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Current Fluctuations in a Superconducting Circuit Carrying a Circulating Current

H. J. FINK* AND A. ZACHARIAS
Bell Telephone Laboratories, Murray Hill, New Jersey
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At 4.2°K and 5.514 Gc/sec persistent currents in lead, vanadium, and niobium are free from fluctuations to less than 1.1×10^{-9}, 4.3×10^{-9}, and 8.9×10^{-10} of full shot noise, respectively.

I. INTRODUCTION

SHOT noise (current fluctuations) can be easily measured in vacuum tubes and semiconductors. In metallic conductors shot noise has not yet been observed, possibly because the current densities have to be kept small to avoid heating. In superconductors, however, the current densities can be made very large and it is not obvious, *a priori*, whether a reduced amount of shot noise is or is not present when a superconducting loop carries a persistent current.

It is generally accepted that shot noise in metallic conductors must be very small because the number of free electrons in a metal is fixed and spontaneous deviations from the Fermi-Dirac distrubtion cannot persist longer than over a time interval corresponding to a few collisions, which is of the order of 10^{-13} to 10^{-14} sec. This was verified by the experiments of Bittel and Scheidhauer[1] who found no noise other than thermal noise at frequencies between 45 cps and 11.5 kc/sec when a dc current was passed through a metallic conductor. The dc current densities in the normal metal have to be kept small compared with current densities which can be achieved in superconductors. For example, in the present experiment, the average current density for lead within a distance of the penetration depth from the surface was approximately 10^8 A/cm^2 at 4.2°K.

The theory of superconductivity by Bardeen, Cooper, and Schrieffer[2] (BCS) leaves room for scattering when a persistent current is flowing in a ring. The momentum vector $\mathbf{q}$ is equal to $\mathbf{k}_1+\mathbf{k}_2$ of a virtual pair of electrons with opposite spin when a net current is flowing, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momentum vectors of the individual electrons. According to BCS, "for each value of $\mathbf{q}$ there is a metastable state with a minimum in free energy and a unique current density. Scattering of individual electrons will not change the value of $\mathbf{q}$ common to virtual states and so can only produce fluctuations about the current determined by $\mathbf{q}$." Because scattering is possible, current fluctuations might also be possible.

According to some workers,[3,4] the present theories do not adequately explain the zero-resistance property of ordinary superconductors at finite temperatures. For the case of "hard" superconductors high-field supercurrents near the "critical" values are actually found to decay with a very long time constant.[5] This decay has been explained by Anderson's flux-creep theory.[6] Flux creep is therefore also a possible source of noise, at least for "hard" superconductors.

No theoretical limits of the order of magnitude of the fluctuations of a superconducting current in a ring have been published. The data of Knol and Volger[7] on superconducting NbN at 6 Mc/sec are insufficient to estimate the upper limit of any possible current fluctuations. In the present paper we have investigated supercurrent fluctuations in rings of Pb, Nb, and V at 4.2°K, and have established an upper limit of any possible current fluctuations at 5.514 Gc/sec expressed in reduced shot noise. This noise reduction factor establishes a lower

* Present address: Atomics International Division of North American Aviation, Inc., Canoga Park, California.

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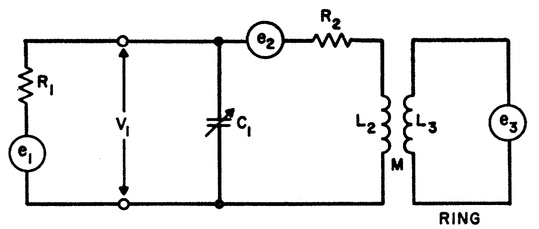

FIG. 1. The equivalent circuit diagram of the input circuit to the radiometer and its noise sources.

limit on the correlation of the current carrying superconducting electrons.

II. EXPERIMENT AND RESULTS

The superconducting rings were machined of commercial Pb, V, and Nb, their impurity content was unknown, and all the rings were of the same size. The cross-sectional radius was 0.089 cm, the average ring radius was 0.262 cm. The lead rings were annealed at room temperature for several weeks and the V and Nb rings were not annealed. It was hoped that defects might increase the noise level. V and Nb are type-II superconductors for which the flux-creep theory[6] is relevant.

A superconducting ring was inserted in a 50-Ω coaxial line which was short circuited at one end. The ring was near the short circuit and the plane of the ring was parallel to the center conductor. After the line was cooled to 4.2°K it was tuned outside the Dewar with a double stub tuner and adjusted for minimum reflected power (about −30 dB) at $f_0=5.514$ Gc/sec, which was the frequency at which the radiometer[8] measured the noise temperature of the equivalent 50-Ω load.

The rings were cooled in the earth magnetic field. An external magnetic field of about 1.4 kG was applied perpendicular to the plane of the rings which was removed after a time interval large compared to the integration time of the radiometer. The measured noise temperatures were recorded continuously. Observations were made of any difference between the output noise intensities when there was no or a small current flowing, and when a large persistent current was flowing. The fluctuations of the magnetic field produced by the persistent current are the basis of observations of the microwave fluctuations of the current. The radiometer was calibrated with a noise lamp, and it was able to detect differences of the noise temperature of the tuned load of smaller than 1°K. The Q of the tuned transmission line loaded by the generator was determined by measuring the frequencies of the half-power points. The frequency bandwidth of the input circuit to the radiometer was large with respect to the bandwidth of the i.f. stage of the radiometer, the latter was about 2 Mc/sec. Figure 1 shows the equivalent circuit diagram of the input circuit to the radiometer. R_1 is the input resistance (50 Ω) of the radiometer which gives rise to a noise voltage $\langle e_1^2\rangle_{av}=4k\gamma T_0R_1B$, where T_0 is room temperature, γT_0 the effective noise temperature of the radiometer, B the bandwidth, and k Boltzmann's constant. C_1 is a variable capacitor and it represents the tuning. $R_2=\sum_i R_{2i}$ is the distributed resistance over the coaxial line which gives rise to a noise voltage $\langle e_2^2\rangle_{av}=4k\sum_i T_iR_{2i}B$, where $\sum_i T_iR_{2i}=\alpha T_0R_2$ by definition. α is a noise temperature reduction factor. L_2 is an effective inductance of the tuned coaxial line which couples to the ring of inductance L_3. L_3 was calculated from the dimension of the ring by assuming that the current is uniform over the whole cross section (for a surface current a small correction has to be made). $\langle e_3^2\rangle_{av}$ is assumed to be $F^2 2eIB(\omega L_3)^2$, where F^2 is the noise reduction factor, I the persistent current, and e the electronic charge. The real part of the impedance Z_3 at ω_0 was neglected because it is very small[9] with respect to the reactive part. Therefore, the Johnson[10] noise due to R_3 was also neglected. When the line is tuned to the frequency ω_0 of the radiometer and R_2 is small, the condition $\omega_0^2C_1L_2(1-K^2)\approx 1$ is satisfied, where K is the coupling constant between the ring of inductance L_3 and the transmission line. It was assumed that the over-all dimensions of the ring are small compared to the wavelength of the radiation. Then K was determined by a separate experiment. The ring was removed from the transmission line and the frequency ω_1, at which the line was matched was measured with exactly the same tuning as above. The resonance frequency ω_1 is determined by the condition $\omega_1^2L_2C_1=1$. From ω_0 and ω_1 the coupling constant K was calculated.

It was found that the additional noise was undetectable when a large persistent current was flowing. This result places an upper limit on the possible noise associated with the superconducting current and this may be expressed in terms of reduced shot noise; $\langle i^2\rangle_{av}=2F^2eIB$, where $\langle i^2\rangle_{av}$ is the mean-squared noise current. One obtains from the analysis on Fig. 1 an expression for the noise reduction factor F^2:

$$F^2=\frac{2k}{e\omega_0L_3Q_0}\frac{1}{K^2}\frac{T_0\beta(\gamma+\alpha)}{I}, \tag{1}$$

where β is a direct observable quantity defined by

$$\beta=\frac{\text{increase in noise temperature due to } I}{\text{noise temperature due to } R_1+R_2}. \tag{2}$$

The quantity $T_0\beta(\gamma+\alpha)$ was found to be about 1°K when the temperature of the load αT_0 was 170°K. The other quantities are: $L_3\approx 6.6\times 10^{-9}H$; $K=0.060$; $Q_0=\omega_0L_2/R_2=$ twice the measured loaded Q when critically coupled to the generator $=790$. The magnitude

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TABLE I. Maximum persistent current I in the Pb, V, and Nb rings; estimated magnetic field H_0 at the center of the rings due to I; and the upper limit of the experimental noise reduction factor F^2.

	I(A)	H_0(G)	F^2
Pb	240	570	1.1×10^{-9}
V	63	150	4.3×10^{-9}
Nb	305	730	8.9×10^{-10}

of the persistent currents was measured in separate experiments. Each of the rings was placed close to the inner wall of the inside of the Dewar. A persistent current was induced by switching on and off the 1.4 kG magnetic field oriented perpendicular to the plane of the ring. The magnetic field due to the current was measured with a Hall probe at a point outside the Dewar along the axis of symmetry of the ring. When the external magnetic field was reversed the magnetic field due to the induced current was also reversed. The magnitude of the measured field was, however, the same regardless of the direction in which the field was applied. From the known distance of the Hall probe from the ring, the dimensions of the ring, and the measured magnetic field the current in the ring was calculated and is shown in Table I. In this calculation it was assumed that the ring can be replaced by a loop of zero cross section and radius 0.262 cm (average radius of ring). The error in the estimated currents is $+0\%$ and -20%. Also shown in Table I are the estimated values of the magnetic field H_0 at the center of the ring from which it may be concluded that in the lead rings a maximum persistent current was established such that the magnetic field due to the persistent current was the critical field on the inner rim of the rings. For the vanadium rings, we measured the resistance along the twofold axis of the rings when a magnetic field was increased as well as decreased parallel and perpendicular to the plane of the rings. A finite resistance appeared when the applied magnetic field was between 150 and 200 G and the measuring current was about 1 mA. The magnetization experiments by DeSorbo[11] on vanadium show that the upper critical field is about 1.55 kG at 2.88°K. Therefore, one may conclude that at 4.2°K, a magnetic field of about 1.4 kG was large enough to quench superconductivity and that in our vanadium rings a maximum persistent current was established when the external field was removed. For niobium, however, the lower and upper critical fields are[12] about 1.4 and 2.6 kG, respectively, and therefore it is uncertain whether or not a maximum persistent current was established.

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Also shown in Table I are the values of F^2 calculated from Eq. (1). In the light of the present experiment the noise reduction factor at 5.514 Gc/sec for the above materials cannot exceed the values for F^2 shown in Table I.

III. CONCLUSIONS

At 5.514 Gc/sec and 4.2°K persistent currents in Pb, V, and Nb are free from fluctuations to less than 1.1×10^{-9}, 4.3×10^{-9}, and 8.9×10^{-10} of full shot noise, respectively. A superconducting current with a limited amount of fluctuations implies a constraint on the occupancies of the superconducting current state by the electrons.

The current $\mathbf{I}$ is:

$$\mathbf{I}=\frac{2e}{l}\sum_{i=1}^{N/2}\mathbf{v}_i=\frac{2e}{m_s l}\sum_{i=1}^{N/2}\mathbf{p}_i=\frac{e\mathbf{P}_s}{lm_s}, \tag{3}$$

where N is the number of superconducting electrons, $\mathbf{v}_i$ and $\mathbf{p}_i$ are the velocity and the momentum of the ith pair of electrons, respectively, e the electronic charge, m_s the mass of the superconducting electron and l the length of the circuit. Because in the present experiment no fluctuations were detected, the total momentum of the superconducting electrons $\mathbf{P}_s$ is a constant for the above superconductors over a time interval of about 1.8×10^{-10} sec to the limit as stated above. These experiments were capable of detecting fluctuations in $\mathbf{p}_i$ only, not in N or $\mathbf{v}_i$ individually. Previous experiments[13] on lead established an upper limit on F^2 of 1.1×10^{-9} of full shot noise at 1.3°K and 2.4 Mc/sec.

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