Low-Energy $\bar{K}N$ Interaction and Y_0^* Regge Pole

S. N. BISWAS AND S. K. BOSE
Tata Institute of Fundamental Research, Bombay, India
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Assuming that the observed $(\Sigma\pi)$ resonance at 1405 MeV is in the $S_{1/2}$ state, and treating it as a Regge particle, the low-energy amplitude is calculated using the Khuri representation. The calculated $\bar{K}N$ scattering length is in agreement with the solution II of Humphrey and Ross.

An extension of the Regge representation, which is suitable for studying the low-energy behavior of scattering amplitudes, has recently been proposed by Khuri.[1] The Khuri representation has been subsequently applied to study the pion-nucleon scattering in the $J=\frac{1}{2}$, $T=\frac{1}{2}$ state by Khuri and Udgaonkar.[2] This method was later extended to treat pion-nucleon scattering in the $J=\frac{3}{2}$, $T=\frac{3}{2}$ state.[3] Similar considerations have been made for neutron-proton scattering in the ${}^3S_1$ state[4] and the problem of pion photoproduction on nucleons.[5] These discussions have shown that approximate solutions for the scattering amplitude, constructed on the basis of a single Regge pole in the direct channel together with simple ansatz regarding the form of the trajectory and the residue function, is capable of roughly reproducing the observed data at low energies. In view of this it is worthwhile to ask if essentially similar discussions can be made for a more complex system. In this paper we attempt an answer to this question. We treat a two-channel problem, namely, the $\bar{K}N$ amplitude in the $T=0$, $S_{1/2}$ state. We take into account only the trajectory associated with the $\Sigma\pi$ resonance[6,7] at 1405 MeV with a width of 50 MeV, the Y_0^*. We find that multichannel effects cause no new difficulty and the entire discussion goes through as in the pion-nucleon case. We also find that the calculated value of the (complex) scattering length for $\bar{K}N$ elastic scattering is in agreement with the solution II of Humphrey and Ross.[8] We should emphasize that our treatment of the $S_{1/2}$ partial wave of the $\bar{K}N$ system will be valid only if the Y_0^* is an S-wave resonance in the $\Sigma\pi$ channel (or an S-wave virtual bound state of $\bar{K}N$ in the sense of Dalitz and Tuan[9]). We may therefore conclude that this assignment of spin and parity to the Y_0^* is not inconsistent with present data.

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The $S_{1/2}$ partial-wave amplitude for $\bar{K}N$ reactions may be written as a matrix in the following way:

$$T=\begin{pmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{pmatrix}. \tag{1}$$

T_{11} corresponds to the process $\bar{K}+N\rightarrow\bar{K}+N$ and is normalized as

$$T_{11}=(1/k)e^{i\delta_1}\sin\delta_1, \tag{2}$$

δ_1 being the (complex) phase shift for elastic $\bar{K}N$ scattering and k the magnitude of c.m. 3-momentum. Similarly, T_{22} is normalized as

$$T_{22}=(1/q)e^{i\delta_2}\sin\delta_2, \tag{3}$$

δ_2 being the $\Sigma\pi$ phase shift and q the c.m. 3-momentum of the $\Sigma\pi$ system. Below the $\bar{K}N$ threshold δ_2 is real. T_{12} corresponds to the reaction amplitude for $\bar{K}+N\rightarrow \Sigma+\pi$, which is related to the corresponding cross section as

$$\sigma(\Sigma^0)\equiv\sigma(\bar{K}+N\rightarrow\Sigma^0+\pi^0)=(4\pi/6)|T_{12}|^2. \tag{4}$$

Using the Khuri representation the contribution of the Y_0^* Regge pole to the elements of the T matrix can be calculated in a straightforward way. The result is

$$T_{11}=\frac{\beta_1}{\alpha(W)-\frac{1}{2}}[e^{(\alpha-\frac{1}{2})\xi_1}+e^{(\alpha-\frac{1}{2})\xi_2}], \tag{5}$$

$$T_{12}=\frac{\beta_{12}}{\alpha(W)-\frac{1}{2}}[e^{(\alpha-\frac{1}{2})\xi_1''}+e^{(\alpha-\frac{1}{2})\xi_2''}], \tag{6}$$

$$T_{22}=\frac{\beta_2}{\alpha(W)-\frac{1}{2}}[e^{(\alpha-\frac{1}{2})\xi_1'}+e^{(\alpha-\frac{1}{2})\xi_2'}]. \tag{7}$$

In (5)–(7) $\alpha(W)$ is the Y_0^* trajectory and β's the corresponding residuum functions. The ξ's are given by the following expressions:

$$\cosh\xi_1=1+2/k^2, \tag{8}$$

$$\cosh\xi_2=[W^2-(m-m_K)^2]/2k^2-1, \tag{9}$$

$$\cosh\xi_1'=1+2/q^2, \tag{10}$$

$$\cosh\xi_2'=[W^2-2m_\Sigma^2-2+m_\Lambda^2]/2q^2-1, \tag{11}$$

$$\cosh\xi_1''=\frac{(m_K+1)^2-m^2-m_\Sigma^2+2(k^2+m^2)^{1/2}(q^2+m_\Sigma^2)^{1/2}}{2|k||q|},\tag{12}$$

$$\cosh\xi_2''=\frac{W^2+m^2-m_K^2-1-2(k^2+m^2)^{1/2}(q^2+m_\Sigma^2)^{1/2}}{2|k||q|}.\tag{13}$$

In (8)–(13), W is the total c.m. energy and m the nucleon mass. m_K, m_Λ, m_Σ denote the masses of the corresponding particles and the pion mass has been set equal to unity. We will now approximate Eqs. (5)–(7) using considerations based on the threshold behavior of the β's. The latter suggests[2] one to write

$$\beta_1(k^2)e^{(\alpha-\frac{1}{2})\xi_1}=C_1,\tag{14}$$

$$\beta_2(q^2)e^{(\alpha-\frac{1}{2})\xi_1'}=C_2,\tag{15}$$

when C_1 and C_2 are real constants. These relations are exact at their respective thresholds and may not be too far off in the low-energy region to which our study is confined. β_{12} is now determined by using the factorization theorem,[10,11] $\beta_{12}^2(W)=\beta_1(W)\beta_2(W)$. This yields

$$\beta_{12}=(C_1C_2)^{1/2}e^{-\frac{1}{2}(\alpha-\frac{1}{2})(\xi_1+\xi_1')}.\tag{16}$$

Using (14)–(16) the expressions for the elements of the T matrix become

$$T_{11}=[C_1/(\alpha-\tfrac{1}{2})][1+e^{(\alpha-\frac{1}{2})(\xi_2-\xi_1)}],\tag{17}$$

$$T_{22}=[C_2/(\alpha-\tfrac{1}{2})][1+e^{(\alpha-\frac{1}{2})(\xi_2'-\xi_1')}],\tag{18}$$

$$T_{12}=\frac{(C_1C_2)^{1/2}}{\alpha-\frac{1}{2}}\exp(\alpha-\tfrac{1}{2})\left(\xi_1''-\frac{\xi_1+\xi_1'}{2}\right)\times[1+e^{(\alpha-\frac{1}{2})(\xi_2''-\xi_1'')}].\tag{19}$$

Let us now consider the Y_0^* trajectory. Following previous discussions[2,3] on the pion-nucleon problem we may write the Y_0^* trajectory in the form

$$\mathrm{Re}\alpha(W)=\tfrac{1}{2}+\epsilon(W-m_{Y_0^*}),\tag{20}$$

$$\mathrm{Im}\alpha(W)=\frac{\epsilon\Gamma}{2}\left(\frac{W-m_\Sigma-1}{m_{Y_0^*}-m_\Sigma-1}\right)^{\alpha_0};\quad \alpha_0=\alpha(W)|_{q^2\to0}.\tag{21}$$

In the above, $m_{Y_0^*}$ and Γ are the mass and width of Y_0^*. For the slope of Y_0^* trajectory we take the usual estimate, i.e., the same as that for the nucleon, $\epsilon\simeq0.4$. We have also written (21) such that the imaginary part of $\alpha(W)$ is nonvanishing above the $(\Sigma\pi)$ threshold. This is the most natural thing to do, as the T matrix develops nonzero imaginary part above the $(\Sigma\pi)$ threshold. Incidentally, the form (20)–(21) for the trajectory together with (17) immediately show that the $\bar{K}N$ scattering length (which is nothing but T_{11} at the $\bar{K}N$ threshold) is complex. Equations (17)–(21) determine the T matrix up to the two unknown constants. Following the discussion in Ref. 3, the constant C_2 may be determined, using unitarity, in terms of ϵ and Γ. This procedure yields:

$$C_2\simeq-\epsilon\Gamma/4q_r\simeq-0.03,\tag{22}$$

q_r being the value of q at the Y_0^* resonance. The constant C_1 remains still undetermined. Studying the behavior of T_{11} near the location of Y_0^* resonance is not of much use, as there is little hope of being able to independently estimate the $(Y_0^*\bar{K}N)$ coupling constant. In this circumstance it is most convenient to supply one additional piece of information, namely, the experimental cross section for the process $\bar{K}+N\to\Sigma^0+\pi^0$ at a given energy. For this we take the result of Humphrey[12] that $\sigma(\Sigma^0)$ is 8.6 mb a laboratory kinetic energy of 35.6 MeV of the incident K meson. This then determines C_1 through (4) and (19). C_1 turns out to be $C_1\simeq-0.05$. The T matrix is now completely determined. The zero-energy scattering length defined as

$$k\cot\delta|_{k\to0}=1/A;\quad T_{11}(W)|_{k\to0}=A\tag{23}$$

may now be easily determined. The result is in units of Fermi

$$A=-0.61+0.71i,\tag{24}$$

which is in good agreement with the solution II of Humphrey and Ross.[8] It may be noted that the real part of our scattering length is somewhat smaller (and the imaginary part larger) than that of Fujii,[13] who has recently discussed the relation between $\bar{K}N$ interaction and the Y_0^* resonance, by treating the latter as a conventional particle. Actually our solution comes closer to that of Humphrey and Ross. We may therefore conclude that treating the Y_0^* as a Regge pole, rather than as a conventional particle, results in an improvement of the agreement between theoretical and experimental scattering length. This enhances the argument in favor of identifying the suggested S-wave $\bar{K}N$ (virtual) bound state with the observed Y_0^*. We should add, however,

TABLE I. Comparison of calculated scattering length with Humphrey-Ross solution II.

Solution	ReA (f)	ImA (f)
Humphrey-Ross I	-0.22 ± 1.07	2.74 ± 0.31
Humphrey-Ross II	-0.59 ± 0.46	0.96 ± 0.17
Fujii A	-0.92	0.43
Fujii B	-0.98	0.40
Present work	-0.61	0.71

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that this conclusion should be accepted with some caution in view of the large uncertainty which is still associated with the Humphrey-Ross solution. These points are summarized in Table I.

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Note on the Electromagnetic Current in SU_3 Symmetry

M. NAUENBERG*
Columbia University, New York, New York
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The possible existence of strongly interacting particles which behave like triplets under the SU_3 symmetry group adds a new contribution to the electromagnetic current, part of which transforms like a scalar under SU_3. We examine some consequences for the electromagnetic properties of baryons and mesons, considered as composite states of these triplets.

IN the SU_3 scheme with baryons and mesons assigned to the octet representation[1] (eightfold way), the electromagnetic current generated by these particles transforms like the Gell-Mann–Nishijima combination of generators $T_3+\frac{1}{2}Y$, where T_3 and Y correspond to the third component of isospin and the hypercharge, respectively. Neglecting the SU_3 symmetry breaking interaction, this transformation property leads to a number of relations among matrix elements of the electromagnetic current,[2] some of which may be compared directly with experiment.[3] However, the conjectured existence of particles which belong to the triplet representation of SU_3[4–6] will add in general a scalar contribution to the electromagnetic current, which can alter some of these relations.[7] Let us denote by $\psi=(\psi_0,\psi_1,\psi_2)$ the three component field associated with, say, a fermion triplet having the charge structure $(q, q+1, q)$ in units of e, where ψ_0 and $(\psi_1\psi_2)$ transform respectively like $I=0$ and $I=\frac{1}{2}$ states under isospin rotation. Its electromagnetic current can be written in the form

$$\bar{\psi}[(q+\tfrac{1}{3})1+T_3+\tfrac{1}{2}Y]\gamma_\mu\psi\,, \tag{1}$$

where the first term transforms like a scalar under SU_3; note that it vanishes in the case $q=-\frac{1}{3}$ only.[4]

If the triplets are regarded as fundamental,[4–6] we expect that their charge structure determines the electromagnetic properties of the observed baryons and mesons. The derivation of these properties is a dynamical problem, and we face the usual complication of not being able to compute reliably the effects of strong interactions. Furthermore, even those relations among electromagnetic current matrix elements obtained on the basis of SU_3 symmetry alone[2] may be violated, because of the existence of a symmetry breaking interaction. With this forewarning, we present here the results of some very simple calculations based on a model of baryons and mesons as bound states of triplets. Actually, the relations that are obtained are valid quite independently of the model, in the limit of exact SU_3 symmetry. To this extent, the model is just a useful mathematical tool to derive consequences of the symmetry of the interaction. On the other hand, if the dynamical approximations are taken seriously, more detailed results emerge. One interesting possibility would be the determination of the charges of the triplets.

In the model,[6] the baryons are an octet bound state $(\bar{\alpha}\beta)$ of a fermion triplet $(\alpha_0\alpha_1\alpha_2)$, and the antiparticles

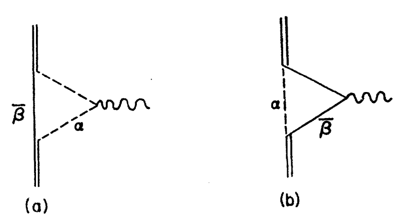

FIG. 1. A baryon $(\alpha\bar{\beta})$ bound state (double line) interacting with a photon (wavy line) through the intermediate α triplet (dashed line) in diagram (a) and $\bar{\beta}$ triplet (heavy line) in diagram (b).

* J. S. Guggenheim Fellow.

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