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Dynamics of Ω^-

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A dynamical calculation is performed for the Ω^- in the $\Xi\bar{K}\to\Xi\bar{K}$ amplitude, using the N/D method. The forces considered are due to the Σ exchange in the crossed channel, as well as the others arising from the far left-hand cut. The latter ones are dealt with in the manner of Balázs. It is seen that the Ω^- can be generated self-consistently with a mass equal to about 1540 MeV.

A RESONANCE has recently been reported[1] in the $\Xi\bar{K}$ system, in the isotopic spin $I=0$ state. From a theoretical point of view, this discovery is extremely important, for it has long been a specific prediction of the Gell-Mann–Ne'eman version[2] of the SU_3-symmetry scheme. It forms the isoscalar member of the well-known group of $P_{3/2}$ baryon-meson resonances forming the tenfold representation of the symmetry group. The oldest among these resonances, namely, the N^*, has been the subject of investigation of various workers. One of the earliest approaches was the Chew-Low static model,[3] which has since been given a better basis in the relativistic dispersion-theoretic treatment of Frautschi and Walecka.[4] The essential result of these calculations is that the simple nucleon-exchange Born diagram can reproduce the general features of the $P_{3/2}$ pion-nucleon resonance. This idea has since been extended by Martin and Wali[5] to cover all the ten resonances as being generated by baryon-exchange forces in the relevant channels. These authors take into account all two-particle baryon-pseudoscalar meson channels, and relate the coupling constants entering the various Born diagrams through SU_3 symmetry to the pion-nucleon coupling constant, in terms of the so-called mixing parameter f. They find that, for reasonable values of this parameter, the tenfold resonances can be approximately understood. On the other hand, there have been two calculations, by Singh and Udgaonkar[6] on N^*, and by Pati[7] on Ξ^*, who have, in addition to baryon-exchange graphs, also considered the distant part of the left-hand cut by the well-known Balázs technique.[8] Their calculations are, however, single-channel ones, and hence their conclusion that the distant part of the left-hand cut has important contributions requires further investigation. A consideration of the Ω^- channel, namely, the $\Xi\bar{K}\to\Xi\bar{K}$ one, will however produce a more clear-cut answer, for there being no other immediate baryon-pseudoscalar meson channel open to it, the multichannel effect is expected to be certainly far less important than in the case of π-N or $\bar{K}$-Λ scattering. In the following, we attempt an understanding of the Ω^- through single-channel Ξ-$\bar{K}$ scattering, taking into account the baryon-exchange diagrams as well as the far left-hand contributions, following the method of Balázs.[8] We find that the far left-hand cut is certainly not unimportant in generating the Ω^-, in agreement with similar conclusions drawn in Refs. 6 and 7.

The kinematics of the problem is identical to that in the case of π-N scattering, which has been dealt with in detail by Frautschi and Walecka.[4] We follow their notation with the pion mass taken as unity. The T matrix may be expressed as

$$T=-A(s,t)+\tfrac{1}{2}(\boldsymbol{q}_1+\boldsymbol{q}_2)B(s,t). \qquad (1)$$

The partial-wave amplitude in the $P_{3/2}$ state is chosen to be

$$\begin{aligned} g_{1+}&=(W^2/q^3)e^{i\delta_{1+}}\sin\delta_{1+}\\ &=\tfrac{1}{32}\pi q^2[\{(W+M_\Xi)^2-M_K^2\}\{A_1(s)+(W-M_\Xi)B_1(s)\}\\ &\quad+\{(W-M_\Xi)^2-M_K^2\}\{-A_2(s)+(W+M_\Xi)B_2(s)\}], \end{aligned} \qquad (2)$$

where W is the total c.m. energy and

$$\{A_l,B_l\}=\int_{-1}^{+1}\{A(s,t),B(s,t)\}P_l(z)dz. \qquad (3)$$

This amplitude can be expressed as

$$g_{1+}=ND^{-1}, \qquad (4)$$

where D has the usual unitarity cut and N incorporates all the other singularities of the amplitude. The function N can be conveniently written as

$$N(s)=N_1^{\Sigma,\Lambda}(s)+N_2(s), \qquad (5)$$

[1] V. E. Barnes, P. L. Connolly, D. J. Crennell, B. B. Culwick, W. C. Delaney *et al.*, Phys. Rev. Letters **12**, 204 (1964).
[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).
[4] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).
[5] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).
[6] V. Singh and B. M. Udgaonkar, Phys. Rev. **130**, 1117 (1963).
[7] J. C. Pati, University of Maryland, 1963 (unpublished).
[8] L. A. P. Balázs, Phys. Rev. **128**, 1939 (1962).

with[9]

$$N_1{}^{\Sigma,\Lambda}(s)=\frac{1}{\pi}\int_{L_1}^{L_2}\frac{\mathrm{Im}g_{1+}{}^{\Sigma,\Lambda}(s')D(s')}{s'-s}ds' \qquad (6)$$

and

$$N_2(s)=\frac{1}{\pi}\int_{-\infty}^{0}\frac{\mathrm{Im}g_{1+}(s')D(s')}{s'-s}ds'. \qquad (7)$$

$N_1(s)$ and $N_2(s)$ correspond, respectively, to contributions arising from the short cut due to single-baryon exchange and contributions from the far-left cut. There are two single-baryon exchange terms, corresponding to exchanges of Σ and Λ. The amplitudes arising out of these diagrams are given (in the notation of Martin and Wali) by

$$(\Xi\bar{K},\Xi\bar{K})_0=3(\Xi\bar{K},\Sigma,\Xi\bar{K})-(\Xi\bar{K},\Lambda,\Xi\bar{K}),$$
$$(\Xi\bar{K},\Xi\bar{K})_1=(\Xi\bar{K},\Sigma,\Xi\bar{K})+(\Xi\bar{K},\Lambda,\Xi\bar{K}), \qquad (8)$$

in isospin states 0 and 1, respectively, and involve the $\Xi\Sigma\bar{K}$ and $\Xi\Lambda\bar{K}$ coupling constants. By invoking SU_3 symmetry, these coupling constants can be related to the π-N coupling constant in terms of the so-called mixing parameter f:

$$g_\Sigma{}^2=g_{\pi\text{-}N}{}^2 \qquad (9)$$
$$g_\Lambda{}^2=\tfrac{1}{3}(1-4f)^2g_{\pi\text{-}N}{}^2.$$

Martin and Wali give a value of $f=0.25$ for a good fit to the observed resonances, whereas Cornwall and Singh[10] get a value of $f=0.4$ from experimental photoproduction results. If we take $f=0.25$, then it is clear that the attraction caused by baryon-exchange forces in the $T=1$ channel is $\frac{1}{3}$ times that in the $T=0$ channel. Anticipating our final result that the Ω^- is a rather strongly bound state, one might expect a resonant $T=1$ state of $\Xi\bar{K}$. However, the forces in the $T=1$ channel are rather sensitive to the value of f, unlike the $T=0$ channel. Any calculation in the $T=1$ state must therefore await more precise information on f, and as a matter of fact definite evidences regarding the existence or otherwise of such a resonance may provide good estimates of f. Calculations in the $T=0$ state are, however, sufficiently insensitive to f. We work, therefore, with $f=0.25$, in which case the Λ coupling vanishes. Consequently, we shall consider the Σ-exchange term only, the contribution of which is

$$g_{1+}{}^\Sigma(s)=(3g_\Sigma{}^2/32\pi g^4)$$
$$\times[\{(W+M_\Xi)^2-M_K{}^2\}(W-M_\Sigma)Q_1(a)$$
$$+\{(W-M_\Xi)^2-M_K{}^2\}(W+M_\Sigma)Q_2(a)], \qquad (10)$$

where

$$a=\{2(M_\Xi{}^2+M_K{}^2)-W^2-M_\Sigma{}^2\}/2q^2+1. \qquad (11)$$

The D function can be written as a once-subtracted dispersion relation involving N:

$$D(s)=1-\frac{s-s_0}{\pi}\int_{\mathrm{Th.}}^{\infty}\frac{q'^3/s'}{(s'-s)(s'-s_0)}N(s')ds', \qquad (12)$$

where s_0 is the subtraction point. We shall now sketch the procedure that is adopted for the evaluation of $N(s)$. Part of it that arises due to the exchange of Σ, i.e., $N_1{}^\Sigma(s)$ can be worked out by inserting a suitable expression for $D(s)$ that is expected to be valid in the region of the short cut. The simple and obvious form of this expression is

$$D(s)=1-(s-s_0)/(s_R-s_0), \qquad (13)$$

where s_R is the position of the resonance to be generated. Substituting this and the expression for $g_{1+}{}^\Sigma(s)$, as given by (10), the integral on the right-hand side of (6) can be done for any S, and thus $N_1{}^\Sigma(s)$ is known for that s. This is how it was evaluated at the matching point (see below). However, $N_1{}^\Sigma(s)$ also occurs in the integral for $D(s)$ and it is essential to know it in a form that exhibits its s dependence in a simple and explicit way for the low-energy region of interest, beginning from the threshold. It is found that the two-pole formula

$$N_1{}^\Sigma(s)=b_1/(s-s_1)+b_2/(s-s_2), \qquad (14)$$

where $s=90$, and $s_2=120$, is quite satisfactory for this purpose. Note that $N_1{}^\Sigma(s)$ depends on s_R through Eq. (13).

The remaining part of the N function, namely, $N_2(s)$, that arises due to the far left-hand cut, can be dealt with following the method of Balázs.[8] With the substitution

$$s\to M_\Xi{}^2+M_K{}^2+2M_\Xi W,$$
$$w\to-(1/x), \qquad (15)$$

one can write $N_2(s)$ as

$$N_2(s)=\frac{1}{\pi}\int_0^{x_L}\frac{\phi(x')}{1+x'w}dx', \qquad (16)$$

where

$$\phi(x')=[\mathrm{Im}g_{1+}(s')D(s')w'] \qquad (17)$$

and

$$x_L=[2M_\Xi/(M_\Xi{}^2+M_K{}^2)]. \qquad (18)$$

The kernel $[1+x'w]^{-1}$ can now be approximated by single straight lines for $0\leqslant x'\leqslant x_L$ and for values of w that fall in the region of interest. Thus, the following approximation can be made:

$$\frac{1}{1+x'w}=\frac{1}{x_1-x_2}\left\{\frac{x'-x_2}{1+x_1w}-\frac{x'-x_1}{1+x_2w}\right\}. \qquad (19)$$

Here x_1 and x_2 are the x' coordinates of the two points at which the various straight lines cut the corresponding curves for $[1+x'w]^{-1}$. Using the above approximation

[9] The limits L_1 and L_2 are, in our units, 80.5 and 129.1, respectively.

[10] J. M. Cornwall and V. Singh, Phys. Rev. Letters **10**, 551 (1963).

for the kernel, $N_2(s)$ can now be approximated as

$$N_2(s)=b_3/(s-s_3)+b_4/(s-s_4)\,, \qquad (20)$$

with $s_3=-10.2$ and $s_4=-844.0$ and b_3 and b_4 as two unknown residues. Equations (14) and (20) can now be substituted in the expression for $D(s)$ [i.e., Eq. (12)] to get

$$D(s)=1-\frac{s-s_0}{\pi}\int_{\mathrm{Th.}}^{\infty}\frac{q'^3/s'}{(s'-s)(s'-s_0)}\times\left[\frac{b_1}{s'-s_1}+\frac{b_2}{s'-s_2}+\frac{b_3}{s'-s_3}+\frac{b_4}{s'-s_4}\right]ds'. \qquad (21)$$

b_3 and b_4 will now be determined by matching the N/D amplitude and its derivative, as discussed above, with the corresponding quantities obtained through the fixed-energy dispersion relations

$$\{A(s,t),B(s,t)\}=\text{Pole terms}+\int\frac{\{A_u(u',s),B_u(u',s)\}}{u'-u}du'+\int\frac{\{A_t(t',s),B_t(t',s)\}}{t'-t}dt'. \qquad (22)$$

We shall approximate these dispersion relations by retaining on the right-hand side only the contributions that come from the crossed-channel Σ pole and direct channel Ω^- pole. We thus obtain

$$g_{1+}(s)=g_{1+}{}^{\Sigma}(s)+g_{1+}{}^{\Omega^-}(s)\,. \qquad (23)$$

$g_{1+}{}^{\Sigma}(s)$ is already given by Eq. (10) and

$$g_{1+}{}^{\Omega^-}(s)=-K\left\{\frac{(W+M\Xi)^2-M_K{}^2}{(W_R+M\Xi)^2-M_K{}^2}\right\}\frac{1}{W-W_R}. \qquad (24)$$

Here K is the residue of the pole due to Ω^- bound state and can be interpreted as the square of the $\Xi\Omega^-\bar{K}$ coupling constant. We could also add the contributions due to ρ, w, and ϕ in Eq. (22), but they can be shown to be quite unimportant. As regards ϕ and w, the situation here is similar to that of Pati,[7] who has recently discussed the Ξ^* in a similar spirit. As for ρ, the relatively unimportant role it plays in such calculation has first been noted in the case of N^*.[6]

We can now match the N/D amplitude and its derivative with, respectively, $g_{1+}(s)$ and its derivative as given by Eq. (23). For this we choose the point $s=70.0$, which we also take to be our subtraction point. Once the matching is done, the N/D amplitude is completely determined and we can look for a zero in the D function, corresponding to the Ω^- bound-state pole. The location $(s_R)_{\mathrm{out}}$ and the residue K_{out} of this pole are then given by

$$D[(s_R)_{\mathrm{out}}]=0\,, \qquad (25)$$

$$K_{\mathrm{out}}=-[(N(s)/D'(s))(1/2\sqrt{s})]_{s=(s_R)_{\mathrm{out}}}. \qquad (26)$$

TABLE I. Variation of output mass and coupling constant with input values of coupling constant for $(S_R)_{\mathrm{in}}=120.0$.

$(s_R)_{\mathrm{in}}$	K_{in}	$(s_R)_{\mathrm{out}}$	K_{out}
120.0	2.0	140.0	5.4
120.0	4.0	134.0	5.6
120.0	8.0	121.0	7.5
120.0	15.0	111.0	8.9

TABLE II. Variation of output mass and coupling constant with the input values of coupling constant for an input mass $(s_R)_{\mathrm{in}}=130.0$.

$(s_R)_{\mathrm{in}}$	K_{in}	$(s_R)_{\mathrm{out}}$	K_{out}
130.0	1.0	135.0	6.1
130.0	2.0	130.0	6.5
130.0	5.0	124.0	7.0
130.0	10.0	116.0	8.0

TABLE III. Variation of output mass and coupling constant with input values of coupling constant for an input mass $(s_R)_{\mathrm{in}}=135.0$.

$(s_R)_{\mathrm{in}}$	K_{in}	$(s_R)_{\mathrm{out}}$	K_{out}
135.0	1.0	129.0	6.5
135.0	2.0	126.0	7.0
135.0	5.0	121.0	7.4
135.0	10.0	116.0	8.2

We shall have generated the Ω^- self-consistently in the amplitude $\Xi\bar{K}\to\Xi\bar{K}$ if the input values of K and s_R, used in Eqs. (13) and (24) coincide with the ones obtained through Eqs. (25) and (26). In practice, this was achieved by choosing various sets of input values of s_R and K, and obtaining the output values through Eqs. (25) and (26) in each case. The results are summarized in Tables I–III, which show that the self-consistency occurs at $s_R=121$ and $K=8$. The experimental mass of Ω^- is, in our units, $s_R=144$. It may be noted that this result is in keeping with calculations on N^*, Ξ^*, etc., where the masses obtained are invariably smaller than those obtained by Martin and Wali,[5] implying that the contributions from the far left-hand cut in generating these resonances is, in all cases, quite important. The masses of all these resonances would presumably turn out to be closer to the experimental masses if one could incorporate the inelastic effects in the D function as evidenced by a recent calculation of Balázs on the ρ meson.[11]

As a last point, we would like to comment briefly on the variations that our results show, as the matching point is changed. For variation of the matching point in the short region between the beginning of the short cut and the origin, the change in the above result is within a few percent. However, when matching at

[11] L. A. P. Balázs, Phys. Rev. **132**, 867 (1963).

points to the right of the short cut, some large variation is observed; the Balázs pole residues sometimes show wild variations, even for slight changes in the position of the matching point. This can be traced to the factor $(W-W_R)^{-1}$ in Eq. (24), which changes sign through infinity as W_R crosses the matching point. It therefore seems advisable to match the amplitudes in the other region mentioned. A different matching procedure in this region, namely, matching the amplitude at two points instead of matching the amplitude and its derivative both at one point, showed no appreciable variation of the result.

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Electroproduction of Protons at 1 and 4 BeV*

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Protons of energies of 110–450 MeV have been produced by bombardment of targets of lithium-6, carbon, aluminum, and copper by electrons of 1- and 4-BeV energy. It is shown that at this energy and angle the electromagnetic interaction of the virtual photons with the nucleus is consistent with an interaction with one nucleon followed by a subsequent scattering process. This is in contrast to the interaction at lower energies (~100 MeV) where the interaction takes place by absorption from two nucleons in the nucleus simultaneously (quasideuteron model). Deuterons have also been observed and are believed to be produced by a pickup process.

APPARATUS

During an experiment at the Cambridge Electron Accelerator on electron-proton scattering with 1- and 4-BeV electrons,[1] we have studied the production of protons from lithium, carbon, aluminum, and copper targets. The apparatus is shown in Fig. 1.

A target of height $\frac{1}{16}$ in. is placed 0.8 in. from the equilibrium orbit of the circulating electron beam. Positively charged particles are focused in a simple quadrupole spectrometer onto a scintillation counter system consisting of two thin defining counters and a counter thick enough to stop 150-MeV protons. The first two counters were used to define a particle trajectory which had crossed the focal plane, and lay within a momentum band $\Delta p/p \simeq 5.4\%$.

The pulse height from the thick counter was displayed on a 400-channel pulse-height analyzer, and was used to separate deuterons from protons and pi mesons. At the smaller momenta, protons stopped therein and gave bigger pulses than all other particles. At the other momenta, protons passed through, giving a smaller pulse height, and the largest pulses were given by deuterons stopping. Positrons and pi mesons had a smaller ionization loss than the protons and gave a still smaller pulse height. A pulse-height distribution for $E_i=4$ BeV, $\phi=63.1°$, $p_p=794$ MeV/c is shown in Fig. 2.

Some of the photoprotons lost energy by nuclear absorption before passing through the final counter. These then gave too small a pulse height and were not identified as protons. A nuclear absorption correction

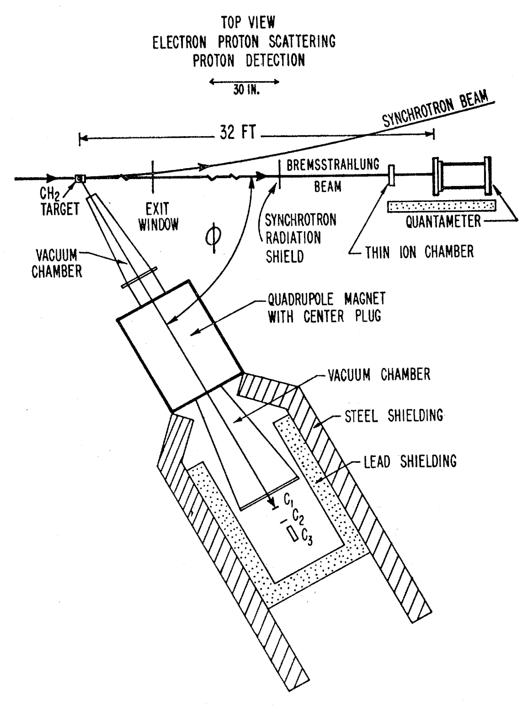

Fig. 1. General layout of the apparatus.

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[1] J. R. Dunning, Jr., K. W. Chen, N. F. Ramsey, J. R. Rees, W. Shlaer, J. K. Walker, and Richard Wilson, Phys. Rev. Letters **10**, 500 (1963).