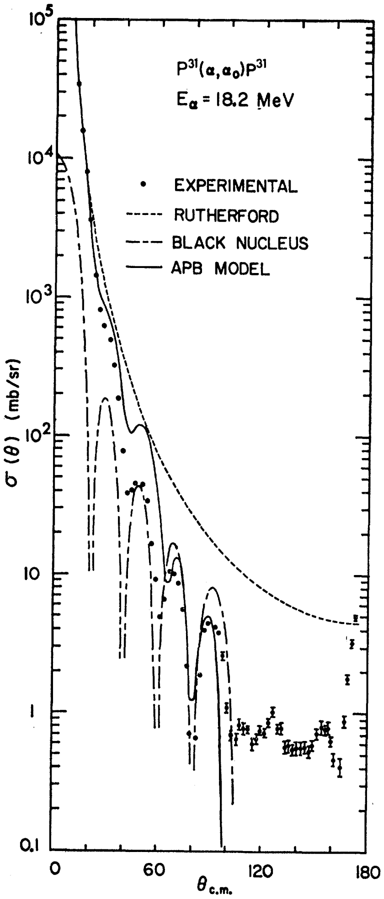

FIG. 1. The Rutherford, black-nucleus, modified APB model (see text), and experimental differential cross sections for the elastic scattering of 18.2-MeV alpha particles from P^{31}. The black-nucleus curve resulted from a compromise fit of the first three maxima with an interaction radius of $R=6.28$ F. The APB curve was fit with a cutoff angular momentum $l_0=9$.

where

$$\sigma_l-\sigma_0=\sum_{l'=1}^{l}\arctan(\eta/l'), \quad (3)$$

$$\eta=(Z_\alpha Z_N e^2/\hbar v), \quad (4)$$

$$|A_l|=0 \quad \text{for} \quad l\leq l_0, \qquad |A_l|=1 \quad \text{for} \quad l>l_0, \quad (5)$$

and l_0 corresponds to the cutoff angular momentum. The *a priori* range of validity would be expected to include those cases of small-angle elastic scattering for which the alpha particle's de Broglie wavelength is much smaller than the total projectile-target radius. This condition is not comfortably satisfied in the present experiment where $\lambda\!\!\!^{-}/R\approx 0.1$. However, a comparison between the experimental angular distribution and that predicted by a semiclassical model did prove interesting.

The solid curve shown in Fig. 1 corresponds to an evaluation of the "fuzzy" APB model of Wall, Rees, and Ford[13] in which $|A_l|$ has the value 0 for $l<l_0$, $\frac{1}{2}$ for $l=l_0$, and 1 for $l>l_0$ with $l_0=9$. The criterion for fitting the data was the reproduction of the angular positions of the maxima. A discrepancy of about a factor of 2 in the magnitudes of the theoretical and measured cross sections exists in the angular interval from 44 to 54°, but for extreme forward angles the agree-

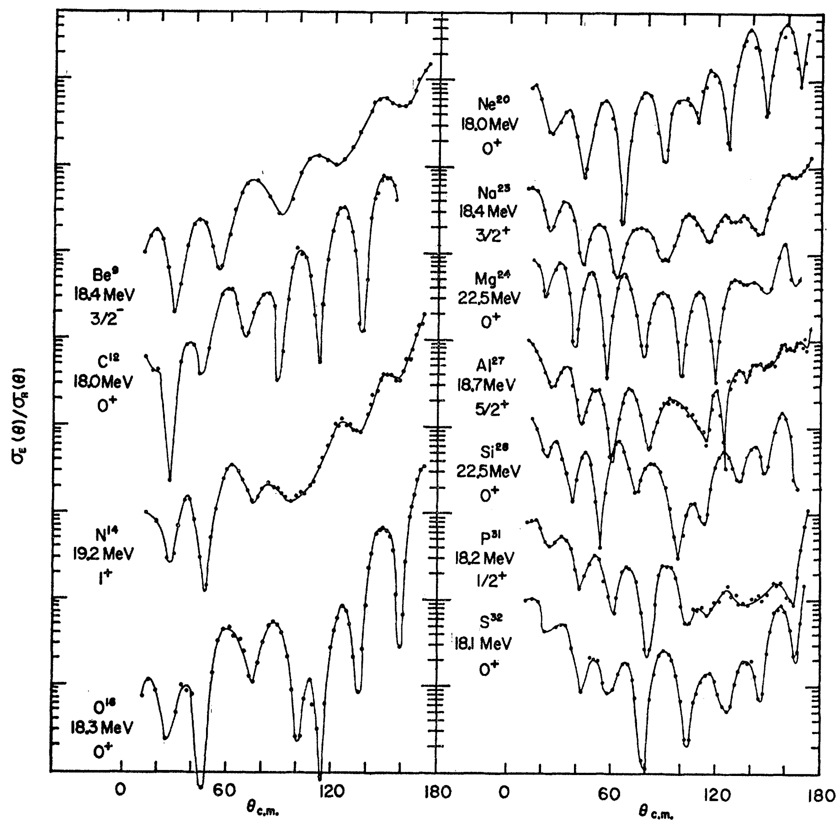

FIG. 2. A compilation of σ_E/σ_R ratios as a function of the alpha-particle scattering angle for a series of light nuclei for which the incident alpha-particle energies are in the range from 18.0 to 22.5 MeV. The data presented is taken from the following sources: Be^9, see Ref. 7; C^{12}, E. Bleuler (private communication); N^{14}, W. D. Ploughe, Ph.D. thesis, Purdue University, 1961 (unpublished) (L.C.: Mic. 61-5755, University Microfilms, Inc., Ann Arbor, Michigan); O^{16}, E. Bleuler (private communication); Ne^{20}, L. Seidlitz, E. Bleuler, and D. J. Tendam, Phys. Rev. **110**, 682 (1958); Na^{23}, preliminary results of an investigation in progress at this laboratory; Mg^{24}, W. W. Eidson (private communication); Al^{27}, O. H. Gailar, Ph.D. thesis, Purdue University, 1956 (unpublished) (L.C.: Mic. 57-616, University Microfilms, Inc., Ann Arbor, Michigan); Si^{28}, W. W. Eidson (private communication); P^{31}, present work; S^{32}, E. Bleuler (private communication).

ment is quite good. The "interaction radius" corresponding to the "best-fitting value" of l_0, in this case 9, determined using Eqs. (1) and (1′) of Ref. 13 is 7.3 F. Taking 2.5 F for the alpha-particle radius and following the procedure of Wall *et al.*,[13] 1.51 F was determined for r_0 which agrees very well with the value of 1.5 F obtained by them.

In Fig. 2 is presented a compilation of σ_E/σ_R ratios as a function of the alpha-particle scattering angle for a series of light nuclei for which the incident alpha-particle energies are in the range from 18.0 to 22.5 MeV. A detailed intercomparison of these curves has suggested the possible existence of certain systematic trends. A pronounced forward-angle diffraction-like pattern is present in every case, but the persistence of this behavior to backward angles appears to be restricted to the $4n$ nuclei. The patterns for the odd-A and odd-odd (N^{14}) nuclei show an irregular or washed-out oscillatory structure at large angles. The Mg^{24} results depart from this trend only in so far as relatively deep minima at about 140° are not observed.[14] This may be due to concurrence of alpha particles scattered from the other stable magnesium isotopes in the target. Limited experimental information suggests that the (α,α) cross section at large angles can under certain circumstances be strongly energy-dependent. Because of the wide variety of bombarding energies used in obtaining the data in Fig. 2, it seems unlikely that the suggested systematic behavior could be a manifestation of this effect.

A possible explanation may be sought in either the diffraction model of Blair[15] or in a recent report by Satchler,[16] both of which take into account collective effects arising from permanent nuclear deformation. Satchler suggests that differences between the elastic scattering from even-even and odd-A nuclei with spin$>\frac{1}{2}$ could be observed whenever the nuclei are permanently deformed. He assumed a nonspherical optical potential, expanded it to various orders in the deformation parameter, and found a second-order correction to the elastic-scattering cross section that appears only for odd-A nuclei. For quadrupole deformations this term is out of phase with the primary term in the elastic-scattering cross section, and tends to fill in the minima of the diffraction pattern at large angles. The diffraction model of Blair as applied to elastic and inelastic alpha-particle scattering predicts a similar odd-even effect for distorted nuclei. It was pointed out by Satchler that although the study of the angular dis-

[13] N. S. Wall, J. R. Rees, and K. W. Ford, Phys. Rev. **97**, 726 (1955).
[14] W. W. Eidson and J. G. Cramer, Jr., Phys. Rev. Letters **9**, 497 (1962).
[15] J. S. Blair, Phys. Rev. **115**, 928 (1959).
[16] G. R. Satchler, Nucl. Phys. **45**, 197 (1963).

tribution of a single nucleus is inconclusive due to the sensitivity of the cross section to the optical-model parameters, a systematic examination of a large number of nuclei should be able to reveal the existence of this effect. The systematic behavior proposed here on the basis of a rather limited amount of data is generally consistent with these predictions, although the predicted absence of this effect for spin-$\frac{1}{2}$ nuclei contradicts the observed qualitative difference between the P^{31} and S^{32} back-angle cross sections. Since these models[15,16] do not treat odd-odd nuclei, the comparison of the N^{14} cross section with those for the other nuclei is not relevant in this connection. Its behavior, however, appears similar to that of the other odd-A nuclei. It is interesting to speculate whether the odd-A nuclei Ne^{21}, Mg^{25}, and Si^{29} would fit into the systematic trend in a manner similar to that observed for Na^{23}, Al^{27}, and P^{31}.

Further systematic experimental investigation of elastic alpha-particle scattering from the lighter nuclei would be useful in gaining a better understanding of the reaction mechanisms involved and in developing a more complete theoretical model for the process. In this connection, it is important that the measurements extend over the full angular range, especially to the extreme backward angles. In those instances where possible and feasible, the energy dependence of the differential cross section should also be investigated.

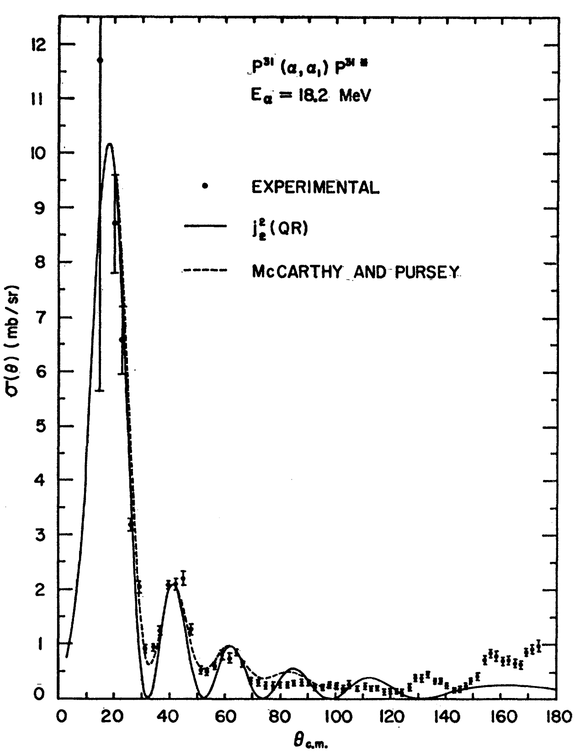

FIG. 3. The inelastic differential cross section for 18.2-MeV alpha-particle scattering to the 1.265-MeV, $\frac{3}{2}^+$ state in P^{31}. The indicated errors are probable errors estimated on the basis of counting statistics and decomposition uncertainties. The solid curve, representing the direct-interaction plane-wave Born-approximation prediction (ABM theory) with $l=2$ and $R=6.52$ F, was normalized at the second maximum. The dashed curve results from an evaluation of Eq. (6) using Eq. (7) with $l=2$, $R=6.62$ F, $\gamma=0.8$, and $\lambda=1.0$ F.

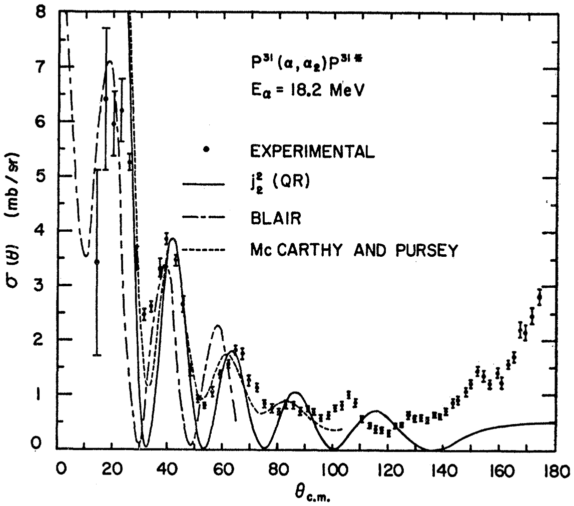

FIG. 4. The inelastic differential cross section for 18.2-MeV alpha-particle scattering to the 2.232-MeV, $\frac{5}{2}^+$ state in P^{31}. The indicated errors are probable errors estimated on the basis of counting statistics and decomposition uncertainties. The solid curve, representing the plane-wave Born-approximation prediction (ABM theory) with $l=2$ and $R=6.52$ F, was normalized at the second maximum. The dashed curve is the same as that shown in Fig. 3 except for the normalization. The Blair-model curve (broken line) corresponds to an evaluation of Eq. (8) using the interaction radius determined from the elastic-scattering data, $R=6.28$ F, and a compromise normalization to the first and second experimental maxima.

B. Inelastic Scattering

The center-of-mass differential cross sections for the scattering of alpha particles from the 1.265 MeV, $\frac{3}{2}^+$ and 2.232 MeV, $\frac{5}{2}^+$ states of P^{31} are shown in Figs. 3 and 4, respectively.[17] Both angular distributions exhibit a distinct oscillatory structure which persists to extreme backward angles, but is less pronounced for angles greater than 70°. A general increase in the cross section at backward angles is common to both angular distributions. However, at extreme forward angles their shapes are somewhat different. The angular distribution associated with the 1.265-MeV state appears to peak more in the forward direction than that of the 2.232-MeV state.

The spin and parity assignments of the 1.265- and 2.232-MeV states of P^{31} have been well established.[1] Consequently, the angular-momentum transfer in inelastic alpha-particle scattering to these states is uniquely determined to be $l=2$. The theoretical inelastic angular distribution according to the direct-interaction theory of Austern, Butler, and McManus[18]

[17] The measured differential cross sections reported in this paper are available in tabulated form from the authors.

[18] N. Austern, S. T. Butler, and H. McManus, Phys. Rev. **92**, 350 (1953).

(ABM theory) for $l=2$ is proportional to $j_2^2(QR)$, the square of the spherical Bessel function of second order, where Q is the magnitude of the momentum transfer, and R is the interaction radius. The solid curves in Figs. 3 and 4 represent the normalized theoretical angular distributions corresponding to $R=6.52$ F. This value of R was obtained through a compromise fitting of the first three maxima of the 1.265-MeV state's angular distribution. The theoretical angular distribution corresponding to the 1.265- and 2.232-MeV excitations of P^{31} were normalized to the experimental maxima at 42 and 40°, respectively. The ratio of the normalization factor of the second to that of the first excited state is 1.83. The value of the interaction radius obtained in the present investigation of P^{31}, $R=6.52$ F, is very close to the value of $R=6.50$ F obtained by Corelli[8] from the inelastic scattering of 18.2-MeV alpha particles from S^{32}. The agreement between the theoretical and experimental angular distributions appears to be better for scattering from the first excited state than from the second excited state. It should be pointed out that the angular distributions are in phase as predicted by the ABM theory.

McCarthy and Pursey[19] have derived expressions for the differential cross section for the inelastic scattering of alpha particles on the basis of a distorted-wave Born approximation assuming empirical distorted wave functions for the incident and scattered particle. In the case of small momentum transfer the cross section can be written

$$\sigma(\theta)\propto\left|\int_0^\infty \exp[-(r-R)^2/\lambda^2] \times j_l[2(kr+i\gamma)\sin(\theta/2)]r^2dr\right|^2, \quad (6)$$

where l is the angular-momentum transfer, k and θ are, respectively, the center-of-mass scattering angle and the average magnitude of the center-of-mass wave vector of the alpha particle. The interaction radius, R, as in the ABM[18] and Blair[15] theories, controls the positions of the extrema in the angular distribution. The anisotropy parameter γ determines the spread of the alpha-particle wave function over the nuclear surface, and influences the peak-to-valley ratios of the angular distribution. The thickness parameter λ expresses the degree of radial localization of the interaction and has a strong effect on the ratio of the forward to backward scattering cross section. The exponential radial weighting function which appears in the integrand of Eq. (6) was arbitrarily chosen by McCarthy and Pursey for reasons of simplicity. It was found convenient in the present investigation to replace this one with another choice,

$$\begin{array}{lll} 1-(r-R)^2/\lambda^2 & \text{for} & (R-\lambda)\le r\le(R+\lambda)\,, \\ 0 & \text{for} & 0<r<(R-\lambda) \text{ and } r>(R+\lambda)\,. \end{array} \quad (7)$$

[19] I. E. McCarthy and D. L. Pursey, Phys. Rev. 122, 578 (1961).

The theoretical expression given in Eq. (6) with the modification indicated in Eq. (7) was fit to the experimental angular distribution corresponding to scattering from the 1.265-MeV state. The fit obtained for $R=6.62$ F, $\lambda=1.0$ F and $\gamma=0.8$ is shown in Fig. 3. The normalization point is at 42°. Over the limited angular interval for which the comparison was made the agreement is quite good. No attempt was made to investigate the uniqueness of this set of parameter values. These values do, however, correspond quite closely to those used by McCarthy and Pursey,[19] $\lambda=0.88$ F and $\gamma=0.8$, in their analyses of the angular distributions for the inelastic scattering of 41-MeV alpha particles from S^{32} and Mg^{24}. The theoretical angular distribution using the same parameter values as in Fig. 3 is shown in Fig. 4 together with the experimental angular distribution for scattering from the 2.232-MeV state. The normalization point is at 40°. The agreement is less satisfactory than for the 1.265-MeV state, particularly at extreme forward angles.

Within the framework of an extended diffraction model, Blair[15] has developed expressions for those (α,α') cross sections in which transitions occur between collective nuclear states. In the weak-coupling approximation under the assumption of a quadrupole nuclear deformation, the cross section for a transition between a ground state of spin I and a final state of spin I' for an odd-A nucleus is given by

$$\sigma(\theta)=\frac{(2I'+1)}{(2I+1)}\frac{(kR^2)^2}{32\pi}\frac{\hbar\omega_2}{C_2}[J_0^2(x)+3J_2^2(x)] \quad (8)$$

for vibrational transitions, and by

$$\sigma(\theta)=(I,2,K,0|I',K)^2\frac{(kR^2)^2}{16\pi}\beta_2^2[J_0^2(x)+3J_2^2(x)] \quad (9)$$

for rotational transitions. The surface phonon energy is $\hbar\omega_2$, C_2 is the surface potential-energy parameter, β_2 is the nuclear-deformation parameter, K is the quantum number associated with the projection of the total angular momentum onto the nuclear symmetry axis, $(I,2,K,0|I',K)$ is a Clebsch-Gordan coefficient, and the other quantities have been identified in connection with Eq. (1). In the latter case, transitions are restricted to the members of the rotational band with the quantum number K which is built upon the ground state. If the interaction radius which has been determined from the elastic angular distribution is used in Eq. (8) or (9), then the normalization of the theoretical expression to the experimental data determines either the vibrational or rotational parameter depending upon the assumed character of the states.

The lower lying levels (<4 MeV) of P^{31} have been analyzed with some success in terms of a weak-coupling vibrational model by Thankappan[5,20] and Pandya.[5]

[20] V. K. Thankappan, Phys. Letters 2, 122 (1962).

The P^{31} nucleus is described as a spherical Si^{30} core (14 protons and 16 neutrons filling the subshells up to $1d_{3/2}$ and $2s_{1/2}$, respectively), whose collective modes are coupled to the $2s_{1/2}$ and $1d_{3/2}$ single-particle states of the odd proton. These investigators concluded that: the ground state is about 90% single-particle $2s_{1/2}$ state, that is, a $2s_{1/2}$ proton coupled to a zero-phonon state of the Si^{30} core; the 1.265-MeV state is an admixture of 50% $d_{3/2}$ single-particle state and 40% $2s_{1/2}$ state coupled to a one-phonon vibrational state of the core; and the 2.232-MeV state is 85% $2s_{1/2}$ state coupled to a one-phonon vibrational state of the core. If these wave functions are assumed to be correct, then the inelastic scattering to the 2.232-MeV state is to a good approximation a pure vibrational transition. The broken curve shown in Fig. 4 then corresponds to the cross section given by Eq. (8). The normalization of necessity corresponded to a compromise fitting of the first and second maxima since the experimental and theoretical curves are slightly out of phase. Normalization determined the quantity $\hbar\omega_2/C_2$ to be 0.0071. Taking the energy of the first excited state of Si^{30} at 2.24 MeV to be the energy of the quadrupole vibrational quanta, $\hbar\omega_2$, and using the latter result, the value of the surface potential-energy parameter, C_2, is 315 MeV. This value may be compared with the irrotational value for P^{31}, $(C_2)_{\mathrm{irrot}}=40.2$ MeV, calculated from a semiempirical expression[21] which was developed on the basis of a liquid-drop model. The vibrational inertial parameter, B_2, may also be calculated since $\omega_2=(C_2/B_2)^{1/2}$. Its value expressed in units of the irrotational value, $(B_2)_{\mathrm{irrot}}$, for P^{31} is 50.

The 1.265-MeV state of P^{31} is, on the other hand, an admixture of single-particle and collective states according to the analysis of Thankappan and Pandya.[5] If it is assumed that the low-lying levels are excited primarily through collective modes and that single-particle excitations are inhibited, as is currently believed to be the case, the cross section for the 1.265-MeV state should be smaller than that for the 2.232-MeV state. The ratio of the differential cross sections integrated over the angular range investigated, $\sigma_{2.232}/\sigma_{1.265}$, was found to be about 2, in qualitative agreement with the above expectation.

The level structure and level decay properties of a number of odd-mass nuclei in the $(1d,2s)$ shell have been studied within the framework of the unified model of Nilsson[22] with varying degrees of success.[23] The analysis of P^{31} by Broude, Green, and Willmott[4] in which the ground and 2.232-MeV states were identified as the two lowest members of the $\Omega=K=\frac{1}{2}$ rotational band (Nilsson's band 9) is of particular interest. If correct, then the inelastic alpha-particle scattering to the 2.232-MeV state could be interpreted as a pure intraband transition. Taking $K=\frac{1}{2}$, it then follows that the normalization of Eq. (9) to the experimental angular distribution yields $|\beta_2|=0.13$, which agrees with the value of $\beta_2=-0.16$ extracted from the results of Broude *et al.*[4] after assuming the spin-orbit coupling to well-depth ratio to have the value of 0.05 suggested by Nilsson. Unfortunately, the justification for assuming a pure intraband transition which would have been provided by the success of the analysis by Broude *et al.*[4] is lost because an error in the original calculations has been reported.[24] The agreement between the experimental and revised theoretical level schemes is less satisfactory than it had been for the original calculation.[25,26] The 1.265-MeV state was found to be of mixed rotational character in both the original and revised calculation. Since the applicability of Eq. (9) is predicated on a pure intraband transition, the significance of the distortion parameter derived from the alpha-particle scattering data is questionable. Moreover, Harris and Seagondollar,[3] on the basis of a comparison of the empirical level scheme of P^{31} with those predicted by a weak-coupling vibrational model and a Nilsson rotational model, suggest that rotational effects which are more or less well established as dominant near $A=25$ may have given way to vibrational effects in the vicinity of $A=31$.

For completeness, it should be pointed out that Chi and Davidson[6] have analyzed the low-lying levels of odd-A nuclei in the mass region from $A=17$ to $A=35$ in terms of an asymmetric, rotating-core model. The level schemes of these nuclei are generally quite well represented by the model. The energies of the lowest three excited states of P^{31} are in very good agreement with the experimental values, and the magnitude of the theoretical deformation parameter is 0.205.

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[21] A. Bohr and B. R. Mottleson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953), see Eqs. (II.6a), (II.6b) in this paper.

[22] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).

[23] An extensive compilation and summary of the results of the studies available as of September 1960 has been presented by H. E. Gove, in *Proceedings of the International Conference on Nuclear Structure at Kingston* (University of Toronto Press, Toronto, 1960), p. 438ff.

[24] L. L. Green, J. C. Willmott, and G. Kayne, Nucl. Phys. **25**, 278 (1961).

[25] E. E. Baart, L. L. Green, and J. C. Willmott, Proc. Phys. Soc. (London) **79**, 237 (1962).

[26] A comparison of the experimental level scheme of P^{31} with the revised version of the theoretical level scheme is shown in Fig. 12 of Ref. 3.