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Ionized Impurity Scattering in Degenerate Many-Valley Semiconductors*

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A theoretical study is presented of the scattering by ionized impurities of conduction electrons in some III–V compounds. In particular, we consider a degenerate material whose conduction band possesses an absolute minimum at the center Γ of the Brillouin zone having a small effective mass and subsidiary minima near the points L. As the carrier concentration increases, the screening length decreases abruptly when the Fermi level reaches the bottom of the subsidiary minima. This effect is dominant despite the onset of intervalley scattering. Thus, the lifetime of the electrons in the central minimum exhibits a sharp increase when the secondary minima begin to fill up. This result accounts for the increase in amplitude of the Shubnikov-de Haas effect in GaSb with increasing carrier concentration.

I. INTRODUCTION

THE present study was motivated by experimental work on the Hall mobility[1–3] and on the Shubnikov-de Haas[4] effect in Te-doped GaSb. In degenerate samples at low temperatures as the concentration of electrons in the conduction band is increased past a critical value $n_c = 1.25\times10^{18}$ cm^{-3}, the Hall mobility and the amplitude of the Shubnikov-de Haas oscillations exhibit abrupt *increases*. In the present work we shall take the band structure of GaSb to be that described by Becker *et al.*[3] The features of the conduction band which are pertinent to our purposes are schematized in Fig. 1. The absolute minimum occurs at the center of the Brillouin zone and is characterized by an isotropic effective mass $m_0 = 0.052m$. (Here, m is the free-electron mass.) Subsidiary minima are located along the [111] directions at the Brillouin zone boundaries and lie at an energy $E_c = 0.08$ eV above the absolute minimum. These four minima have ellipsoidal effective-mass tensors and a density of states effective mass $m_d = 17.3m_0$. To avoid unnecessary complications, we shall assume an electron in one of these minima to have a spherical effective mass $m_1 = 4^{-2/3}m_d = 6.87m_0$. Because of this large ratio of effective masses, the transport properties are primarily those of the lighter electrons in the central valley.

Scattering of the conduction electrons is, in degenerate samples and at low temperatures, dominated by the ionized impurities. Ionized impurity scattering has been discussed by several authors.[5] It is known that when the carriers are degenerate, the Coulomb interaction $ze^2/\kappa r$ of an electron with an impurity of charge ze embedded in a medium of dielectric constant κ is modified by the dynamical readjustment of the electron distribution and becomes approximately $(ze^2/\kappa r)\exp(-r/\lambda_D)$. Here the Debye screening length λ_D is given by $\lambda_D^{-2} = 4\pi e^2 g(\zeta)/\kappa$, where $g(\zeta)$ is the density of electron states per unit volume and per unit energy evaluated at the Fermi level ζ. We shall show that, when the band structure of Fig. 1 is adequately taken into account in treating this screening, the seemingly anomalous behavior referred to above is found to be, in fact, quite normal. As the carrier concentration is increased through a small range about n_c and the subsidiary valleys are populated, the screening length plummets and, despite the onset of intervalley scatter-

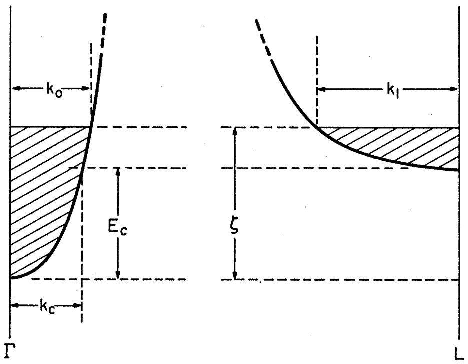

FIG. 1. Conduction band model. The parameter values assumed for GaSb are: $E_c = 0.08$ eV, $n_c = 1.25\times10^{18}$ cm^{-3}, $m_0 = 0.052$ m, $m_1 = 6.87m_0 = 0.36$ m, $(k_L/k_c) = 26.9$.

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[1] A. Sagar, Phys. Rev. **117**, 93 (1960).

[2] A. J. Strauss, Phys. Rev. **121**, 1087 (1961).

[3] W. M. Becker, A. K. Ramdas, and H. Y. Fan, J. Appl. Phys. Suppl. **32**, 2094 (1961).

[4] W. M. Becker and H. Y. Fan, Bull. Am. Phys. Soc. **8**, 246 (1963).

[5] See, e.g., the review article by F. J. Blatt in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1957), Vol. 4, pp. 199–366.

ing, leads to an increase in the lifetime and mobility of the light electrons. The relevant data for GaSb are accounted for.

The immediate theoretical task, to which we address ourselves in this paper, is to treat the scattering by ionized impurities of the light-mass central-valley electrons of the model conduction band of Fig. 1. We forgo a full many-body treatment such as that, for example, of Langer.[6] For simplicity we calculate the lifetime but not the mobility of these electrons, and moreover use the Born approximation. We emphasize that the lifetime we calculate in this paper is not to be identified with the relaxation time that appears in the mobility. The latter must take into account the fact that relaxation of the momentum of the charge carriers is more effectively accomplished by large-angle scattering than by small-angle scattering. However, the collision broadening of the Schubnikov-de Haas oscillations is a function of the lifetime, as discussed in Sec. III. The relaxation time and lifetime are comparable in magnitude and have similar dependence on carrier concentration. The matrix elements of the scattering potential are obtained by using the wave number-dependent dielectric function within the self-consistent field approximation. This is done in Sec. II. A numerical application of the results of Sec. II to the case of GaSb is the object of Sec. III. The first Born approximation is valid here under conditions which differ from those ordinarily encountered in metals. This question will be the subject of a future publication. The theory to be developed is rather general. It is applicable as it stands to several III–V compounds besides GaSb, and additional or different sets of valleys (e.g., along [100] directions) can be included with modest effort. It is also clear that so marked a change in screening with carrier density as we find should affect properties other than those we shall discuss here.

II. THEORY

We wish to study the scattering of electrons by a set of N_i impurity centers per unit volume distributed at random. We shall assume these centers to be ionized donors of charge ze relative to that of the host ions replaced. In the absence of screening by the conduction electrons the potential energy of one electron in the Coulomb field of the donors would be

$$U(\mathbf{r})=\sum_{\nu=1}^{N_i\Omega} V(\mathbf{r}-\mathbf{R}_\nu)=-\sum_{\nu=1}^{N_i\Omega}\frac{ze^2}{\kappa|\mathbf{r}-\mathbf{R}_\nu|}. \qquad (1)$$

We shall refer to $V(\mathbf{r})$ as the bare Coulomb interaction with a donor. In Eq. (1), κ is the dielectric constant of the host crystal, $\mathbf{R}_\nu$ is the position of the νth impurity ion, and Ω is the volume of the crystal. In subsequent development we shall need to know the matrix elements of $V(\mathbf{r})$ for the wave functions which characterize the stationary states of electrons in the conduction band of our model. They will be written

$$\langle j'\mathbf{k}'|V(\mathbf{r})|j\mathbf{k}\rangle=\sum_{\mathbf{q}} V(\mathbf{q})\langle j'\mathbf{k}'|\exp(i\mathbf{q}\cdot\mathbf{r})|j\mathbf{k}\rangle, \qquad (2)$$

where $V(\mathbf{q})$ is the Fourier transform of $V(\mathbf{r})$ defined by

$$V(\mathbf{q})=\frac{1}{\Omega}\int d^3\mathbf{r}\exp(-i\mathbf{q}\cdot\mathbf{r})V(\mathbf{r})=-\frac{4\pi ze^2}{\Omega\kappa q^2}. \qquad (3)$$

The index j is a label for the different valleys; $j=0$ designates a state in the main minimum, and $j=1,2,3,4$ designates states in the different subsidiary minima. If we regard the energy as a multivalued function of $\mathbf{k}$ in the reduced zone, we remark that the subsidiary minima all belong to the same sheet of the energy surface. The results of this paper do not depend on whether the central minimum does or does not lie on the same sheet as do the secondary minima. We shall at first take the matrix elements of $\exp(i\mathbf{q}\cdot\mathbf{r})$ to be

$$\langle j\mathbf{k}'|\exp(i\mathbf{q}\cdot\mathbf{r})|j\mathbf{k}\rangle=\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}, \qquad (4)$$

and

$$\langle j\mathbf{k}'|\exp(i\mathbf{q}\cdot\mathbf{r})|0\mathbf{k}\rangle=\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}M, \qquad (5)$$

with

$$M=\int_{\Omega_0} d\mathbf{r}\,u^*_{j,\mathbf{k}+\mathbf{q}}(\mathbf{r})u_{0\mathbf{k}}(\mathbf{r})\approx\int_{\Omega_0} d\mathbf{r}\,u^*_{j}(\mathbf{r})u_0(\mathbf{r}). \qquad (6)$$

Strictly speaking, the matrix elements in Eqs. (4) and (5) are nonvanishing whenever $\mathbf{k}'-\mathbf{k}-\mathbf{q}$ is either zero or a finite vector of the reciprocal lattice. The modification of our results stemming from the inclusion of Umklapp processes will be made at the end of this section. The region of integration in Eq. (6) is the volume Ω_0 of the primitive cell. We have taken the Bloch state $|j\mathbf{k}\rangle$ to be normalized to unity over the volume of the crystal and its periodic part $u_{j\mathbf{k}}$ to be normalized in the primitive cell. As indicated, the $u_{j\mathbf{k}}(\mathbf{r})$ are to be approximated by the periodic functions at the minima. Another matrix element which arises is that associated with transitions between two different subsidiary minima ($j'\neq j\neq 0$). However, we shall see later that we need not concern ourselves with its value. For the purpose of references, we shall designate all such matrix elements by the symbol M'. We remark that for $j\neq 0$ and $\mathbf{q}$ small there are $\mathbf{k}$ for which the $\mathbf{k}'$ required by Eq. (4) lie outside the first Brillouin zone, and that these would in a strict reduced zone scheme be counted among the U processes. In these transitions, $\mathbf{k}$ and $\mathbf{k}'$ belong to the same subsidiary valley but to halves separated in the reduced zone scheme by a reciprocal lattice vector. We shall here count such as N processes, in effect using a repeated zone scheme[7] and taking care not to double count in sums over valley indices.

Calculations of the energy band structure of GaSb

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[6] J. S. Langer, Phys. Rev. **120**, 714 (1960); **124**, 1003 (1961).

[7] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, New York, 1960), p. 182.

are nonexistent at the present time. Thus we are not in a position to obtain a numerical value for either M or M'. However, band calculations for III–V compounds exhibiting a band structure similar to that of GaSb are available.[8] From the results of this reference we find that for all III–V compounds having subsidiary minima in the [111] direction $|M|^2$ is of the order of 0.5. The corresponding values of $|M'|^2$ are smaller. A summary of estimates is given in Table I.

TABLE I. Matrix elements M for some III–V compounds.[a]

Material	$V(0)$	$\Sigma\, c_j^2$ Γ	$\Sigma\, c_j^2$ L	M	$\lvert M\rvert^2$
BN	-2.87	1.11	1.08	$0.507-0.538i$	0.546
SiC	-2.44	1.11	1.06	$-0.531+0.475i$	0.508
AlP	-2.0	1.19	1.05	$-0.313-0.697i$	0.584

[a] $V(0)$ is the space average of the crystal potential in Ry. The c_j are the coefficients of symmetrized plane waves in conduction band wave functions after orthogonalization to the core states. The values of M listed were computed by retaining only the plane wave part of the conduction wave functions. Since, as indicated by the values of $\Sigma\, c_j^2$, core states contribute only about 10% of the normalization, this is adequate for our purposes. For details, see Ref. 8.

The consequence of the screening provided by the conduction electrons is that the Fourier component of the screened potential $U_s(\mathbf{q})$ for a process that involves a change in the crystal momentum in the amount $\mathbf{q}$ is given by the relation

$$U_s(\mathbf{q})=\frac{U(\mathbf{q})}{\epsilon(\mathbf{q})}=-\frac{4\pi ze^2}{\kappa\Omega q^2\epsilon(\mathbf{q})}\sum_{\nu=1}^{N i\Omega}\exp(-i\mathbf{q}\cdot\mathbf{R}_\nu)\,,\qquad(7)$$

where $\epsilon(\mathbf{q})$ is the static dielectric function.[9] Within the framework of the random-phase approximation

$$\epsilon(\mathbf{q})=1-\frac{4\pi e^2}{\kappa\Omega q^2}\sum_{j\mathbf{k}j'\mathbf{k}'}\frac{f_0(E_{j'\mathbf{k}'})-f_0(E_{j\mathbf{k}})}{E_{j'\mathbf{k}'}-E_{j\mathbf{k}}}\times|\langle j'\mathbf{k}'|\exp(i\mathbf{q}\cdot\mathbf{r})|j\mathbf{k}\rangle|^2.\qquad(8)$$

The function $\epsilon(\mathbf{q})$ can be decomposed into several sums over pairs of states. The first two sums are over pairs in which both members come from the same minimum (i.e., $j'=j$). Each of these terms yields a contribution to $\epsilon(\mathbf{q})$ which is of the form of the dielectric constant for a single spherical band obtained by Lindhard.[10] There are also sums involving transitions between the different subsidiary valleys in the conduction band. Such terms are also of this form, except that they correspond to a change in a crystal momentum equal to the distance in $\mathbf{k}$ space between two subsidiary minima. These contributions are negligible as compared to the previous ones. The remaining sums involve pairs, one member of which is to be found in the main minimum while the other is in one of the subsidiary minima. The details of this calculation are relegated to Appendix A. We find that for intravalley scattering within the main minimum an accurate approximation is

$$\epsilon(\mathbf{q})=1+(4e^2/\pi\kappa\hbar^2q^2)\times\{m_0k_0p(q/2k_0)+4m_1k_1p(q/2k_1)\}\,,\qquad(9)$$

where

$$p(x)\equiv\tfrac{1}{2}+\frac{1-x^2}{4x}\ln\left|\frac{1+x}{1-x}\right|\,,\qquad(10)$$

and, if ζ is the Fermi energy measured from the bottom of the main minimum, k_0 and k_1 are the radii of the pieces of the Fermi surface in the central and subsidiary valleys, respectively. We have

$$k_0=(2m_0\zeta/\hbar^2)^{1/2}\,,\qquad(11a)$$

$$\begin{aligned}k_1&=[2m_1(\zeta-E_c)/\hbar^2]^{1/2}\,, &&\text{if}\quad \zeta>E_c\\ &=0\,, &&\text{if}\quad \zeta\leqslant E_c\,.\end{aligned}\qquad(11b)$$

It is convenient to define a critical wave vector k_c by

$$E_c\equiv(\hbar^2k_c^2/2m_0)\,,\qquad(12)$$

The critical concentration n_c is that for which $\zeta=E_c$. The crystal momentum transfers q for intravalley scattering are of the order of k_c which will in general be very small compared to the distance in reciprocal space $s\equiv|\mathbf{s}_j|$ from the central minimum to the jth subsidiary minimum. For GaSb, $k_c=3.3\times10^6$ cm^{-1}, while $s=8.9\times10^7$ cm^{-1}. For intervalley scattering from the central valley to one of the secondary valleys it is sufficient to take $\epsilon(\mathbf{q})=1$. In fact, for GaSb, $\epsilon(\mathbf{s}_j)$ differs from unity by approximately 3×10^{-5}.

The calculation of the lifetime is straightforward and is outlined in Appendix B. The lifetime $\tau(\mathbf{k})$ of an electron in state $|0,\mathbf{k}\rangle$ at the Fermi surface is given by:

$$\frac{1}{\tau(\mathbf{k})}=\frac{8\pi N_i}{\hbar^3}\left(\frac{ze^2}{\kappa}\right)^2\left\{\frac{m_0}{k_0}\int_0^{2k_0}\frac{q\,dq}{[q^2\epsilon(q)]^2}+\sum_{j=1}^{4}2|M|^2m_1\times\frac{[(m_1/m_0)(k_0^2-k_c^2)]^{1/2}}{[|\mathbf{s}_j-\mathbf{k}|^2-(m_1/m_0)(k_0^2-k_c^2)]^2}\right\}.\qquad(13)$$

To the extent to which we can neglect $k_0\sim0(k_c)$ as compared to s the lifetime becomes isotropic. The integral in Eq. (13) can be evaluated analytically if we adopt the Thomas-Fermi approximation to $\epsilon(q)$, which consists of replacing $p(x)$ by unity. In this approximation the Thomas-Fermi screening constant $k_s=(1/\lambda_D)$ is given by

$$k_s^2=(4e^2/\pi\kappa\hbar^2)(m_0k_0+4m_1k_1)\,.\qquad(14)$$

In Fig. 2 we exhibit a plot of k_s as a function of the carrier concentration n for the mass ratio $(m_1/m_0)=6.87$. This plot is independent of the values of E_c and n_c. We note that the *rate* of increase of k_s with n is infinite at $n=n_c$ and from Eq. (14) that the larger the

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[8] F. Bassani and M. Yoshimine, Phys. Rev. **130**, 20 (1963).

[9] D. Pines, *The Many-Body Problem* (W. A. Benjamin, Inc., New York, 1961).

[10] J. Lindhard, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **8**, (1954).

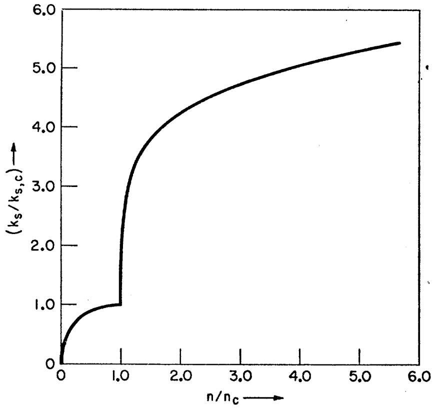

FIG. 2. Fermi-Thomas screening vector k_s divided by its value $k_{s,c}$ at the critical density n_c, calculated from Eq. (14) for the mass ratio $(m_1/m_0)=6.87$.

number of secondary minima and the mass ratio, the larger is the total change in k_s.

Making the Fermi-Thomas approximation and neglecting (k_0/s) we have

$$\frac{1}{\tau(k_0)}=\frac{1}{\tau_0(k_0)}\left\{1+4|M|^2\left(\frac{k_s}{s}\right)^4\right.$$

$$\left.\times\left[1+4\left(\frac{k_0}{k_s}\right)^2\right]\left(\frac{m_1}{m_0}\right)^{3/2}\left(1-\frac{k_c^2}{k_0^2}\right)^{1/2}\right\}, \quad (15)$$

where

$$\frac{1}{\tau_0(k_0)}\equiv\frac{16\pi N_i}{\hbar^3}\left(\frac{ze^2}{\kappa}\right)^2\frac{m_0k_0}{k_s^2(k_s^2+4k_0^2)}. \quad (16)$$

$\tau_0(k_0)$ is the lifetime which would obtain if intervalley scattering to the subsidiary minima were neglected.[11]

In treating the U processes it is convenient to regard the central and subsidiary minima as belonging to the same sheet of the reduced zone energy surface, as is the result of band calculations, and to suppress valley indices. We then have

$$\langle\mathbf{k}'|\exp(i\mathbf{q}\cdot\mathbf{r})|\mathbf{k}\rangle=\delta_{\mathbf{k}'+\mathbf{h},\mathbf{k}+\mathbf{q}}\int_{\Omega_0}d^3\mathbf{r}$$

$$\times\exp(i\mathbf{h}\cdot\mathbf{r})u_{\mathbf{k}'}^*(\mathbf{r})u_{\mathbf{k}}(\mathbf{r}), \quad (17)$$

where $\mathbf{h}$ is any reciprocal lattice vector. It is also useful to note that the periodicity in $\mathbf{k}$ space of the Bloch functions[12] implies

$$u_{\mathbf{k}+\mathbf{h}}(\mathbf{r})=\exp(-i\mathbf{h}\cdot\mathbf{r})u_{\mathbf{k}}(\mathbf{r}). \quad (18)$$

All appearances of U processes in Eq. (8) are in matrix elements associated with large crystal momentum transfers $\mathbf{q}$. Since these make negligible contributions to the screening, the dielectric function $\epsilon(\mathbf{q})$ is, for the purposes of this paper, quite unaffected by U processes.

We now take up the matrix elements [Eq. (2)] of the bare interaction for initial states $\mathbf{k}$ lying in the central valley. For intravalley transitions the U processes may be ignored because the integral in Eq. (17) is (0.1) or less and $V(q)/\epsilon(q)$ decreases with increasing q. For an intervalley transition to $\mathbf{k}'$ near $\mathbf{s}_j$ we must include in the sum over $\mathbf{q}$ of Eq. (2) both $\mathbf{k}'-\mathbf{k}\approx\mathbf{s}_j$ and the Umklapp $\mathbf{k}'-\mathbf{k}-2\mathbf{s}_j\approx-\mathbf{s}_j$. Since k_c is small compared to s, we have

$$V(\mathbf{k}'-\mathbf{k})\approx V(\mathbf{k}'-\mathbf{k}-2\mathbf{s}_j)\approx V(s).$$

The matrix element of $\exp(i\mathbf{q}\cdot\mathbf{r})$ for the direct transition is just Eq. (5). In the integral of Eq. (17) for the U processes we use Eq. (18) and the fact that, since the Bloch function is real at a minimum, $u_{-\mathbf{s}_j}^*(\mathbf{r})=u_{\mathbf{s}_j}(\mathbf{r})$. We then have

$$\int_{\Omega_0}d^3\mathbf{r}\,\exp(-2i\mathbf{s}_j\cdot\mathbf{r})u^*_{\mathbf{k}'}(\mathbf{r})u_{\mathbf{k}}(\mathbf{r})=\int_{\Omega_0}d^3\mathbf{r}u^*_{\mathbf{k}'-2\mathbf{s}_j}(\mathbf{r})u_{\mathbf{k}}(\mathbf{r})$$

$$\approx\int_{\Omega_0}d^3\mathbf{r}u^*_{-\mathbf{s}_j}(\mathbf{r})u_0(\mathbf{r})=M^*. \quad (19)$$

Accordingly, neglecting k_c as compared to s, the matrix element Eq. (2) for intervalley scattering is

$$\langle j,\mathbf{k}'|V|0,\mathbf{k}\rangle=(M+M^*)V(s)=2(\mathrm{Re}M)V(s). \quad (20)$$

Consequently, Eq. (13) and Eq. (15) are to be replaced by

$$\frac{1}{\tau(\mathbf{k})}=\frac{8\pi N_i}{\hbar^3}\left(\frac{ze^2}{\kappa}\right)^2\left\{\frac{m_0}{k_0}\int_0^{2k_0}\frac{qdq}{[q^2\epsilon(q)]^2}+\sum_{j=1}^{4}8(\mathrm{Re}M)^2m_1\right.$$

$$\left.\times\frac{[(m_1/m_0)(k_0^2-k_c^2)]^{1/2}}{[|\mathbf{s}_j-\mathbf{k}|^2-(m_1/m_0)(k_0^2-k_c^2)]^2}\right\}, \quad (21)$$

and

$$\frac{1}{\tau(k_0)}=\frac{1}{\tau_0(k_0)}\left\{1+16(\mathrm{Re}M)^2\left(\frac{k_s}{s}\right)^4\right.$$

$$\left.\times\left[1+4\left(\frac{k_0}{k_s}\right)^2\right]\left(\frac{m_1}{m_0}\right)^{3/2}\left[1-\left(\frac{k_c}{k_0}\right)^2\right]^{1/2}\right\}, \quad (22)$$

respectively. Reference to Table I shows that for the III–V compounds inclusion of U processes increases the intervalley scattering rate to about twice that due to

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[11] In the same approximation, the mobility is readily found to be

$$\frac{1}{\mu}=\frac{2\pi N_im_0^2}{\hbar^3k_0^3|e|}\left(\frac{ze^2}{\kappa}\right)^2\{\ln(1+y^{-1})-(1+y)^{-1}\},$$

where $y\equiv(k_s/2k_0)^2$. See, e.g., N. F. Mott and H. Jones, *The Theory of the Properties of Metals and Alloys* (Clarendon Press, Oxford, 1936), p. 294. An exact calculation shows that the behavior of the mobility as a function of carrier concentration is qualitatively the same as for the lifetime. However, for the parameters appropriate to GaSb (see Fig. 1) the mobility increases by a factor of 2 in the range $1<(n/n_c)<3$ while the corresponding change in the lifetime is by a factor of nearly 10.

[12] G. H. Wannier, *Elements of Solid State Theory* (Cambridge University Press, New York, 1959), *pp*, 144–148.

N processes alone. In striking contrast, the intervalley scattering rate for elemental group IV semiconductors becomes exactly zero. This happens by reason of symmetry since in the diamond lattice the relevant conduction band states, at Γ and L, have opposite parity under inversion and the impurity scattering potential used here is spherically symmetric.

III. NUMERICAL RESULTS FOR GaSb

In this section we present the results of numerical calculations for GaSb. In Fig. 3 we show the scattering rate divided by its value $(1/\tau_c)$ at the critical density as a function of carrier density. Values were obtained from Eq. (21), taking $z=+1$, $\mathrm{Re}M=0.5$, neglecting (k_0/s) as compared to unity, and choosing $\kappa=15.9$ for computational convenience since an accurate value is not known. The contribution of intervalley scattering to the transition rate increases rapidly with increasing carrier concentration. However, even at the greatest concentrations considered (namely, $n=5.66n_c$) the intervalley contribution to the scattering rate is less than 1% of the total. This fact is essential for the validity of our simple treatment, since otherwise we should have to take explicit account of virtual transitions within the subsidiary valleys and it is doubtful that these can be treated adequately by the Born approximation. As the carrier concentration is increased from n_c to $1.42n_c$, the lifetime for electrons in the central valley increases by a factor of about 6. The amplitude A of the Shubnikov-de Haas oscillations is proportional to $\exp(-2\pi/\omega_c\tau)$, where ω_c is the cyclotron frequency of the electrons in the central valley. This damping of the oscillations can be interpreted as arising from broadening of the Landau levels by collisions, and hence the $(1/\tau)$ appropriate here is the total rate of transition out of a state, without regard to momentum transfer. We have implicitly assumed that it is sufficient to calculate this rate in zero magnetic field. If ionized impurities were to provide the sole scattering mechanism, then, at a magnetic field of 3×10^4 G, A would increase by a factor of 113 as n increases from n_c to $1.42n_c$. However, this factor would be reduced if not all the Te in solution gives rise to ionized donor centers. Suppose that the fraction α of all Te atoms in solution which is ionized and the total cross section σ_n for scattering by a neutral impurity were independent of concentration. Then the ratio $[A(n)/A(n_c)]$ of amplitudes at carrier concentrations n and n_c would be

$$\frac{A(n)}{A(n_c)}=\exp\left[\frac{2\pi}{\omega_c\tau_c}\left\{1-\frac{\tau_c}{\tau(k_0)}\right\}\right]\times\exp\left[-\frac{2\pi(1-\alpha)\hbar\sigma_n}{\omega_c\alpha m_0}(k_0n-k_cn_c)\right].$$

The first exponential arises from ionized impurity scattering and increases with n, while the second exponential is for $n>n_c$ always less than unity and decreases with increasing n. From this we see that examination of $A(n)$ as a function of n at constant magnetic field and temperature can, in principle, yield information on the neutral impurity scattering and degree of ionization of elements such as Te and Se in GaSb. It does not seem possible to pursue this point theoretically at present.

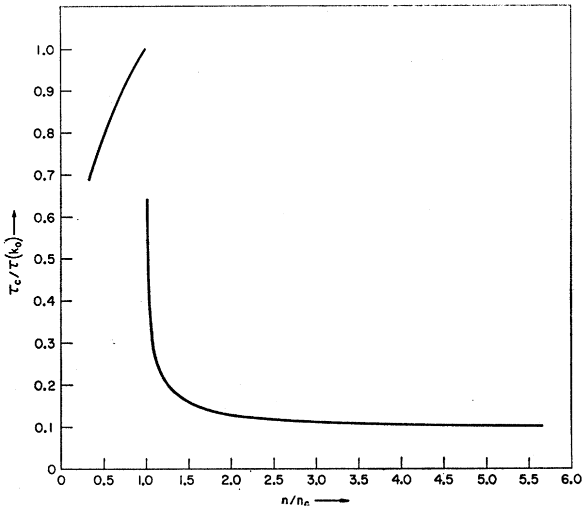

FIG. 3. Ionized impurity scattering rate $[1/\tau(k_0)]$ divided by its value $(1/\tau_c)$ at the critical density.

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APPENDIX A

The dielectric function (8) can be calculated in a straightforward but tedious way. Let us define

$$S_{jj'}(\mathbf{q})=\sum_{\mathbf{kk}'}\frac{f_0(E_{j'\mathbf{k}'})-f_0(E_{j\mathbf{k}})}{E_{j'\mathbf{k}'}-E_{j\mathbf{k}}}\times|\langle j'\mathbf{k}'|\exp(i\mathbf{q}\cdot\mathbf{r})|j\mathbf{k}\rangle|^2. \quad \text{(A1)}$$

Then,

$$\epsilon(\mathbf{q})=1-\frac{4\pi e^2}{\kappa q^2}\left[S_{00}(\mathbf{q})+\sum_{j=1}^{4}S_{jj}(\mathbf{q})+|M|^2\sum_{j=1}^{4}S_{0j}(\mathbf{q})+|M'|^2\sum_{j\neq j'\neq 0}S_{jj'}(\mathbf{q})\right]. \quad \text{(A2)}$$

One can easily show that

$$S_{00}(\mathbf{q})=-\frac{m_0k_0}{\pi^2\hbar^2}p\left(\frac{q}{2k_0}\right), \tag{A3}$$

$$S_{jj}(\mathbf{q})=-\frac{m_1k_1}{\pi^2\hbar^2}p\left(\frac{q}{2k_1}\right),\quad j=1,2,3,4, \tag{A4}$$

and

$$S_{jj'}(\mathbf{q})=-\frac{m_1k_1}{\pi^2\hbar^2}p(|\mathbf{q}+\mathbf{s}_j-\mathbf{s}_{j'}|/2k_1), \tag{A5}$$

with $j\neq j'\neq 0$. The quantity $S_{0j}(\mathbf{q})$ presents some difficulty because of the presence in the denominators of the difference of the kinetic energies of two electron states possessing different effective masses. We obtain the following result:

$$\begin{aligned}S_{0j}(\mathbf{q})=&-\frac{m_0k_1}{\pi^2\hbar^2q_j}\\&\{K_+^{(j)}p(K_+^{(j)}/k_1)+K_-^{(j)}p(|K_-^{(j)}|/k_1)\}\\&+\frac{m_1k_0}{\pi^2\hbar^2q_j}\{k_+^{(j)}p(k_+^{(j)}/k_0)-k_-^{(j)}p(k_-^{(j)}/k_0)\}.\end{aligned} \tag{A6}$$

Here we have used the symbols

$$q_j=|\mathbf{q}-\mathbf{s}_j|, \tag{A7}$$

$$\begin{aligned}K_\pm^{(j)}=q_j[1-(m_0/m_1)]^{-1}\pm\{q_j^2[1-(m_0/m_1)]^{-2}\\-[1-(m_0/m_1)]^{-1}[q_j^2-(2m_0E_c/\hbar^2)]\}^{1/2},\end{aligned} \tag{A8}$$

and

$$\begin{aligned}k_\pm^{(j)}=\pm q_j\left(\frac{m_0}{m_1-m_0}\right)\\+\left\{q_j^2\left(\frac{m_0}{m_1-m_0}\right)^2+\frac{m_0}{m_1-m_0}\left(q_j^2+\frac{2m_1E_c}{\hbar^2}\right)\right\}^{1/2}.\end{aligned} \tag{A9}$$

If $|\mathbf{q}|\ll|\mathbf{s}_j|$, both $k_\pm^{(j)}$ and $K_\pm^{(j)}$ become approximately independent of j. Since for ordinary circumstances $k_\pm^{(j)}$ and $K_\pm^{(j)}$ are much larger than k_0 and k_1 we can neglect the third and fourth terms in the parentheses in Eq. (A2). This yields the approximate result given in Eq. (9) of the text.

The second case of interest is that in which $\mathbf{q}\sim\mathbf{s}_j$ for some j. The term S_{0j} contains large contributions which cancel each other while $S_{0j'}$ (with $j'\neq j$) gives rise to contributions of the order of 3×10^{-5} to the dielectric function. Thus, in this region $\epsilon(\mathbf{q})\approx1$.

APPENDIX B

The calculation of the lifetime is carried out within the framework of the Born approximation. We use the relation

$$\begin{aligned}\frac{1}{\tau(\mathbf{k})}=\sum_{\mathbf{q}j}(2\pi/\hbar)|\langle j,\mathbf{k}+\mathbf{q}|U_s(\mathbf{r})|0,\mathbf{k}\rangle|^2\\\times\delta[E_{j,\mathbf{k}+\mathbf{q}}-E_{0,\mathbf{k}}].\end{aligned} \tag{B1}$$

The matrix elements involved are replaced by the approximate expressions in Eqs. (4) and (5) of the text. The sum over j extends over all the valleys including the central one. One can then separate the contribution into two sums. The first takes into account of intravalley scattering within the central minimum and the second considers the intervalley scattering from the central valley to the subsidiary minima. The intravalley scattering contribution to $\tau^{-1}(\mathbf{k})$ is

$$\frac{1}{\tau'(k)}=\frac{8\pi N_im_0}{\hbar^3k}\left(\frac{ze^2}{\kappa}\right)^2\int_0^{2k}\frac{qdq}{[q^2\epsilon(q)]^2}. \tag{B2}$$

The contribution arising from intervalley scattering is

$$\frac{1}{\tau''(\mathbf{k})}=\sum_j\frac{8\pi N_i}{\hbar^3}\left(\frac{ze^2}{\kappa}\right)^2|M|^2\frac{m_1}{|\mathbf{s}_j-\mathbf{k}|}\int_{q-}^{q+}\frac{qdq}{[q^2\epsilon(q)]^2}, \tag{B3}$$

where

$$q_\pm=|\mathbf{s}_j-\mathbf{k}|\pm\left[\frac{m_1}{m_0}(k^2-k_c^2)\right]^{1/2}. \tag{B4}$$

If we now consider that when $|\mathbf{q}|\sim|\mathbf{s}_j|$, $\epsilon(q)\approx1$, we obtain the result given in Eq. (13) of the text.