contributions.[17] Another interesting set of reactions involving ϕ and ω-productions are $\pi^+ + P \to N^{*++} + \phi(\omega)$. Recently, Meshkov *et al.*[18] analyzed these reactions together with some of the partner processes in unitary symmetry. Assuming that the SU (3) matrix elements are not perturbed by the mass splitting interactions, they derived the cross-section sum rule

$$F_a\sigma_a = F_b\sigma_b + 3F_c\sigma_c - 3F_d\sigma_d\,, \qquad (13)$$

where σ_a, σ_b, σ_c, and σ_d are the cross sections for the reactions $K^+ + P \to N^{*++} + K^{*0}$, $\pi^+ + P \to N^{*++} + \rho^0$, $\pi^+ + P \to N^{*++} + \omega_8$, and $\pi^+ + P \to Y_1^{*+} + K^{*+}$, respectively. F_a, F_b, F_c, and F_d are the respective kinematic factors. Experimentally the ratio $\sigma(\pi^+ + P \to N^{*++} + \phi)/\sigma(\pi^+ + P \to N^{*++} + \omega)$ is much less than unity, which leads to

$$\sin^2\theta \simeq [F_a\sigma_a + 3F_d\sigma_d - F_b\sigma_b]/3F_f\sigma_f\,,$$

where σ_f and F_f are, respectively, the cross section and kinematical factor for the reaction $\pi^+ + P \to N^{*++} + \omega$. For what it is worth, the best estimate of θ, from the data presented by Meshkov *et al.*[18] turns out to be

$$\theta \simeq 23° \sim 27°$$

which is close to the solution θ_1 obtained here.

To summarize, we have attempted to obtain the ϕ-ω-mixing angle θ independently of the Gell-Mann–Okubo mass formula for the vector-meson octet. The considerations presented here indicate mixing angles smaller than the Okubo-Sakurai value, based on the use of the mass formula. The lower mixing angles obtained here correspond to only a small violation (less than 10%) of the mass-square formula. Experimental information on the various decay rates and branching ratios, that we feel, will be of great interest in testing some of the ideas and results presented here are listed in Table I.

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[17] There seems to be good indications from Ref. 8 that both K and K^* exchange contribute to these reactions.

[18] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **12**, 87 (1964).

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Reciprocal Bootstrap Relationship of the Octet Baryon and the Decuplet Baryon*

YASUO HARA†
California Institute of Technology, Pasadena, California

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The reciprocal bootstrap relationship of the octet of $(\frac{1}{2})^+$ baryons and the decuplet of $(\frac{3}{2})^+$ baryons is studied making use of the static approximation. If we regard the octet baryon as a $B_8\Pi_8$ bound state due to octet and decuplet baryon exchange, we obtain $\gamma_{10} \approx 7d^2$ and $d/f \approx 2.2$, where γ_{10} is the $\bar{B}_{10}B_8\Pi_8$ coupling constant and d and f are the d and f coupling constants of $\bar{B}_8B_8\Pi_8$ coupling. If octet vector meson exchange processes are included and if we assume the vector theory (gauge theory) of strong interactions, we obtain $\gamma_{10} < 7d^2$ and $d/f < 2.2$. If we regard the decuplet baryon as a $B_8\Pi_8$ bound state due to octet baryon exchange, we obtain $\gamma_{10} \approx 4d^2$ for the ratio $d/f = 2.2$.

ABOUT two years ago, Chew proposed the idea of a reciprocal bootstrap relationship between the nucleon and the (3,3) resonance.[1] Since that time, the accumulated experimental data, especially the recent discovery of the Ω^- particle,[2] seem to have proved the validity of the first-order broken eightfold way.[3–5] Here we would like to check the idea of a reciprocal bootstrap relationship between the octet of $(\frac{1}{2})^+$ baryons and the decuplet of $(\frac{3}{2})^+$ baryons, making use of the static approximation.[6] This is interesting since the bootstrap

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† On leave of absence from Physics Department, Tokyo University of Education, Tokyo, Japan.

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[2] V. E. Barnes, P. L. Connolly, D. J. Crennell, B. B. Culwick, *et al.*, Phys. Rev. Letters **12**, 204 (1964).

[3] M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report CTSL-20, 1961 (unpublished).

[4] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[5] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 947 (1962).

[6] This problem has been discussed by V. Singh (to be published) making use of the static model, but he has made mistakes in the treatment of the coupled N/D method. This problem has also been discussed by I. S. Gerstein and K. T. Mahanthappa, but their result is contradictory with a Chew-type coupled N/D method. I. S. Gerstein and K. T. Mahanthappa, Nuovo Cimento **32**, 239 (1964).

mechanism is expected by several physicists[7] to explain the origin of the eightfold way.

The reciprocal bootstrap relationship of the octet and the decuplet has been discussed by Martin and Wali.[8] Though they have included relativistic and recoil effects, it is rather difficult to see the validity of the reciprocal bootstrap relationship in their formalism. On the other hand, it is easy to see the validity of the reciprocal bootstrap relationship if we use the static approximation. We consider that it would be instructive to check the idea of the reciprocal bootstrap relationship making use of the static approximation. Though the static model seems to be a very crude model, the results due to the static model[9,10] have been shown to agree fairly well with the results due to the relativistic N/D method.[11]

In this article, we consider the limit of unitary symmetry for the sake of simplicity. At first, we neglect vector meson exchange processes. It has been shown[9,12,13] that only an octet of $(\frac{1}{2})^+$ baryons and a decuplet of $(\frac{3}{2})^+$ baryons are possible bound (or resonant) states of $B_8\Pi_8$ system in this approximation for $1\lesssim d/f\lesssim 2$, where d and f are the d and f coupling constants[3] of the $ps(pv)\bar{B}_8B_8\Pi_8$ coupling[14] and

$$(d+f)^2=f_{\pi NN}^2=g_{\pi NN}^2(m_P/2m)^2,$$

$(g_{\pi NN}^2\approx 15)$. Thus, the scattering amplitudes in the octet $(\frac{1}{2})^+$ states can be written in the pole approximation as

$$f_{ij}(\omega)=\frac{a_{ij}}{\omega}+\frac{b_{ij}}{\omega+\epsilon}+\frac{c_{ij}}{\omega+\omega^*},\quad (1\leq i,\, j\leq 2) \qquad (1)$$

where ω is the meson energy, $\omega^*=m(B_{10})-m(B_8)$, and $\mathrm{Im}(f^{-1})=-\rho(\omega)[\rho(\omega)\approx(\omega^2-1)^{3/2}$ at low energies$]$. The indices 1 and 2 which are assumed by i and j in (1) stand for two octet states 8_1 and 8_2,

$$8_1=\alpha|8_S\rangle+\beta|8_A\rangle,$$
$$8_2=-\beta|8_S\rangle+\alpha|8_A\rangle,$$

where

$$\alpha=(\sqrt{6}+1)/(12+2\sqrt{6})^{1/2}$$

and

$$\beta=\sqrt{5}/(12+2\sqrt{6})^{1/2}.$$

The constant ϵ is a conventional one which discriminates the pseudopole term due to the octet of baryons in the u channel from the true pole term due to the octet of baryons in the s channel and is taken to be equal to zero at the end of the calculation. In Eq. (1), a_{ij}, b_{ij}, and $c_{ij}(=c_i\delta_{ij})$ are as follows:

$$(a_{ij})=-\begin{pmatrix}p^2, & pq\\ pq, & q^2\end{pmatrix},$$

$$p=(20/3)^{1/2}\alpha d+2\sqrt{3}\beta f,$$

$$q=-(20/3)^{1/2}\beta d+2\sqrt{3}\alpha f,$$

$$(b_{ij})=\begin{pmatrix}\alpha, & \beta\\ -\beta, & \alpha\end{pmatrix}\begin{pmatrix}\frac{2}{3}d^2+2f^2, & 0\\ 0, & (10/9)d^2-2f^2\end{pmatrix}\begin{pmatrix}\alpha, & -\beta\\ \beta, & \alpha\end{pmatrix}.$$

and

$$(c_{ij})=\begin{pmatrix}(1+\sqrt{6})/3, & 0\\ 0, & (1-\sqrt{6})/3\end{pmatrix}\gamma_{10},$$

where γ_{10} is the reduced half-width of the decuplet baryon defined as $\lim_{\omega\to\omega^*}\{(\omega^*-\omega)f_{10}(\omega)\}$. In order to determine d and f as a function of γ_{10}, we use the N/D method. As usual, we write $f_{ij}(\omega)=N_{ik}(\omega)D_{kj}^{-1}(\omega)$, where

$$N_{ij}(\omega)=\frac{b_{ik}D_{kj}(-\epsilon)}{\omega+\epsilon}+\frac{c_iD_{ij}(-\omega^*)}{\omega+\omega^*}, \qquad (2)$$

and

$$D_{ij}(\omega)=\delta_{ij}-\frac{(\omega+\omega^*)}{\pi}\int_1^\Lambda d\omega'\frac{\rho(\omega')N_{ij}(\omega')}{(\omega'+\omega^*)(\omega'-\omega)}. \qquad (3)$$

The cutoff energy Λ is determined by the condition,[15] $\det|D_{ij}(0)|=0$.

As will be shown later, the terms in Eqs. (2) and (3) that contain b_{ik} are not important. Therefore, let us neglect these terms as a first approximation. Then, as is well known,[12] the coupled equations (2) and (3) can be solved easily $[a_{11}=-c_1,\ a_{12}=a_{21}=a_{22}=0$; see Eq. (4)$]$ and we find

$$d/f=3(1+\sqrt{6})/5\approx 2.07$$

and

$$\gamma_{10}\approx 8d^2.$$

For the ratio $d/f=2.07$, we have $b_{11}=0.99d^2$, $b_{12}=-0.22d^2$, and $b_{22}=0.79d^2$ which are much smaller than $c_1\approx 9d^2$. [We can compare b_{ij} and c_i without considering the difference of the positions of the poles. See Eq. (4).]

Next let us solve Eqs. (2) and (3) without neglecting the terms that contain b_{ik}. However, we will neglect terms of $O(b^2)$, since $|b_{ik}|\ll|c_j|$. After a straightforward

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[7] See, for example, R. H. Capps, Phys. Rev. Letters **10**, 312 (1963); E. Abers, F. Zachariasen, and A. C. Zemach, Phys. Rev. **132**, 1831 (1963); S. Glashow, *ibid.* **130**, 2132 (1963); R. E. Cutkosky and P. Tarjanne, *ibid.* **132**, 1354 (1963).

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[11] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

[12] R. E. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963).

[13] Y. Hara, Phys. Rev. **133**, B1565 (1964).

[14] In the following, we use units such that $\hbar=c=$(octet pseudoscalar meson mass, $m_P)=1$, where $m_P^2=(4m_K^2+3m_\pi^2+m_\eta^2)/8=(410\text{ MeV})^2$. The octet baryon mass $m=m(B_8)=(2m_N+2m_\Xi+3m_\Sigma+m_\Lambda)/8=1150$ MeV. The decuplet baryon mass $m(B_{10})=(m(\Omega)+2m(\Xi^*)+3m(Y^*)+4m(N^*))/10=1385$ MeV. The octet vector meson mass squared $m_V^2=(4m(K^*)^2+3m(\rho)^2+m(\phi)^2)/8=(860\text{ MeV})^2$.

[15] In Eq. (3) we have assumed that $\Lambda_1=\Lambda_2=\Lambda$. This assumption seems to be physically reasonable.

calculation, we find

$$-(a_{ij})=-\lim_{\omega\to 0}\{\omega(f_{ij}(\omega))\}=-\lim_{\omega\to 0}\{\omega(N_{ik}(\omega)D_{kj}^{-1}(\omega))\}$$

$$=\begin{pmatrix}c_1, & 0\\ 0, & c_2\end{pmatrix}\begin{pmatrix}D_{22}(0), & -D_{12}(0)\\ -D_{21}(0), & 0\end{pmatrix}\times\frac{1}{D_{22}(0)}$$

$$+\begin{pmatrix}b_{11}, & b_{12}\\ b_{21}, & b_{22}\end{pmatrix}\begin{pmatrix}1, & D_{12}(-\epsilon)\omega^*/\epsilon\\ D_{21}(-\epsilon)\omega^*/\epsilon, & D_{22}(-\epsilon)\omega^*/\epsilon\end{pmatrix}\begin{pmatrix}D_{22}(0), & -D_{12}(0)\\ -D_{21}(0), & 0\end{pmatrix}\times 1/D_{22}(0)$$

$$\approx\begin{pmatrix}c_1+b_{11}, & b_{12}\\ b_{21} & 0\end{pmatrix}, \tag{4}$$

where $D_{11}(\omega)\approx-\omega/\omega^*$, $D_{21}(0)\approx 0$, and

$$D_{12}(0)\approx -b_{12}D_{22}(0)/c_1.$$

From (4), we obtain

$$d/f\approx 2.2 \quad\text{and}\quad \gamma_{10}\approx 7d^2. \tag{5}$$

Next, let us consider the decuplet of $(\frac{3}{2})^+$ baryons. For $-1\lesssim\omega\lesssim 2$, the pseudopole near $\omega=0$ should dominate the numerator functions, giving

$$N_{10}(\omega)=(2/9)(8d^2+24df)/\omega,$$

while we assume

$$D_{10}(\omega)=1-(\omega/\omega^*)-i\rho(\omega)N_{10}(\omega)\theta(\omega-1).$$

Thus, the reduced half-width of the decuplet of $(\frac{3}{2})^+$ baryons is predicted to be

$$\gamma_{10}=\omega^*N_{10}(\omega^*)=(2/9)(8d^2+24df)\approx 4d^2. \tag{6}$$

If we take account of the fact that the static model is a crude one, we may say that our results (5) and (6) certainly suggest the validity of the reciprocal bootstrap relationship.

Let us consider the vector meson exchange effect. The integrated contribution to the $(\frac{1}{2})^+$ octet states from two vector meson cuts is roughly (see Appendix A)

$$f_{ij}^V(\omega)=\int_{V_8\,\text{cuts}}d\omega'\frac{G_{ij}^V(\omega')}{\omega'-\omega}\approx-\frac{1}{4\pi}g_{ij}\frac{1}{2k^2}\ln\left(1+\frac{4k^2}{m_V^2}\right), \tag{7}$$

where $k^2=\omega^2-1$ and the g_{ij} is

$$(g_{ij})=\begin{pmatrix}-(10/\sqrt{6})d_V-6f_V, & -(10/3)^{1/2}d_V\\ -(10/3)^{1/2}d_V, & (10/\sqrt{6})d_V-6f_V\end{pmatrix}f_{\Pi\Pi V}, \tag{8}$$

In (8), $f_{\Pi\Pi V}$ is the $\Pi_8\Pi_8V_8$ coupling constant and d_V and f_V are the d and f coupling constants of $\bar{B}_8B_8V_8$ tensor coupling. If we neglect ω^*/Λ and m_V/Λ, the vector meson exchange process does not contribute to the decuplet pole residue. However, it contributes to the pole residues of the octet baryon poles if $d_V\neq 0$. If the octet vector meson contributions dominate the dispersion integral of the electromagnetic form factors, $d_V\approx 3f_V$ and $f_Vf_{\Pi\Pi V}/4\pi\approx(\mu_p-\mu_n)/16m$, where μ_p and μ_n are the anomalous magnetic moments of the proton and the neutron. In this case, vector meson exchange decreases the d/f ratio and reduces the γ_{10}/d^2 ratio qualitatively (see Appendix B). Thus, instead of (5) we obtain

$$d/f\lesssim 2.2,$$

and

$$\gamma_{10}\lesssim 7d^2, \tag{5'}$$

and $\gamma_{10}\lesssim 7d^2$ should be compared with $\gamma_{10}\approx 4d^2$.

In conclusion, the idea of a reciprocal bootstrap relationship seems to be still valid in this extended case.

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APPENDIX A

We have obtained the expression (7) as follows[16]:

$$f_{ij}^V(\omega)\approx\{4m^2(A_1+\omega B_1)+k^2(-A_0+2mB_0)\}/(32\pi m^2k^2)$$
$$\approx(-A_0+2mB_0)/(32\pi m^2)$$
$$\approx -g_{ij}\ln(1+4k^2/m_V^2)/(8\pi k^2).$$

APPENDIX B

We approximate $f_{ij}^V(\omega)$ as follows[16]:

$$f_{ij}^V(\omega)\approx\frac{id_{ij}}{\omega+i\xi}-\frac{id_{ij}}{\omega-i\xi},$$

where ξ is a real constant, and d_{ij}'s are all positive except d_{22}. If we neglect b_{ij}, we obtain

$$\begin{pmatrix}a_{11}, & a_{12}\\ a_{21}, & a_{22}\end{pmatrix}=\begin{pmatrix}c_1D_{22}(0), & -c_1D_{12}(0)\\ -c_2D_{21}(0), & c_2D_{11}(0)\end{pmatrix}\bigg/(D_{11}(0)+D_{22}(0)),$$

where $D_{12}(0)<0$, $D_{21}(0)>0$, $D_{11}(0)<0$, and $D_{22}(0)>0$. Thus, we find $a_{11}>c_1$ and $a_{12}>0$. Therefore, we find

$$7d^2>\gamma_{10} \quad\text{and}\quad d/f<2.2.$$

[16] For the notations, see S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).