Ball Lightning*

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A plasmoid model for ball lightning is examined. The usual virial theorem shows that confinement by self-field alone is inconsistent with conservation laws for energy and momentum; a generalization shows that the presence of air pressure removes this inconsistency and gives an upper bound to the stored energy. This upper bound is much less than the energies reported for some occurrences. For permissible energies the kinetic temperature and density of the plasma can be chosen so that it will not be degraded by internal Coulomb collisions or dissipated by cyclotron radiation for some seconds. It is however necessary to insulate the plasma from the air. A self-field that is able to do this will give up the total stored energy to ohmic heat in the air boundary in a much shorter time than is reported. It is concluded that the plasmoid model is impossible and that energy must be supplied to the ball during its existence if the order of magnitude of the reported energies and times are accepted. Therefore a new model is examined. The high dc electric fields associated with lightning storms are invoked as energy source, and an idealized nonlinear conduction problem is shown to admit ball-like solutions. This leads to a ball lightning model of a low-current glow discharge in an atmospheric dc field. A region of higher conductivity results in a local increase of the electric field and current density sufficient to produce a glow discharge, which provides the higher conductivity and is thus self-consistent. If this model is appropriate, then ball lightning has no relevance to controlled-fusion plasma research.

LUMINOUS balls of some 10- to 100-cm diam, hovering or drifting through the air for seconds without contact with other bodies and vanishing either silently or with a bang, have appeared at many times and places according to reports.[1] It is interesting to suppose that these observations are meteorological rather than psychological in nature, and to set ourselves the problem: What physical model is consistent with most of these observations?

Here we first consider the possibility of a plasmoid confined by a combination of self-field and atmospheric pressure. It is well known that self-field alone cannot confine a plasmoid for many sound-transit times; this is a consequence of the virial theorem. With external gas pressure, confinement is no longer impossible, and we will use a generalization of the virial theorem to see how much energy can be stored in a plasmoid. We also ask what kind of plasma can persist for the observed times, in spite of Coulomb scattering and cyclotron radiation. Then we examine the supposed boundary between the plasmoid and the surrounding air. We conclude that the reported energies cannot be internally stored because of the virial theorem, and that even the energies consistent with the virial theorem would be quickly dissipated either in the plasma or its boundary.

This leads us to attribute the reported energies to external sources, the most obvious of which is the earth's dc electric field. Therefore, we introduce a nonlinear conduction model for ball lightning. It is shown that a current-dependent conductivity idealizing the transition from Townsend to glow discharge regimes leads to a nonlinear conduction equation with nonunique solutions to the boundary value problem for stationary current flows. For a certain critical range of applied electric fields there are possible both uniform dark and ball-shaped luminous current patterns. According to this model, ball lightning consists of a limited dc glow discharge surrounded by a Townsend discharge. A rise or fall in the external field causes either catastrophic or quiet disappearance of the ball.

I. THE DATA

There is enough variation in the descriptions of ball lightning to suggest considerable variety in the thing itself: perhaps as much variety as we see in clouds. Most reports give estimates of size and duration, and a sample of these is presented in Figs. 1 and 2. Some reports are quite circumstantial. Thus, there is an engraving by an eyewitness who with Lomonosov was visiting the laboratory of D. Richman when Richman was killed by his lightning apparatus.[2] Again, there exists a most informative sequence of frames from a color motion picture showing lightning repeatedly striking a depth-charge water column and leaving each time a series of balls (bead lightning) in its path.[3]

Most reports link ball lightning with ordinary lightning. They agree that it appears bright even in daylight.

The energy content of ball lightning is crucial for

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[1] For collections of ball lightning testimonials, see: M. Rosenweld, Z. Meterol. **8**, 24 (1954). J. R. McNally, Preliminary Report Ball-Lightning, 2nd Ann. Meeting, Div. Plasma Phys. Am. Phys. Soc., Gatlinburg, Tennessee, November 1960 (unpublished); H. W. Lewis, Sci. Am. **208**, 107 (1963).

[2] P. N. Chivinskiy, Priroda **43**, 27 (1954). P. E. Viemeister, *Lightning Book* (Doubleday & Company, Inc., New York, 1961).

[3] Naval Ordinance Laboratory film strip kindly supplied by E. Dewan. We thank Dr. Dewan for providing us with much literature on the subject and especially for many stimulating conversations.

deciding between models. According to one report,[4] about 15 l of water in a rain barrel were still uncomfortably warm some 20 min after a ball lightning plunged into the barrel. It is difficult to account for this and other reports unless an energy of more than 10^6 J accompanies some ball lightning.

II. MODELS

What is the constitution of a ball lightning? What processes form it, maintain it, and end it?

We can sort the models that have been invented as we would stoves.

For example, we must choose between *chemical* models, gas burners so to speak, and *electrical* models, bearing charges or currents of electricity and containing significant electric or magnetic fields.

Then again, some are *self-powered*, and run on internal resources of energy until the reserves are consumed, while others are *externally-powered*, and draw their energy from external reservoirs until these are depleted or the coupling is disrupted.

Furthermore we must decide whether, in the main, the matter within the ball lightning is *stationary*, *oscillatory*, or *turbulent*; and in the case of electrical models the analogous decision must be made about electromagnetic quantities: dc, ac, or noisy.

One check of a ball lightning model is whether it is possible to produce the model in the laboratory. Another is whether it tells us where to look for ball lightning in nature.

Nauer's ball lightning[5] is an externally-powered, chemical model.

Kapitza's ball lightning[6] is an externally-powered, electrical, stationary, ac design. It has been produced by Richie[7] by focusing microwaves of 3-cm wavelength; for the reported ball lightning sizes, wavelengths of about 1 m would be required. These have not been found to accompany lightning storms in nature in significant quantities.

Hill's ball lightning, like a miniature thundercloud, is an electrical, internally-powered, dc, turbulent model.[8]

Stekol'nikov's ball lightning, a plasmoid, is an electrical, internally-powered, dc, stationary model.[9]

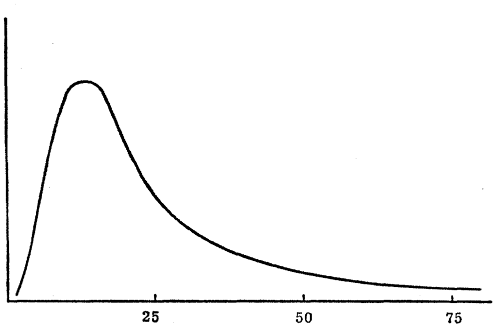

FIG. 1. Frequency distribution of ball lightning diameters by McNally (Ref. 1).

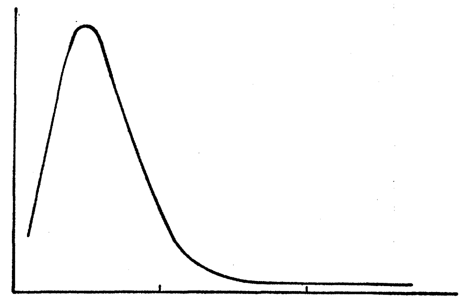

FIG. 2. Frequency distribution for ball lightning lifetimes (Ref. 1).

III. SELF-CONFINED PLASMA MODELS

It is a well-known consequence of the virial theorem that a plasma cannot be confined by its own magnetic field for many particle-transit times, but in the case of ball lightning there is present an external atmospheric pressure. If this sufficed to permit a long-time self-magnetic confinement, there could be very interesting applications of this model. We therefore consider a model for ball lightning made of a plasma core confined by its self-field and atmospheric pressure. We take up first some elementary statements about the plasma temperature, plasma duration, and the plasma-air boundary. For definiteness we suppose a radius of 10 cm.

It would have to be a very special kind of plasma that goes into such ball lightning. Let n be the particle density in plasma, Θ the kinetic temperature, and

$$T = k\Theta/e,$$

the temperature in units of potential. In the n-T plane of Fig. 3, the plasma must lie near the line

$$p \sim neT \sim 1 \text{ atm} \tag{1}$$

and, at any rate, must certainly lie below this line for mechanical equilibrium. We shall treat this in greater detail a little later on, using a virial theorem to show that the total energy density of an air-confined plasmoid cannot exceed a small multiple of the internal energy density of the ambient air. Thus, the total plasma energy is of order $pV \sim 10^3$ J, rather than the MJ sometimes reported. We now wish to estimate the time that even

[4] B. L. Goodlet, J. Inst. Elec. Engs. **81**, 1, 32, 55 (1937).
[5] H. Nauer, Z. Angew. Phys. **5**, 441 (1953).
[6] P. L. Kapitza, Dokl. Akad. Nauk SSSR **101**, 245 (1955).
[7] D. J. Ritchie, J. Inst. Elec. Engs. **9**, 202 (1963).
[8] E. L. Hill, J. Geophys. Res. **65**, 1947 (1960).
[9] I. S. Stekol'nikov, *Fizika Molnii i Grozozash hita* (Moscow-Leningrad, 1943). V. D. Shafranov, Zh. Eksperim. i Teor. Fiz. **36**, 478 (1959) [English transl.: Soviet Phys.—JETP **9**, 333 (1959)].

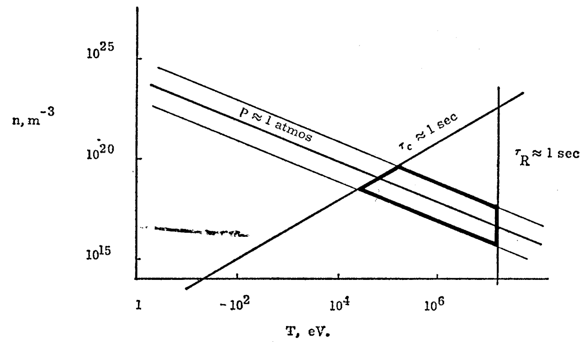

FIG. 3. Bounds of temperature and density for self-confined models, given by the requirement that the Coulomb-collision time (τ_c) and the radiation-losses time (τ_R) ought to be smaller than the observed lifetime (seconds), and that $nkT \sim 1$ atm for mechanical equilibrium. These restrictions reduce the range of variation of T and (n) to within the indicated trapezoid.

this moderate amount of energy can be stored in circulating plasma currents. These currents may dissipate energy through collisions, which are most important at low-electron energies, and through magnetic or cyclotron radiation, which is most important at high-electron energies, and we shall seek a compromise that gives a duration of at least 1 sec.

After a time,

$$\begin{aligned} \tau_c &\lesssim 1/[\pi n v(e^2/2\pi\epsilon_0 v m^2)^2] \\ &\sim 10^{12} T^{3/2}/n, \end{aligned} \tag{2}$$

(MKS units) the plasma electrons will have forgotten their initial conditions because of Coulomb collisions with ions, and so the plasma must lie under the line $\tau_c \sim 1$ sec in Fig. 3. The two conditions (1) and (2) already put the plasma into a semi-infinite band. But, in addition, the plasma electrons cannot be too relativistic or their radiation time

$$\tau_R \sim 0.2 \times 10^{24} R^2/T^3 \tag{3}$$

will be less than 1 sec. Thus, the plasma lies to the left of the line $\tau_R \sim 1$ sec, and therefore in the outlined trapezoid.

The range of particle energies in this trapezoid is from about 10^5 to 10^7 eV. Our exclusion of energies outside this range does not imply that energies within this range are possible, as we have only taken a small number of dissipative processes into account at the present stage of detail of the model. However, the most serious problems turn out to lie in the plasma-air boundary. The upper limit, though relativistic for electrons, is well below estimated cloud potentials[10] of 10^8 V, so the production of this model during a lightning flash-over is not too far-fetched.

However, more vexing than the still somewhat remote question of production is the maintainance of the density difference between the atmosphere ($n \sim 10^{26}$ m^{-3}) and the ball ($n < 10^{20}$ m^{-3}). An interface of contact between the dense atmosphere and the energetic plasma will cool the plasma within several transit times. The model must be provided with what amounts to a vacuum jacket, a region surrounding the plasma from which the external atmosphere is excluded, and which can therefore only be occupied by the self-field of the plasma. Moreover, pursuing the implications of this model still further, the boundary of the air that rests on this field must be ionized and the field must be rapidly changing in time for the penetration of the field by the air to be impeded. Let σ_a be a mean conductivity of the air boundary and let ω measure the time dependence of the magnetic field, whether ac (like the field produced by a system of currents rotating with angular velocity ω) or noisy (system of currents with a power spectrum of turbulence peaked at ω). Since the ionized air would be brightly visible at these high currents, the skin-depth Δr is evidently limited by

$$\Delta r \sim (\mu_0 \sigma_a \omega)^{-1/2} \lesssim 10 \text{ cm},$$

taking the reports into account; the amplitude of the self-field in the boundary region must be such that

$$B^2/2\mu_0 \sim 1 \text{ atm}, \quad B \sim 10^{-2} \text{ Wb/m}^2,$$

and current surface density,

$$i \sim B/\mu_0 \sim 10^4 \text{ A/m}^2,$$

must flow in the air. Thus, the power $\sim i^2 r^2/\sigma \Delta r$ dissipated in the skin consumes the entire energy $\sim B^2 r^3/\mu_0$ on a time-scale of the order

$$\tau \sim (\mu_0 \sigma_a/\omega)^{1/2} r > 1 \text{ sec}, \tag{5}$$

taking the reports into account. Now the air conductivity σ_a is determined by the air temperature T_a which in turn is related by energy conservation to the applied field B and to transport coefficients such as thermal conductivity κ_a of the surrounding air. For slightly ionized air[11] ($T_a < 1$ eV)

$$\kappa_a \sim 10^4 (T_a)^{1/2} \text{ J/m sec eV}, \; T_a \text{ in eV},$$

and the rate of heat flow to the surrounding air is at least

$$W \gtrsim \kappa_a T_a r \sim 10^4 T_a^{3/2} r,$$

actually being greater due to convection. Thus, since we must have $W \times 1 \text{ sec} \lesssim 10^3$ J (the total energy compatible with mechanical equilibrium) the highest air temperature can hardly exceed the estimate

$$T_a \sim (W/10^4 r)^{3/2} \lesssim 1 \text{ eV};$$

[10] B. F. J. Schonland, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 22, p. 356.

[11] A. Bulent Cambel, *Plasma Physics and Magnetofluid Mechanics* (McGraw-Hill Book Company, Inc., New York, 1963), p. 182.

for higher air-boundary temperatures the heat conduction will be disastrous. In this temperature range the electrical conductivity of the air is roughly[12]

$$\sigma_a \sim 0.1 e^{12T_a}\ \mathrm{ohm^{-1}\ m^{-1}}.$$

The expression (4) for the skin depth in air now tells us that

$$\omega \gtrsim 1/(10^2(\mu_0\sigma_a)^{1/2}) \sim 10^{+5}e^{-6T_a}\ \mathrm{sec^{-1}}. \qquad (6)$$

Alas, for the given range of T_a this is absurdly inconsistent with the condition (5) for the power dissipated in the air skin-depth:

$$\omega < 10^{-2}\mu_0\sigma_a \cdot \mathrm{sec^{-1}} \sim 10^{-9}e^{12T_a};$$

when the skin is sufficiently thin, it is too lossy.

This applies to any model in which self-field separates the ball from the surrounding air.

IV. THE VIRIAL THEOREM

In this section we establish a relation between the internal energy and the external pressure for a plasmoid. Similar results have been already obtained,[13] but they neglected either relativistic energies or external gas pressure.

Continuity of energy-momentum for any extended system is expressed by

$$\partial_\mu T^\mu{}_\nu = 0.$$

We fix our attention on a particular space-like hyperplane Σ: $x^0=0$ with normal $n_\mu=(1,0,0,0)$. For any 3-volume V in Σ we take the second moment of the mass distribution in V, the moment of inertia

$$I^{ij}(V) \equiv \int_V dV x^i x^j T^{00}, \quad i,j=1,2,3,$$

and form the trace

$$I(V) \equiv I^{ii}(V) = \int dV r^2 T^{00}.$$

We iterate the continuity equation for energy momentum to show that the energy density and the stress are related by

$$\partial^2 T^{00}/\partial t^2 = \partial^2 T^{ij}/\partial x^i \partial x^j.$$

By direct differentiation and several integrations by parts, it follows from this that for any volume V with boundary surface S,

$$\begin{aligned}\frac{d^2I}{dt^2} &= \int_V dV r^2 \frac{\partial^2 T^{00}}{\partial t^2} = \int_V dV r^2 \frac{\partial^2 T^{ij}}{\partial x^i \partial x^j} \\ &= -\int_S dS_j r^2 \frac{\partial T^{ij}}{\partial x^i} - 2\int dV x^i \frac{\partial T^{ij}}{\partial x^j} \\ &= -\int_S dS_j r^2 \frac{\partial T^{0j}}{\partial x^0} - 2\int dS_j x^i T^{ij} + 2\int_V dV T^{ii}.\end{aligned}$$

Thus,

$$\frac{d^2I}{dt^2} + \frac{d}{dt}\int_S r^2 \mathbf{W}\cdot d\mathbf{S} + 2\int_S dS_j x^i T^{ij} = 2\int dV T^{ii}, \quad (7)$$

where $\mathbf{W}$ is the energy-flux density vector, the Poynting vector for the field plus the contribution of the particles. There are now several circumstances in which the two total time derivatives can be dropped; the system might be stationary, or if it is periodic a time average over a period can be carried out. In these cases there is a simple relation among time averages,

$$\mathrm{Av}\int_V dV T^{ii} = \mathrm{Av}\int_S dS_j x^i T^{ij}, \qquad (8)$$

which we call the virial theorem.

Inspection shows the derivation to be valid for quantum mechanical systems, yielding (7) as an operator relation, involving integrals of field operators.

For example, let V be occupied by a plasma and an electromagnetic field, described by some distribution function N and vector potential A_μ. For each of the particles we have a 4-velocity $u^\mu = dx^\mu/d\tau$, a rest mass m_0, and a contribution to T^{ii} which lies between the kinetic energy and its double.

$$2(\gamma-1)m_0c^2 > \gamma m_0 u^i u^i > (\gamma-1)m_0c^2 = \text{kinetic energy}.$$

Hence, for the particles in V as a whole,

$$T^{ii(p)} = \alpha' K, \quad 1<\alpha'<2,$$

where K designates their total kinetic energy. (For nonrelativistic particles, $\alpha' \to 2$; for ultrarelativistic, $\alpha' \to 1$.) For the field it is well known that

$$T^{ii(f)} = T^{00(f)}.$$

Hence, the sum $E(V)$ of the particle kinetic energy and the field energy for the stationary or periodic plasma within V must obey

$$\mathrm{Av}E(V) = \alpha \mathrm{Av}\int_S dS_j x^i T^{ij}, \quad \tfrac{1}{2}<\alpha<1$$

even for relativistic plasmas. Suppose that a "plasmoid" occupies a volume V bounded by a surface S outside

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[12] A. Bulent Cambel, *Plasma Physics and Magnetofluid Mechanics* (McGraw-Hill Book Company, Inc., New York, 1963), p. 171.

[13] G. Schmidt, Phys. Fluids **3**, 481 (1960). M. N. Rosenbluth and G. W. Stuart, Phys. Fluids **6**, 452 (1963). The proof of the present essentially trivial extension of the published forms of the virial theorem is given only for the sake of completeness.

which the self-fields vanish,

$$A^{\mu}=0 \text{ on } S,$$

and at which the particle stress is a pure pressure, that of the external atmosphere:

$$T^{ij}=p\delta^{ij} \text{ on } S.$$

Then,

$$\int_S dS^i x_i T^{ij}=p\int_S dS_j x^j=3pV.$$

Thus, the virial theorem becomes a strict relation between the stored energy (exclusive of rest mass) and the boundary gas pressure.

$$E(V)=2\alpha pV,\quad \tfrac{1}{2}<\alpha<1. \tag{9}$$

Again, suppose the self-field is permitted to extend beyond the surface S, but all sources of the field are still within the volume V. Can energy be stored in the outer field in excess of the upper bound $3pV$? No: Now the virial theorem shows that

$$E\equiv E(V)+\mathrm{Av}\int_{V'} dVT^{00(f)}<3pV$$
$$+\mathrm{Av}\int_S dS_j x^i T^{ij(f)}+\mathrm{Av}\int_{V'} dVT^{00(f)}$$

(where V' is the complement or outside of V); furthermore,

$$\partial_\mu T^{\mu\nu(f)}=0 \text{ in } V',$$

due to the absence of currents or charges in V', so that applying the virial theorem exclusively to the fields in V' we deduce

$$\mathrm{Av}\int_{V'} dVT^{00(f)}=-\mathrm{Av}\int_S dS_j x^i T^{ij},$$

the minus sign expressing the difference in the direction of the normal to S where it is regarded as boundary of V' rather than V. Thus the greatest stored energy in the plasmoid including self-fields remains

$$E<3pV,$$

where V is only the volume in which the plasma distribution is different from its surrounding atmosphere even if the self-field extends much further.

The kinetic energy, whether ordered or disordered, slow or relativistic, and the electromagnetic energy, whether electrostatic or magnetic, of particles confined by pressure within a volume V reveal themselves by a counter pressure at the boundary of at least $E/3V$, at most $2E/3V$.

If only the collective (Vlasov) electromagnetic field is used in computing the field energy density $T^{00(f)}$, and the strong self-fields near each charged particle are neglected, then $E(V)$ does not include stored chemical energy, which may be estimated separately. At the low density we have found necessary for the long plasma life, the chemical energy is a negligible correction.

V. VIOLATIONS OF THE VIRIAL INEQUALITY

In case the plasmoid is not stationary or periodic the strict virial theorem may not apply and the energy E may exceed $3pV$; but not for long!

Suppose that the second term in (7) can be neglected (e.g., no power input to V) and that the field contribution to the third term, a moment of the stress, can also be neglected. If there exist constant E_0, I_0, τ such that the inequalities

$$E-3pV>E_0>0,$$
$$I<I_0$$

hold for a time τ, then

$$\tau<(2I_0/E_0)^{1/2}.$$

This shows about how long our virial inequality can be violated. For the case of a ball of particles of radius R and high-temperature T, corresponding to thermal velocity v_T, $I_0=MR^2$ is a safe upper bound for I, yielding $\tau\lesssim R/v_T\sim$ transit time for particles or sound waves.

VI. THE NONLINEAR CONDUCTIVITY MODEL

The large energies reported for ball lightning phenomena cannot be stored within the reported ball lightning volumes in the forms of kinetic, thermal or electromagnetic energy. If the reports are correct then it is most likely that the bulk of the energy is externally supplied during the observed phenomena, presumably electrically. Since strong static electric fields exist at the surface of the earth during the large majority of reported observations, which are generally associated with lightning storms, it is natural to look to these fields for the source of the ball lightning energy. Therefore one ought to examine externally-powered dc models for ball lightning.

This completely alters the magnitudes of the fields and flows involved. The only way an external dc electric field can give energy to a volume surrounded by air is through electric currents in the air. Since the surrounding air is not reported to be luminous, these currents must be small, in the Townsend regime. We are led therefore to consider a different ball lightning model: a localized glow discharge completely surrounded by a Townsend discharge.

A violation of the laws of conservation of charge and electric flux might be feared when a volume of higher current and lower electric field is to be surrounded by lower current and higher field. This is not necessarily the case. Let us approximate air as a medium with conductivity $\sigma=\sigma(j)$ depending on the current density j. The continuity of charge, Ohm's law, and the assumption of stationary flow lead to the equations

$$\nabla\cdot\sigma E=0\,,$$
$$\nabla\times E=0\,. \tag{10}$$

Evidently it is convenient to suppose σ a function of the square of the magnitude of the electric field rather than the current density, and to express the electric field in terms of a potential: $\sigma=\sigma(E^2)$, $\neq E=-\nabla\varphi$. Then we have a partial differential equation for φ:

$$\sigma\nabla^2\varphi=\sigma'\partial_i\varphi\partial_j\varphi\partial_i\partial_j\varphi\,, \tag{11}$$

where

$$\sigma=\sigma(E^2)\,,$$
$$\sigma'=\partial\sigma/\partial(E^2)\,.$$

This equation is nonlinear in the extreme, and familiar methods and uniqueness theorems do not seem to hold.

For the case that concerns us σ is a multivalued function of E^2, and we first idealize it by supposing a constant low conductivity σ_0 for the Townsend regime, a constant higher conductivity σ_1 for the glow regime: Fig. 4.

For simplicity suppose the ball is situated in infinite space, with boundary conditions

$$E\to E_0\,,\quad |x|\to\infty$$
$$\varphi\sim E_0\cdot x\,,$$

where E_0 is in the Townsend regime. Obviously one

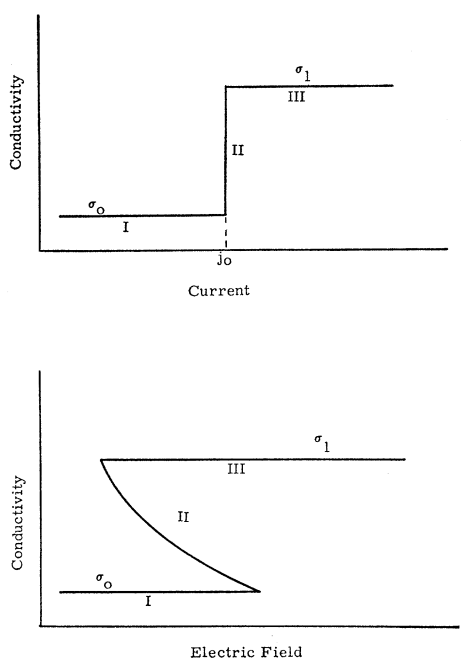

FIG. 4. Idealized plot of conductivity versus current density and electric field. The assumption of this behavior for σ allows us to find an exact nontrivial and physically interesting solution to the conduction-model equations.

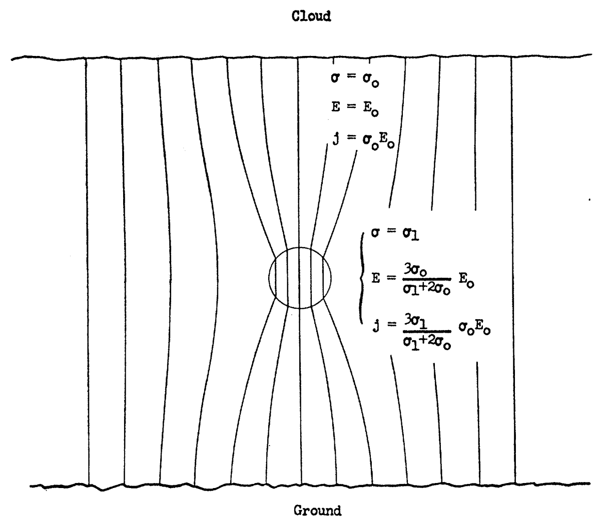

FIG. 5. The nonlinear conductivity model. The lines of current are indicated.

solution of the equation is then a uniform Townsend discharge

$$\varphi\equiv E_0\cdot x\,,\quad \sigma=\sigma_0$$

throughout all space. If the system were linear, this would be the sole solution, but it is a typical pathology for nonlinear equations that they possess more than one solution for given boundary conditions. Here, for example, we have the solution

$$\varphi=E_1\cdot x,\quad \sigma=\sigma_1,\quad r<1\,,$$
$$\varphi=E_0\cdot x+D\cdot\nabla(1/4\pi\epsilon_0 r),\quad \sigma=\sigma_0,\quad r>1\,, \tag{12}$$

consisting of a spherical glow discharge with uniform field surrounded by a Townsend regime in which the field is that of a dipole D parallel to an asymptotic constant field E_0. The dipole D is adjustable so that both the tangential component of E and the normal component of j are continuous at $r=1$.

Thus, a sphere of conducting gas produces a local convergence of electric lines of force and current that maintains it in its conducting state. For the idealized conductivity assumed here, a solution of the nonlinear problem has been constructed from the well-known linear problem of a dielectric sphere in a uniform external field (Fig. 5).

Order of magnitude estimates show that the energy dissipated in the outer Townsend regime could be carried away by even quite gentle convection currents ($\sim$1 m/sec).

To fix the shape and size of ball lightning requires more physics than is contained in the simple laws of continuity and conductivity we have posited. Any sphere, indeed any ellipsoid could be used to construct a solution of this nonlinear conduction problem. Evidently the nonlinear equation (11) possesses scale invariance $x \rightarrow ax$, $E \rightarrow E$, $\sigma \rightarrow \sigma$, $\sigma' \rightarrow a^{-2}\sigma'$. This scale invariance is destroyed when further details of the transports involved are brought in.

Clearly the external field strength E_0 must lie in a rather well-defined range for such solutions. If E_0 is too small, it cannot be raised above the breakdown field by the presence of a more conducting sphere (which even if $\sigma_1/\sigma_0 \rightarrow \infty$ can at most triple the external field, it is well known). If E_0 is too large there will be a breakdown between the distant sources of the field. If the field should fall during the existence of such a ball, it would disappear quietly, because of the small energies involved; if the field should rise, however, the ball may give way to a high-power discharge.

Thus the nonlinear conduction model is helpful in understanding some of the reported circumstances of ball lightning, and in suggesting further theoretical and experimental work.

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Effect of Applied Electric Fields on the Electron Spin Resonance of Fe^{3+} and Mn^{2+} in α-Al_2O_3

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The effect of applied electric fields in broadening the electron spin resonance lines of the isoelectronic ions Fe^{3+} and Mn^{2+} in α-Al_2O_3 has been observed. This is interpreted as an unresolved line "splitting" which is linear in the applied field. The change in the spin Hamiltonian D parameter with applied field parallel to the c axis $(\partial D/\partial E)$ is 1.00 ± 0.09 for Fe^{3+} and 1.03 ± 0.09 for Mn^{2+} in units of 10^{-5} G cm/V. The existence of this effect implies that these ions replace Al^{3+} ions substitutionally in the α-Al_2O_3 lattice. A simplified calculation indicates that ionic displacement plays an important role in producing the effect in these S-state ions and thatthe resultant relative change in the even component of the axial crystalline field appears to be much larger for Mn^{2+} than Fe^{3+}.

AS has been pointed out by Bloembergen,[1] if one places a paramagnetic ion in a crystal site which lacks inversion symmetry, he can, in general, expect to produce a change in the electron spin resonance (ESR) spectrum upon application of an external electric field. In this paper, the broadening of ESR lines due to the isoelectronic pair of ions Fe^{3+} and Mn^{2+} $(3d^5)$ in Al_2O_3 is reported, and the effect is interpreted in terms of an unresolved splitting of the lines.

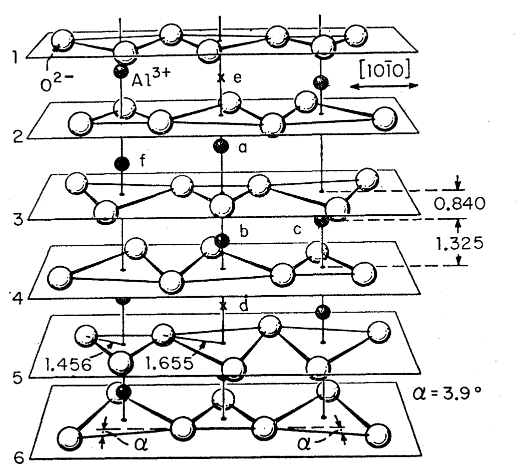

FIG. 1. A portion of the α-Al_2O_3 lattice showing the four types of Al^{3+} sites (after Geschwind and Remeika, Ref. 3).

In the α-Al_2O_3 (corundum) lattice, the Al^{3+} ions are located in sites which lack inversion symmetry as is seen in Fig. 1. This representation of the corundum lattice is taken from an article by Geschwind and Remeika[2] as adapted by Artman and Murphy.[3] The Al^{3+} ions are surrounded by a greatly distorted octahedral distribution of O^{2-} ions such that the point symmetry is reduced to C_3 and thus lacks inversion symmetry. As can be seen, however, sites a and f in Fig. 1 are related to one another by inversion symmetry, as are sites b and c, and all Al^{3+} sites in the lattice are so related in these two types of pairs.

In ruby, the linear electric field pseudosplitting of certain ESR lines arising from Cr^{3+} in such sites has been exhibited and measured.[4,5] In actuality, the lines from one type of site merely shift their position while those from the inversion-related site shift by the same amount in the opposite direction. Since it is possible to substitute other iron group transition metal ions into this lattice, we considered it would be helpful in elucidating the atomic mechanism to examine the effect

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