

## Systematics of Fission Asymmetry with Respect to Nuclear Charge and Neutron Contents\*

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Swiatecki's asymmetry formula was applied to charge and neutron asymmetry in fission. The constants involved were determined from the average proton and neutron contents of fragments from the thermal neutron fission of  $U^{235}$  and  $Pu^{239}$ . The average proton and neutron contents were then deduced from the asymmetry equations for the fragments of the compound nuclei  $Th^{233}$ ,  $U^{234}$ ,  $U^{238}$ ,  $U^{239}$ ,  $Cm^{242}$ , and  $Cf^{252}$ . The relationship between the average charge and mass of the light and heavy fragments leads to two almost parallel straight lines, which almost coincide with the  $Z_p$  versus  $A$  lines for  $U^{235}$  ( $n, F$ ), implying that the relationship between the most probable values of fragment charge and mass is independent of the fissioning nucleus; in other words, this relationship is almost universal. The average charge and mass of the light fragments vary within much wider limits in comparison to the heavy-fragment charge and mass which vary in a much narrower range, signifying that the fission process is more selective with respect to the heavy-fragment composition. A neutron added to the original nucleus goes preferentially (90%) into the heavy fragment and transfers some charge (15% of one charge unit) from the light to the heavy fragment; whereas, a proton added to the original nucleus goes into the light fragment and transfers from the heavy to the light fragment a little more charge (10% of one charge unit) as well as a substantial amount (1.25 units) of neutrons. These effects are perfectly additive with respect to both the number of neutrons and/or protons added (or subtracted), and permit one to predict the average charge and neutron contents of the two fragments for any fissile nuclide, if these quantities are known for any other reference nuclide.

### INTRODUCTION

THE systematics of fission asymmetry with respect to nuclear charge and neutron contents could be studied directly if satisfactory experimental mass yield, charge dispersion, and  $Z_p$  versus mass curves for various nuclei were available. The application of the integral transform procedure, reported previously,<sup>1</sup> would then provide direct results on average charge and neutron contents of the light and heavy fission fragments and from these results systematic effects of the compound nucleus composition on the average composition of the fragments would be inferred. But, unfortunately, at present the needed reliable experimental information is available only for the thermal neutron fission of  $U^{235}$  and, to some extent, for that of  $Pu^{239}$ . This difficulty can, however, be overcome by making use of Swiatecki's asymmetry formula and determining the constants involved by means of the average proton and neutron contents of fragments from the thermal neutron fission of  $U^{235}$  and  $Pu^{239}$ .

Swiatecki<sup>2</sup> has shown that the liquid-drop model predicts for low-energy fission an asymmetry proportional to  $[(Z^2/A)_a - Z^2/A]^{1/2}$ , where  $Z$  and  $A$  stand, respectively, for the charge and mass number of the compound nucleus undergoing fission. The parameter  $Z^2/A$ , which plays an important role in the theory of nuclear fission is a measure for the ratio of Coulombic to surface energy, and  $(Z^2/A)_a$  is a limiting value for this parameter below which the symmetrical saddle-

point shape becomes unstable against asymmetric distortions. Swiatecki has also shown that this expression for the asymmetry holds quite generally and is independent of some hypotheses made usually on the behavior of the nuclear fluid (incompressibility, etc.). He defines the asymmetry with respect to mass as asymmetry =  $(\bar{A}_H - \bar{A}_L)/A$ ,  $\bar{A}_H$  and  $\bar{A}_L$  being, respectively, the average mass of the heavy and that of the light fragment after prompt neutron boil-off. Thus:

$$(\bar{A}_H - \bar{A}_L)/A = c_a [(Z^2/A)_a - Z^2/A]^{1/2}, \quad (1)$$

$c_a$  being a constant of proportionality. The average masses are also related by  $\bar{A}_H + \bar{A}_L = A - \nu$ , where  $\nu$  is the average number of prompt neutrons emitted per fission. A plot of  $(\bar{A}_H - \bar{A}_L)^2/A^2$  versus  $Z^2/A$  gives a straight line for various fissile nuclei, and the constants are found from the slope and intercept, to be:  $c_a = 0.090$  and  $(Z^2/A)_a = 40.2 \pm 0.7$ ; the average value of  $\nu = 2.8$  is used for all nuclei.

### ASSUMPTIONS

(1) In view of the generality of Swiatecki's asymmetry expression we will assume that the proton and neutron asymmetries can be defined in an analogous manner, namely:

$$\begin{aligned} (\bar{Z}_H - \bar{Z}_L)/Z &= c_z [(Z^2/A)_z - Z^2/A]^{1/2}, \\ (\bar{N}_H - \bar{N}_L)/N &= c_n [(Z^2/A)_n - Z^2/A]^{1/2}, \end{aligned} \quad (2)$$

where  $\bar{Z}_H$ ,  $\bar{Z}_L$  and  $\bar{N}_H$ ,  $\bar{N}_L$  are the average proton and neutron contents of the two fragments, while  $(Z^2/A)_z$  and  $(Z^2/A)_n$  are the limiting values of the parameter  $Z^2/A$  for two fictitious nuclides that undergo fission with zero charge and neutron asymmetry, respectively.

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<sup>1</sup> M. Talât-Erben and Binay Güven, *Phys. Rev.* **129**, 1762 (1963).

<sup>2</sup> W. J. Swiatecki, *Phys. Rev.* **100**, 936 (1955); **101**, 651 (1956).

The average quantities are also related by the conservation conditions (which are not linearly independent):

$$\begin{aligned} \bar{Z}_H + \bar{Z}_L &= Z, & \bar{N}_H + \bar{N}_L &= N - \nu = A - Z - \nu, \\ \bar{Z}_H + \bar{N}_H &= \bar{A}_H, & \bar{Z}_L + \bar{N}_L &= \bar{A}_L. \end{aligned} \quad (3)$$

(2) The average proton and neutron contents of the fission fragments cannot be determined directly in low-energy fission. However, as shown in a previous work,<sup>1</sup> they can be calculated from the experimental isobaric yield curve, if the most probable charge  $Z_p$  as a function of  $A$  and the charge dispersion curve are known. This knowledge is available for the thermal neutron fission of  $U^{235}$ . In this case the charge dispersion curve is the Gaussian<sup>3,4</sup>

$$P_A(Z) = (0.9\pi)^{-1/2} \exp[-(Z - Z_p)^2/0.9]. \quad (4)$$

On the other hand, according to the bivariate normal distribution approach<sup>5,6</sup> to the fission process,  $Z_p$  versus  $A$  consists of two parallel straight lines, one for the light group and another for the heavy group of fission products. A least-squares calculation based on Wahl's<sup>4</sup> "empirical"  $Z_p$  values leads to the equations

$$\begin{aligned} Z_p &= 0.424 A - 2.256 & (\text{Light group}), \\ Z_p &= 0.410 A - 2.901 & (\text{Heavy group}). \end{aligned}$$

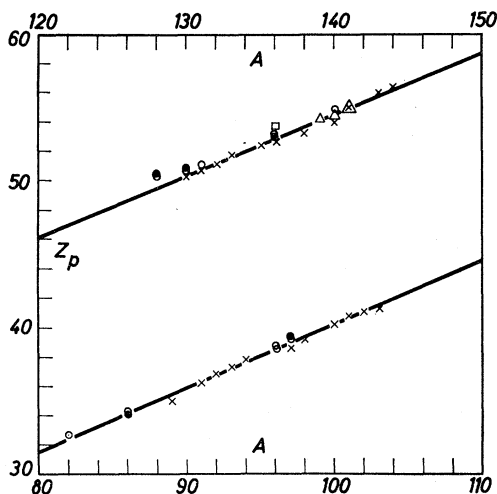


FIG. 1. A plot of the experimental  $Z_p$  values as a function of the mass number  $A$  for different fissile nuclides. The straight lines are those obtained by a least-square computation based on Wahl's "empirical" values for  $U^{235}(n, F)$ . Note that the lines are parallel as the bivariate normal distribution approach to the fission process requires.  $\times$   $U^{235}$ ;  $\circ$   $U^{238}$ ;  $\bullet$   $Pu^{239}$ ;  $\square$   $Cm^{242}$ ;  $\triangle$   $Cf^{252}$ .

<sup>3</sup> I. F. Croall, J. Inorg. Nucl. Chem. **16**, 358 (1961).

<sup>4</sup> A. C. Wahl, R. L. Ferguson, D. R. Nethaway, D. E. Troutner, and K. Wolfsberg, Phys. Rev. **126**, 1112 (1962).

<sup>5</sup> H. B. Levy and D. R. Nethaway, University of California Radiation Laboratory Report No. UCRL-6948, 1962 (unpublished).

<sup>6</sup> M. Talât-Erben, in *Chemistry Research and Chemical Techniques Based on Research Reactors* Tech. Repts. Series No. 17 (International Atomic Energy Agency, Vienna, 1963).

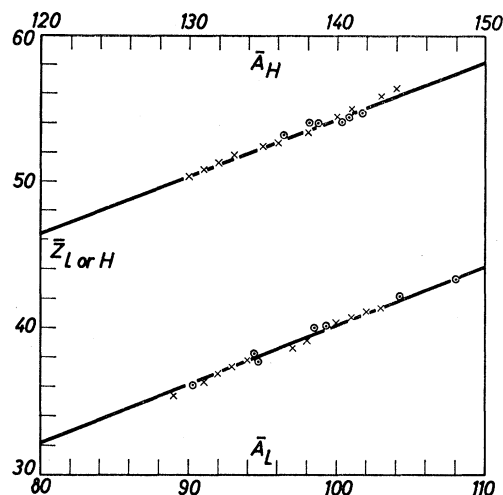


FIG. 2. A comparative plot of the average charge versus mass (circles) and of the most probable charge versus mass number (crosses). The circles correspond to values derived from the charge and neutron asymmetry of fragments from the fission of the compound nuclei  $Th^{233}$ ,  $U^{234}$ ,  $U^{236}$ ,  $U^{238}$ ,  $U^{239}$ ,  $Pu^{240}$ ,  $Cm^{242}$ , and  $Cf^{252}$ ; some points have been omitted for clarity. The crosses are Wahl's "empirical" points for  $U^{235}(n, F)$ . The straight lines are those deduced from the constancy of the ratios  $\bar{Z}_L/\bar{A}_L$  and  $\bar{Z}_H/\bar{A}_H$ ; note that they are almost parallel and practically coincide with the straight lines of Fig. 1, implying that the relationship between the most probable charge and mass is universal. Note also that the circles for the heavy fragments scatter over a much narrower range, indicating that the fission process is more selective with respect to the heavy fragment composition.

We will assume that the same equations are applicable to the thermal neutron fission of  $Pu^{239}$ , a second reference nuclide that will be used in the evaluation of the constants as explained below. The fact that this assumption is not far from reality is illustrated in Fig. 1, where the known  $Z_p$  values for this and other fissile nuclei are plotted against  $A$ . The validity of this assumption is, in addition, justified by the *a posteriori* observation that a plot of the average (the most probable) fragment charge versus the average (the most probable) fragment mass deduced for various fission processes, reproduces almost the same lines of Fig. 1 (See Fig. 2). Some small deviations from the straight lines, which might be due to fine structure effects, should not affect the results since all the discussion presented herein deals with *average* values.

(3) We will assume also that the same charge dispersion curve is applicable to the thermal neutron fission of  $Pu^{239}$ . This assumption also cannot be too far from reality, since Blann<sup>7</sup> found that the charge dispersion of the products from the fission of  $Au^{197}$  with 112-MeV  $C^{12}$  ions, a process radically different from the thermal neutron fission of  $U^{235}$ , follows exactly the same Gaussian.

<sup>7</sup> H. M. Blann, thesis, University of California Radiation Laboratory Report No. UCRL-9190, 1960 (unpublished); Phys. Rev. **123**, 1356 (1961).

**EXPRESSIONS FOR THE AVERAGE PROTON AND NEUTRON CONTENTS OF FISSION FRAGMENTS**

From Eqs. (2) and (3) the following formulas are derived for the average proton and neutron contents of products of a fissile compound nucleus ( $Z, A$ ):

$$\begin{aligned}\bar{Z}_H &= (Z/2)\{1 + c_z[(Z^2/A)_z - Z^2/A]^{1/2}\}, \\ \bar{Z}_L &= (Z/2)\{1 - c_z[(Z^2/A)_z - Z^2/A]^{1/2}\}, \\ \bar{N}_H &= (N/2)\{1 + c_n[(Z^2/A)_n - Z^2/A]^{1/2}\} - (\nu/2), \\ \bar{N}_L &= (N/2)\{1 - c_n[(Z^2/A)_n - Z^2/A]^{1/2}\} - (\nu/2).\end{aligned}\quad (5)$$

**EVALUATION OF THE CONSTANTS**

As it is seen from Eqs. (2), a plot of  $(\bar{Z}_H - \bar{Z}_L)^2/Z^2$  and  $(\bar{N}_H - \bar{N}_L)^2/N^2$  versus  $Z^2/A$  will give two straight lines, and the four constants that appear in (5) can be evaluated from the slopes and intercepts of these lines. Obviously, for obtaining better estimates, as many points as possible should be considered; this requires a knowledge of the average quantities  $\bar{Z}$  and  $\bar{N}$  for as many fissile nuclides as possible. We have calculated the average proton and neutron contents of fragments from various fissile nuclides by using the experimental isobaric yield curves reported in the literature,<sup>8</sup> and applying to them the integral transform method explained elsewhere.<sup>1</sup> The average quantities calculated as weighted means of the charge and neutron contents using the isotopic and isotonic yields as the weights must satisfy the conservation conditions (3). However, we found that, except for  $U^{236}$  and  $Pu^{240}$ , the experimental fission yield curves are inadequate since the average quantities deduced from them do not fulfill the conditions (3). Therefore, the straight lines were drawn through the two most reliable points defined by the data of Table I.

These data give  $Z=92.10$  and  $\nu=2.43$  for  $U^{236}$ , and  $Z=93.62$  and  $\nu=2.83$  for  $Pu^{240}$ , and are satisfactory. For other cases, as Swiatecki points out, no systematic attempt has been made to adjust  $A - \bar{A}_H - \bar{A}_L$  to agree with available information on the number of emitted neutrons, and the reported curves are inadequate.

From Table I, the following numerical values are deduced for the constants:

$$\begin{aligned}c_z &= 0.089, & (Z^2/A)_z &= 39.65, \\ c_n &= 0.103, & (Z^2/A)_n &= 39.50.\end{aligned}$$

TABLE I. Average charge and neutron contents of fission fragments of the reference nuclides used in the evaluation of the asymmetry constants.

	$\bar{Z}_L$	$\bar{Z}_H$	$\bar{N}_L$	$\bar{N}_H$
${}_{92}U^{236}$	38.15	53.95	56.69	84.78
${}_{94}Pu^{240}$	39.79	53.83	59.51	84.04

<sup>8</sup> E. P. Steinberg and L. E. Glendenin, in *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 7, p. 3.

In accordance with Swiatecki we take  $\nu=2.8$ . These values are close to those found for mass asymmetry as would be expected. The probable error on  $(Z^2/A)_z$  and  $(Z^2/A)_n$  is again  $\pm 0.7$ , so that these parameters may be considered as identical. However, the differences in the numerical values of  $c_a$ ,  $c_z$ , and  $c_n$  should be considered as significant since the corresponding mass, proton, and neutron asymmetries are not identical.

The average mass of the heavy fragment  $\bar{A}_H$  as well as that of the light one  $\bar{A}_L$  can be calculated from Eqs. (3) and (5); the expressions thus derived are not identical to those given by Swiatecki since mass, charge, and neutron asymmetries differ. However, the results calculated from (3) and (5) by using the numerical values of the constants based on the data of Table I are in good agreement with those calculated directly from Swiatecki's formulas, indicating that the values obtained for the constants are satisfactory. It should be noted that the uncertainty in the observed values of  $\bar{A}_H$  and  $\bar{A}_L$  is of the order of  $\pm 1$  or  $\pm 2$  mass units (even more in the case of  $U^{239}$ ), whereas the difference between the values calculated directly and those calculated indirectly is of the order  $\pm 0.5$ .

**EFFECTS OF CHANGES IN THE COMPOSITION OF THE FISSION NUCLEUS ON THE COMPOSITION OF FRAGMENTS**

The charge, neutron, and mass asymmetries of the most probable fragments are affected by a change in the proton and/or neutron contents of the fissile nucleus. The changes in the proton and neutron contents of the heavy fragment corresponding to unit change in proton and neutron contents of the fissile nucleus are given by:

$$\begin{aligned}\partial \bar{Z}_H / \partial Z &= \frac{1}{2} - c_z Z^2 (2A - Z) (2A)^{-2} s_z^{-1} + (c_z/2) s_z, \\ \partial \bar{Z}_H / \partial N &= c_z Z^3 (2A)^{-2} s_z, \\ \partial \bar{N}_H / \partial Z &= -c_n Z (A - Z) (2A - Z) (2A)^{-2} s_n^{-1}, \\ \partial \bar{N}_H / \partial N &= \frac{1}{2} - c_n Z^2 (A - Z) (2A)^{-2} s_n^{-1} + (c_n/2) s_n,\end{aligned}\quad (6)$$

where

$$s_{z,n} = [(Z^2/A)_{z,n} - Z^2/A]^{1/2}.\quad (7)$$

The corresponding quantities for the light fragment can be deduced from the conditions:

$$\begin{aligned}\frac{\partial \bar{Z}_H}{\partial Z} + \frac{\partial \bar{Z}_L}{\partial Z} &= 1, & \frac{\partial \bar{Z}_H}{\partial N} + \frac{\partial \bar{Z}_L}{\partial N} &= 0, \\ \frac{\partial \bar{N}_H}{\partial Z} + \frac{\partial \bar{N}_L}{\partial Z} &= 0, & \frac{\partial \bar{N}_H}{\partial N} + \frac{\partial \bar{N}_L}{\partial N} &= 1, \\ \frac{\partial \bar{A}_H}{\partial Z} + \frac{\partial \bar{A}_L}{\partial Z} &= 1, & \frac{\partial \bar{A}_H}{\partial N} + \frac{\partial \bar{A}_L}{\partial N} &= 1.\end{aligned}\quad (8)$$

## Results

All of the partial derivatives listed above were calculated numerically for various fissile nuclides. An examination of the results leads to the following observations:

(1) The nuclides capable of undergoing low-energy fission (neutron-induced or spontaneous) are divided into two groups. One of these groups comprises  $\text{Th}^{233}$ ,  $\text{U}^{234}$ ,  $\text{U}^{236}$ ,  $\text{U}^{238}$ ,  $\text{U}^{239}$ , and  $\text{Pu}^{240}$ ; the other includes  $\text{Cm}^{242}$  and  $\text{Cf}^{252}$ . Any partial derivative of any average quantity (charge, neutron contents, or mass) has an almost constant value within each group; in other words, no systematic change is observed for the derivatives.

 FISSION OF  $\text{Th}^{233}$ ,  $\text{U}^{234}$ ,  $\text{U}^{236}$ ,  $\text{U}^{238}$ ,  $\text{U}^{239}$ , AND  $\text{Pu}^{240}$ 

(2) In this group, if one neutron is added to the fissile nucleus, the neutron contents of the most probable heavy fragment increase by about 0.90, while this increase is only about 0.10 unit for the most probable light fragment; thereby the neutron asymmetry increases by  $0.90 - 0.10 = 0.80$  unit.

(3) The addition of one neutron to the fissile nucleus affects also the proton contents of the most probable fragments in such a way that the charge of the heavy fragment increases by 0.15, while that of the light fragment decreases by 0.15 charge unit.

(4) If one proton is added to the fissile nucleus, the neutron contents of the most probable heavy fragment decrease by about 1.25 units, while those of the light fragment increase by the same amount; thus the neutron asymmetry decreases by 2.5 units.

(5) On adding one proton to the fissile nucleus, the charge of the most probable heavy fragment decreases slightly, by about 0.10 charge unit, while the charge of the light fragment increases by 1.10 units, thus the fission asymmetry with respect to charge decreases by about  $-0.10 - (+1.10) = -1.20$  units of charge.

The foregoing results may be summarized as follows: A neutron added to the original nucleus goes preferentially (90%) into the heavy fragment, and in doing so it takes with it some charge (15% of one charge unit) at the expense of the light fragment. On the other hand,

TABLE II. Effects of unit change in the proton and/or neutron contents of the fissile nucleus on the composition and asymmetry of the most probable fission fragments.

Addition of one neutron ( $\Delta N=1, \Delta Z=0$ )	Heavy fragment	Light fragment	Asymmetry change
$\Delta \bar{N}$	+0.90	+0.10	+0.80
$\Delta \bar{Z}$	+0.15	-0.15	+0.30
Addition of one proton ( $\Delta Z=1, \Delta N=0$ )			
$\Delta \bar{N}$	-1.25	+1.25	-2.50
$\Delta \bar{Z}$	-0.10	+1.10	-1.20

TABLE III. Effects of unit change in the proton and/or neutron contents of the fissile nucleus on the composition and asymmetry of the most probable fission fragments of  $\text{Cm}^{242}$  and  $\text{Cf}^{252}$ .

Addition of one neutron ( $\Delta N=1, \Delta Z=0$ )	Heavy fragment	Light fragment	Asymmetry change
$\Delta \bar{N}$	+1.06	-0.06	+1.12
$\Delta \bar{Z}$	+0.27	-0.27	+0.54
Addition of one proton ( $\Delta Z=1, \Delta N=0$ )			
$\Delta \bar{N}$	-2.03	+2.03	-4.06
$\Delta \bar{Z}$	-0.57	+1.57	-2.14

a proton added to the original nucleus goes preferentially to the light fragment, and in doing so it takes with it a little more charge (10% of one charge unit) and a substantial amount (1.25 units) of neutrons at the expense of the heavy fragment. (See Table II.)

The data of Table II apply to the fission of the following compound nuclei:  $\text{Th}^{233}$ ,  $\text{U}^{234}$ ,  $\text{U}^{236}$ ,  $\text{U}^{238}$ ,  $\text{U}^{239}$ , and  $\text{Pu}^{240}$ .

 SPONTANEOUS FISSION OF  $\text{Cm}^{242}$  AND  $\text{Cf}^{252}$ 

Essentially the same conclusions apply to this group, but the numerical values for the partial derivatives are somewhat different. (See Table III.)

The effects on the fragment composition caused by unit change in the proton and/or neutron contents of the compound nucleus are perfectly additive with respect to both the number of neutrons and that of protons added (or subtracted), and permit one to predict the average charge and neutron contents of the two fragments for any fissile nuclide, if these quantities are known for any other reference nuclide.

## CORRELATIONS BETWEEN AVERAGE QUANTITIES

In the foregoing discussion a series of compound nuclei were considered, which produce fragments of various compositions. It would be of interest to see whether there is any regular relationship between the average quantities for the products. An equation relating any two of the three quantities  $\bar{Z}$ ,  $\bar{N}$ ,  $\bar{A}$  cannot be derived analytically, because only two of them are independent, while two variables,  $Z$  and  $N$  (or  $A$ ), have to be eliminated. However, the average quantities can be calculated for each member of the series and compared afterwards. Such a comparison leads to the following results.

(1)  $\bar{Z}$  versus  $\bar{N}$ : For nonisotopic fissile nuclei the average charge of the light fragment increases with neutron contents within wide limits. In the case of isotopic nuclei  $\bar{Z}_L$  decreases with  $\bar{N}_L$ , but both  $\bar{Z}_L$  and  $\bar{N}_L$  remain in a very narrow region, indicating a high composition selectivity of the light fragment with respect to both charge and neutron contents.

TABLE IV. Charge per nucleon of the fissioning nucleus and of its average light and heavy fission fragments. In the computation of the data based on Table II,  $U^{236}$  was used as the reference nuclide.

Compound nucleus	$Z/(Z+N)$	$\bar{Z}_L/(\bar{Z}_L+\bar{N}_H)$		$\bar{Z}_H/(\bar{Z}_L+\bar{N}_H)$	
		from: Eqs. (5)	Table II	from: Eqs. (5)	Table II
Th <sup>233</sup>	0.389	0.399	0.400	0.386	0.385
U <sup>234</sup>	0.395	0.406	0.405	0.392	0.393
U <sup>236</sup>	0.394	0.402	0.402	0.389	0.389
U <sup>238</sup>	0.390	0.399	0.399	0.385	0.386
U <sup>239</sup>	0.389	0.397	0.398	0.384	0.383
Pu <sup>240</sup>	0.396	0.404	0.402	0.390	0.391
Cm <sup>242</sup>	0.401	0.408	0.405	0.390	0.392
Cf <sup>252</sup>	0.393	0.399	0.399	0.385	0.388
Average	0.393	0.402	0.401	0.387	0.388

The average charge of the heavy fragment is almost independent of the neutron contents, indicating a high selectivity with respect to charge, associated with an indifference with respect to neutron contents.

(2)  $\bar{Z}$  versus  $\bar{A}$ : A very regular relationship exists between  $\bar{Z}$  and  $\bar{A}$  for both the light and the heavy fragment. As Table IV shows, the ratio  $Z/A$ , which may be called the charge density, or charge per nucleon, has a constant value for each group of fragments, regardless

of the nature of the fissioning nucleus. The mean value of the constant is 0.402 for the light fragment and 0.387 for the heavy fragment. In all cases the charge density of the fissile nucleus is larger than the charge density of the heavy fragment and smaller than that of the light fragment, giving no support to the "unchanged charge ratio" postulate,<sup>9</sup> also in cases other than that of the thermal neutron fission of  $U^{235}$ .

The relationship between the average charge and mass of the light and heavy fragments leads to two almost parallel straight lines (Fig. 2), which almost coincide with the  $Z_p$  versus  $A$  lines for  $U^{235}(n,F)$ , implying that the relationship between the most probable values of fragment charge and mass is independent of the fissioning nucleus; in other words, this relationship is *universal*.

The average charge and mass of the light fragment vary within much wider limits (8 charge units and 18 mass units, respectively), in comparison to the heavy fragment charge and mass which vary in a much narrower range (2 charge units and 6 mass units, respectively), signifying that the fission process is more selective with respect to the heavy-fragment composition.

<sup>9</sup> N. Sugarman and A. Turkevich, private communication to Professor Coryell; C. D. Coryell, M. Kaplan, and R. D. Fink, *Can. J. Chem.* **39**, 646 (1961).

## $(d,He^3)$ Reaction on $Ca^{40}$ and the Titanium Isotopes\*

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The  $(d,He^3)$  reaction on  $Ca^{40}$  and isotopically enriched targets of  $Ti^{46}$ ,  $Ti^{47}$ ,  $Ti^{48}$ ,  $Ti^{49}$ , and  $Ti^{50}$  has been investigated. Absolute differential cross sections for a number of transitions were obtained over an angular range from 11 to 33°. The angular distributions are compared with distorted-wave calculations which use the optical-model-potential parameters derived from the elastic scattering of 21-MeV deuterons on the same target materials. The data for the elastic scattering are given together with the optical-model fits. The  $Ca^{40}(d,He^3)K^{39}$  reaction was used to obtain the normalization coefficients necessary to extract spectroscopic factors from the comparison of the measured and calculated differential cross sections. The results show that the excitation energies of the  $d_{3/2}$  and  $s_{1/2}$  proton-hole states are surprisingly low and increase with the number of  $f_{7/2}$  neutrons in the nucleus. In the case of  $Sc^{45}$ , the  $d_{3/2}$  hole state which contains the full  $d_{3/2}$  strength is found at an excitation energy of less than 50 keV. The excitation energy of the  $d_{3/2}$  hole state increases to 800 keV for  $Sc^{47}$  and to about 2.4 MeV in  $Sc^{49}$ . The excitation energy of the  $s_{1/2}$  hole state increases from 0.92 MeV in  $Sc^{45}$  to 2.1 MeV in  $Sc^{48}$ .

### 1. INTRODUCTION

IN recent years a number of papers have reported experiments on reactions in which a neutron from the target nucleus is captured by the bombarding nucleus. These experiments have been used in particular

to obtain information on the ground-state wave functions of the target nucleus and, in some cases, on the final nucleus. Only a small number of experiments on reactions involving the capture of a proton from the target nucleus have been reported.<sup>1-3</sup> It has been

\* Work performed under the auspices of the U. S. Atomic Energy Commission.

<sup>1</sup> J. L. Yntema, T. H. Braid, B. Zeidman, and H. W. Broek, in *Proceedings of the Rutherford Jubilee International Conference*,