laboratory using the linear accelerator as a pulsed source of neutrons.

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Quantum Theory of Elementary Particles*

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Quantum numbers which may possibly be identified with strangeness S, baryon number B, and isospin I are found to be natural consequences of the generalized field theory of a spinning particle developed in earlier papers, the theory requiring that $S+2I+2J$ is even, as observed. The generalized Dirac equation for fermions leads to the correct values of B, S, I_3, and J, and approximately the correct masses for the states n, p, Ξ^0, Ξ^-, N_{13}^{*+}, N_{13}^{*0}, the lowest known states of $I=\frac{1}{2}$, $J=\frac{1}{2}$, or $\frac{3}{2}$. The generalized Dirac equation for bosons similarly describes these quantities for the K and K^* mesons. The generalized Kemmer equation for fermions yields the correct values of B, S, I_3, J, and the masses for the Λ^0, Y_0^*, and Y_{03}^* if the spin of the Y_0^* is $\frac{1}{2}$, and the generalized Kemmer equation for bosons similarly leads to the correct masses, spins, and isospins for the $S=0$ states ϕ, f, ω, η, ρ, and predicts $I=1$, $S=0$ states at 1-BeV spin 1 (χ_1?), 1.24-BeV spin 2(B?), 450-MeV spin 0(ζ?), and $I=0$ states at 965 MeV (spin 1) and 926 MeV (spin 0). The only arbitrariness in the theory lies in the choice of the two mass parameters for each equation, and in the choice of which combination of two independently conserved currents allowed by each equation is identified with the electric current. The theory satisfies a correspondence principle with the classical relativistic equation of motion of a symmetric top, and yields a prescription for describing states of higher quantum numbers. It then predicts the spin of the Y_0^{**} state as $\frac{5}{2}$, correctly describes the spin and mass of the N_{15}^* state, predicts a series of N^* states 166 MeV apart of progressively increasing spin, and describes other states, the properties of which have not yet been investigated.

1. INTRODUCTION

IN our attempts to understand elementary particles and nuclear forces, for several decades we have been making an assumption that is not forced on us either by the principles of relativity theory or by the requirements of quantum theory. This assumption ultimately has to do with the *shape* of an elementary particle, but in the relativistic quantum theory of a point-particle, a concept such as shape does not enter. It is therefore necessary to examine the classical limit of relativistic field theory—the relativistic classical mechanics of a spinning particle—where the motion of the spin of even a point-particle can be described only when we know its moments of inertia about axes along, and perpendicular to, its spin axis. In the absence of any information about the structure of the particles it is necessary to treat the particle as a point with, however, a finite amount of spin-angular momentum associated with it. This requires nonzero moments of inertia if the angular velocity is to remain finite, and these may be prescribed as parameters which are a measure of the "shape" of the particle.

In the corresponding quantum theory we have ignored these questions, arguing that the angular velocity is not an observable and that it is sufficient to associate a spin-angular momentum with the particle, and look for equations of motion which lead to irreducible representations of the Lorentz group for different spin values. These equations, in particular those of Dirac and Kemmer, are also based on the assumption that the spin and rest mass of a particle are always constant parameters.

In view of the well-established correspondence between classical and quantum physics it seems surprising that dynamical variables and parameters such as angular velocity and moment of inertia, so important in classical mechanics, play no role in quantum theory. It has, therefore, seemed reasonable to conduct a reinvestigation of the relation between the Dirac equation and the classical equations of motion to see at what point the correspondence was lost. For many years it has

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been recognized that the classical equations of motion reflect, in an imperfect way, the essential properties that make the Dirac equation such an accurate description of nature. In particular, pair production *zitterbewegung*, and the gyromagnetic ratio of the electron all appear in the classical theory if we do not inhibit the free exchange of angular momentum between spin and orbital modes, as some incomplete statements of the classical equations are prone to do.

Further investigation has revealed that the operator which corresponds to the angular velocity $\boldsymbol{\omega}$ in the classical theory does in fact have its counterpart in the Dirac theory. It is nothing but a constant times the Pauli spin operator $\boldsymbol{\sigma}$, and the equation $\mathbf{J}=\frac{1}{2}\hbar\boldsymbol{\sigma}$ for the spin has as its classical limit $\mathbf{J}=I\boldsymbol{\omega}$, where I is the moment of inertia about the spin axis. Quantum mechanically, there seems to be no need to break $\mathbf{J}$ up into factors of dimension I and $\boldsymbol{\omega}$, and we have become accustomed to thinking of the Pauli spin operator as the spin itself, apart from the factor $\frac{1}{2}\hbar$. Angular velocity and angular momentum are thereby assumed to be parallel to each other, and indistinguishable apart from a constant factor.

The nonrelativistic classical equations of motion of course do not require that the angular momentum and angular velocity of a body should be parallel to each other, unless the body is rotating about a principal axis. The relativistic classical equations of a point-particle with spin lead to the surprising result that the angular momentum and angular velocity are not required to be parallel even if the particle *is* rotating around a principal axis. The reason for this is essentially the same as the reason why the ordinary momentum and velocity of a spinning particle are not required to be parallel in classical theory, and are represented by quite different operators in the Dirac theory. A distribution of matter rotating about a principal axis will acquire products of inertia from relativistic effects if the axis itself rotates, and in the limit of a point-particle at rest this leads to the relation between spin-angular momentum $\mathbf{J}$ and angular velocity $\boldsymbol{\omega}$

$$\mathbf{J}=I\boldsymbol{\omega}+(K/c)(\boldsymbol{\omega}\times(d\boldsymbol{\omega}/dt)), \tag{1.1}$$

K being a parameter proportional to the moment of inertia of the particle about any axis at right angles to $\boldsymbol{\omega}$ (assuming axial symmetry). In fact, for a free particle, Eq. (1.1) has a solution in which $\boldsymbol{\omega}$ precesses around the constant vector $\mathbf{J}$ with angular velocity $\boldsymbol{\omega}'=(c/K\omega^2)\mathbf{J}$ according to Euler's equations

$$I'(d\boldsymbol{\omega}/dt)+\boldsymbol{\omega}\times\mathbf{s}=0, \tag{1.2}$$

where $I'=K\omega^2/c$ is the moment of inertia about axes orthogonal to $\boldsymbol{\omega}$.

For $\mathbf{v}\neq 0$, the corresponding relativistic classical equations of motion for a symmetrical top are[1]

$$\begin{aligned} \dot{J}_{\mu\nu}&=-(v_\mu p_\nu-v_\nu p_\mu)=I\dot{\omega}_{\mu\nu}-K(\omega_{\mu\sigma}\ddot{\omega}_{\sigma\nu}-\omega_{\nu\sigma}\ddot{\omega}_{\sigma\mu}),\\ \dot{M}&=(K/2Ic)\ddot{\omega}_{\mu\nu}\dot{J}_{\mu\nu}, \end{aligned} \tag{1.3}$$

where

$$v_\mu p_\mu+Mc=0,\quad \omega_{\mu\nu}v_\nu=0 \tag{1.4}$$

and

$$M=m-(K/4c)\omega_{\mu\nu}\ddot{\omega}_{\mu\nu}, \tag{1.5}$$

where m is a constant. Thus, even for $\mathbf{v}=0$, there are extra contributions to the mass and spin given by

$$\delta M=-(K/2c^3)\boldsymbol{\omega}\cdot(d^2\boldsymbol{\omega}/dt^2),\quad \delta\mathbf{J}=(K/c)\boldsymbol{\omega}\times d\boldsymbol{\omega}/dt,$$

which, for the motion described by Eq. (1.2) give

$$J=J_\omega+J_\Omega,$$

where

$$\delta M=(J_\Omega/\hbar)m_0,\quad \omega'=2m_0c^2/\hbar, \tag{1.6}$$

and J_ω is the component of $I\boldsymbol{\omega}$ in the direction of $\mathbf{J}$.

If we set $K=0$, the classical equations reduce to

$$v_\mu p_\mu+mc=0,\quad J_{\mu\nu}=I\omega_{\mu\nu},\quad \omega_{\mu\nu}v_\nu=0$$

or

$$W=\mathbf{v}\cdot\mathbf{p}+mc^2(1-\beta^2)^{1/2},\quad \mathbf{J}=I\boldsymbol{\omega}. \tag{1.7}$$

It follows as a consequence of the equations of motion, that m is a constant of the motion. These equations are to be compared with the Dirac or Kemmer equations, in which m is also a constant.

$$(i\epsilon_\mu p_\mu+mc)\psi=0,\quad J_{\mu\nu}=-i\hbar\epsilon_{\mu\nu},$$

where

$$\begin{aligned} \epsilon_{\mu\nu}&\equiv\alpha(\epsilon_\mu,\epsilon_\nu)\\ (\epsilon_{\mu\nu},\epsilon_\sigma)&=\epsilon_\mu\delta_{\nu\sigma}-\epsilon_\nu\delta_{\mu\sigma}, \end{aligned} \tag{1.8}$$

and α is a constant. [The Dirac equation is given by the choice

$$\epsilon_\mu=\gamma_\mu,\quad \alpha=\tfrac{1}{4},\quad J_{\mu\nu}=-\tfrac{1}{4}i\hbar\gamma_{\mu\nu}, \tag{1.9}$$

and the Kemmer equation by the choice

$$\epsilon_\mu=\beta_\mu,\quad \alpha=1,\quad J_{\mu\nu}=-i\hbar\beta_{\mu\nu}, \tag{1.10}$$

where γ_μ, β_μ are the Dirac and Kemmer operators, respectively, and $\gamma_{\mu\nu}\equiv(\gamma_\mu,\gamma_\nu)$, $\beta_{\mu\nu}\equiv(\beta_\mu,\beta_\nu)$.] From (1.8), it follows that

$$(\epsilon_{\mu\nu},\epsilon_{\sigma\tau})=-(\epsilon_{\mu\sigma}\delta_{\nu\tau}+\epsilon_{\nu\tau}\delta_{\mu\sigma}-\epsilon_{\mu\tau}\delta_{\nu\sigma}-\epsilon_{\nu\sigma}\delta_{\mu\tau}).$$

As shown in Ref. 2, the basic wave equation which we adopt for a free particle is suggested by the classical equations (1.4) and (1.5).

$$(i\epsilon_\mu p_\mu+Mc)\psi=0, \tag{1.11}$$

[1] H. J. Bhabha and H. C. Corben, Proc. Roy. Soc. (London) **A178**, 273 (1941); S. Shanmugadhasan, Can. J. Phys. **30**, 226 (1952).

[2] I. H. C. Corben, Proc. Natl. Acad. Sci. U. S. **48**, 1559 (1962); II. H. C. Corben, Proc. Natl. Acad. Sci. U. S. **48**, 1746 (1962); III. H. C. Corben, Nuovo Cimento **28**, 202 (1963); IV. H. C. Corben, Phys. Rev. Letters **10**, 555 (1963); V. H. C. Corben, Phys. Rev. **131**, 2219 (1963).

TABLE I. B, S, I_3, J and mass values for solutions of Eq. (1.17). For n, Ξ^0 states the values of S and I_3 are for the limit in which mass differences are neglected. The theoretical values for the mass levels are based on the choice $m=1328$ MeV$=8m_0$. Antiparticles of all those listed appear when the signs of both ϵ and $\gamma_4\eta_4$ are reversed. Each state occurs twice, with the same values of η_5, B, S, I_3, J but with $\tau=\pm 1$.

Representation of β_μ	$\gamma_4\eta_4$	ϵ	η_5	B	S	I_3	Mass (units m)	Mass (MeV)	Spin J	Particle	Mass (MeV) experimental
1×1	−1	1	1	1	−2	$-\frac{1}{2}$	1	1328	$\frac{1}{2}$	Ξ^-	1321
5×5	1	−1	1	1	−2	$-\frac{1}{2}$	1	1328	$\frac{1}{2}$	Ξ^-	1321
			−1	1	−2	$\frac{1}{2}$	$a+(1-2a-2a^2)^{1/2}$	1292	$\frac{1}{2}$	Ξ^0	1316
			−1	−1	0	$\frac{1}{2}$	$a-(1-2a-2a^2)^{1/2}$	−959	$\frac{1}{2}$	$\bar{n}$	−940
			−1	−1	0	$\frac{1}{2}$	$-(1+a)$	−1494	$\frac{3}{2}$	$\bar{N}_{13}^{*0}$	−1517
10×10	−1	−1	1	1	0	$\frac{1}{2}$	$(1+2a)^{1/2}$	1485	$\frac{3}{2}$	N_{13}^{*+}	1517
			−1	1	−2	$\frac{1}{2}$	$a+(1-2a-2a^2)^{1/2}$	1292	$\frac{1}{2}$	Ξ^0	1316
			1	1	0	$\frac{1}{2}$	$(1-4a)^{1/2}$	939	$\frac{1}{2}$	p	938
			1	−1	0	$-\frac{1}{2}$	$-(1-4a)^{1/2}$	−939	$\frac{1}{2}$	$\bar{p}$	−938
			−1	−1	0	$\frac{1}{2}$	$a-(1-2a-2a^2)^{1/2}$	−959	$\frac{1}{2}$	$\bar{n}$	−940
			1	−1	0	$-\frac{1}{2}$	$-(1+2a)^{1/2}$	−1485	$\frac{3}{2}$	$\bar{N}_{13}^{*-}$	−1517
			−1	−1	0	$\frac{1}{2}$	$-(1+a)$	−1494	$\frac{3}{2}$	$\bar{N}_{13}^{*0}$	−1517

where[3]

$$M=m-m_0\epsilon_{\mu\nu}\lambda_{\mu\nu}. \tag{1.12}$$

The $\lambda_{\mu\nu}$ satisfy among themselves the same commutation relations as do the $\epsilon_{\mu\nu}$:

$$\lambda_{\mu\nu}\equiv\alpha(\lambda_\mu,\lambda_\nu), \tag{1.13}$$

$$(\lambda_{\mu\nu},\lambda_\sigma)=\lambda_\mu\delta_{\nu\sigma}-\lambda_\nu\delta_{\mu\sigma},$$

so that

$$(\lambda_{\mu\nu},\lambda_{\sigma\tau})=-(\lambda_{\mu\sigma}\delta_{\nu\tau}+\lambda_{\nu\tau}\delta_{\mu\sigma}-\lambda_{\mu\tau}\delta_{\nu\sigma}-\lambda_{\nu\sigma}\delta_{\mu\tau}).$$

In addition, we postulate that

$$(\epsilon_\mu,\lambda_\nu)=0, \tag{1.14}$$

so that

$$(\epsilon_{\mu\nu},\lambda_{\sigma\tau})=0,\quad(\epsilon_\mu,\lambda_{\nu\sigma})=0,\quad(\epsilon_{\mu\nu},\lambda_\sigma)=0.$$

The spin operator of the particle described by Eq. (1.11) is now

$$J_{\mu\nu}=-i\hbar(\epsilon_{\mu\nu}+\lambda_{\mu\nu}), \tag{1.15}$$

since it follows from (1.11) that the components of

$$J_{\mu\nu}+x_\mu p_\nu-x_\nu p_\mu$$

are constants of the motion.

Apart from the superficial similarity between Eqs. (1.11), (1.12), and the classical equations (1.4), (1.5), we note that, if $i\hbar\dot{X}$ is defined as (X,H), where H is the invariant operator on the left-hand side of Eq. (1.11), it follows that

$$\dot{J}_{\mu\nu}=-i(\epsilon_\mu p_\nu-\epsilon_\nu p_\mu)$$
$$=-i\hbar\dot{\epsilon}_{\mu\nu}+2m_0c(\epsilon_{\mu\sigma}\lambda_{\sigma\nu}-\epsilon_{\nu\sigma}\lambda_{\sigma\mu}), \tag{1.16}$$
$$\dot{M}=-(im_0/\hbar)\lambda_{\mu\nu}\dot{J}_{\mu\nu}.$$

[3] Readers unimpressed by classical limits and the correspondence principle may think of the extra term in Eq. (1.12) as an interaction that we are "guessing," a term which splits the otherwise degenerate mass levels.

The correspondence between these quantum equations of motion and the classical equations (1.3) is established by writing

$$I\omega_{\mu\nu}=-i\hbar\epsilon_{\mu\nu},$$
$$\hbar K\ddot{\omega}_{\mu\nu}=-2im_0cI\lambda_{\mu\nu},$$
$$v_\mu=i\epsilon_\mu.$$

Independently of the choice (1.9) or (1.10) for the ϵ_μ, we may choose

$$\lambda_\mu=\gamma_\mu',\quad\alpha=\tfrac{1}{4},\quad\lambda_{\mu\nu}=-\tfrac{1}{4}i\hbar\gamma_{\mu\nu}'$$

or

$$\lambda_\mu=\beta_\mu',\quad\alpha=1,\quad\lambda_{\mu\nu}=-i\hbar\beta_{\mu\nu}',$$

where the γ_μ', β_μ' commute with the γ_μ, β_μ. We are therefore led to the four following possibilities: two equations for fermions (with $\epsilon_\mu=\gamma_\mu$, $\lambda_\mu=\beta_\mu$ or with $\epsilon_\mu=\beta_\mu$, $\lambda_\mu=\gamma_\mu$) and two equations for bosons (with $\epsilon_\mu=\gamma_\mu$, $\lambda_\mu=\gamma_\mu'$, or with $\epsilon_\mu=\beta_\mu$, $\lambda_\mu=\beta_\mu'$). Each of the fermion equations

$$[i\gamma_\mu p_\mu+mc-\tfrac{1}{4}m_0c\gamma_{\mu\nu}\beta_{\mu\nu}]\psi=0, \tag{1.17}$$

$$[i\beta_\mu p_\mu+m'c-\tfrac{1}{4}m_0'c\beta_{\mu\nu}\gamma_{\mu\nu}]\psi=0, \tag{1.18}$$

describes states of spin $J_{\mu\nu}=-i\hbar[\frac{1}{4}\gamma_{\mu\nu}+\beta_{\mu\nu}]$, i.e., $\mathbf{J}=\hbar[\frac{1}{2}\boldsymbol{\sigma}+\boldsymbol{\Sigma}]$ (with J_{23}, J_{31}, $J_{12}=\mathbf{J}$, γ_{23}, γ_{31}, $\gamma_{12}=2i\boldsymbol{\sigma}$, β_{23}, β_{31}, $\beta_{12}=i\boldsymbol{\Sigma}$). The spin of a particle state described by either (1.17) or (1.18) is therefore $\frac{1}{2}$ (for $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}=0$ or -2) or $\frac{3}{2}$ (for $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}=1$).

Similarly the two boson equations obtained from (1.11) and (1.12) are

$$[i\gamma_\mu p_\mu+m''c-\tfrac{1}{16}m_0''c\gamma_{\mu\nu}\gamma_{\mu\nu}']\psi=0, \tag{1.19}$$

$$[i\beta_\mu p_\mu+m'''c-m_0'''c\beta_{\mu\nu}\beta_{\mu\nu}']\psi=0, \tag{1.20}$$

the first describing particles of spin

$$\mathbf{J}=\tfrac{1}{2}\hbar(\boldsymbol{\sigma}+\boldsymbol{\sigma}'),$$

i.e., spin zero or unity, and the second describing particles of spin

$$\mathbf{J}=\hbar(\mathbf{\Sigma}+\mathbf{\Sigma}'),$$

i.e., spin 0, 1, 2.

In the following sections we examine in turn the properties of the solutions of these four equations in the rest systems of the various particle states.

2. BARYONS OF STRANGENESS 0, ±2

The solutions of Eq. (1.17) in the rest system have been given in Ref. 2 (III and V) and are reproduced here in Table I and Fig. 1. Equation (1.17) describes particle states of spin $\frac{1}{2}$ or $\frac{3}{2}$, and, as will become apparent, states of isospin $\frac{1}{2}$. The six lowest states known to fall into these categories are[4] p, n, Ξ^0, Ξ^-, N_{13}^{*+}, N_{13}^{*0} and the six states given by Eq. (1.17) are identified with these, together with their antiparticles.

The conserved probability density four-vector is given by

$$s_\mu=ic\psi^*\gamma_4\eta_4\gamma_\mu\psi \tag{2.1}$$

($\eta_\mu=2\beta_\mu^2-1$) so that the probability density $\psi^*\eta_4\psi$ is not positive definite and may be normalized to ±1.

$$\tau=\int\psi^*\eta_4\psi dV=\pm1. \tag{2.2}$$

For any particular state the sign of τ is automatically determined. The independently conserved charge-current density may be taken to be

$$j_\mu=\tfrac{1}{2}ie_0c\epsilon\psi^*\gamma_4\eta_4(1+\eta_5)\gamma_\mu\psi, \tag{2.3}$$

where e_0 is the proton charge and $\epsilon=\pm1$. The choice of the sign of ϵ is discussed below. Neutral states are characterized by $\eta_5=-1$, and charged states by $\eta_5=+1$, the total charge in units of e_0 being given by

$$Q=\frac{-i}{e_0c}\int j_4dV=\tfrac{1}{2}\epsilon\int\psi^*\eta_4(1+\eta_5)\psi dV. \tag{2.4}$$

The quantity

$$B\equiv\int\psi^*\gamma_4\psi dV \tag{2.5}$$

is found to be positive for positive-energy states and negative for negative-energy states. In the rest system, the various eigenstates may be characterized by the eigenvalues ±1 of the operator $\gamma_4\eta_4$. Hence, for such states with wave functions normalized according to Eq. (2.2), we have $B=\pm1$, the sign being that of the rest energy. Thus, B [Eq. (2.5)] may be interpreted as the baryon number.

[4] The notation is that of M. Roos, Rev. Mod. Phys. **35**, 314 (1963).

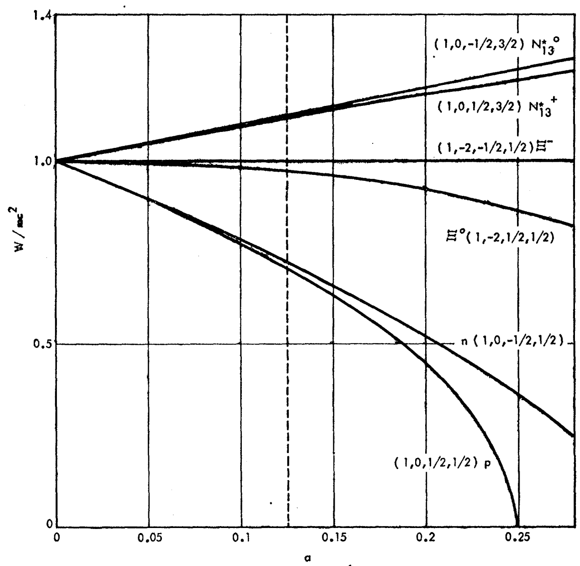

FIG. 1. Positive mass levels of Eq. (1.17) in units mc^2 and as functions of $a=m_0/m$. The quantum numbers represent B, S, I_3, J, respectively. The vertical dashed line indicates the value of a used in Table I.

We now define

$$Y=B+S=\tfrac{1}{2}\epsilon\int\psi^*\{1-\eta_5+\eta_4(1+\eta_5)\}\psi dV, \tag{2.6}$$

$$I_3=\tfrac{1}{4}\epsilon\int\psi^*\{\eta_5-1+\eta_4(1+\eta_5)\}\psi dV, \tag{2.7}$$

so that, from Eq. (2.4),

$$Q=\tfrac{1}{2}(B+S)+I_3. \tag{2.8}$$

Further, from (2.6), (2.7)

$$I_3=\tfrac{1}{2}(B+S)\eta_5, \tag{2.9}$$

where η_5 now represents the eigenvalue of η_5 for the state ψ. Thus, $I_3=\frac{1}{2}(B+S)$ for charged states and $-\frac{1}{2}(B+S)$ for neutral states, in agreement with the usual assignments.

For charged states ($\eta_5=+1$) we have

$$\begin{aligned}Q=B+S&=\epsilon\int\psi^*\eta_4\psi dV\\&=\int\psi^*(\epsilon\gamma_4\eta_4)\psi dV\\&=\lambda B,\end{aligned} \tag{2.10}$$

for eigenstates of the operator $\epsilon\gamma_4\eta_4$, which has eigenvalues $\lambda=\pm1$. Thus, charged states $\lambda=1$ characterizes states of charge $Q=Y=B$, with $B=\pm1$, whereas $\lambda=-1$ characterizes states of $S=-2B$, $Q=Y=-B$. We therefore see how strangeness appears as a natural consequence of the basic equation (1.17).

In the classical Dirac theory of the electron, the electric charge is given by

$$Q=\epsilon\int\psi^*\psi dV\,,$$

where $\epsilon=-1$ for positive energy states and $\epsilon=+1$ for negative energy states. In the present theory, however, the choice of the sign of ϵ is more subtle. Each representation of the β_μ gives rise to a set of states for one sign of $\gamma_4\eta_4$ and to the corresponding antiparticle states for the opposite sign of $\gamma_4\eta_4$. In addition, each state occurs in two different representations of the β_μ. We therefore choose the sign of $\epsilon(\epsilon=+1)$ for the one positive energy state ($\gamma_4=1$, $\eta_4=-1$, $\eta_5=+1$) that occurs in the 1×1 representation of the β_μ. The spin of this state is $\frac{1}{2}$, and since $\lambda=-1$ it follows immediately from the definitions (2.4), (2.5), (2.6), and (2.7) that $Q=-1$, $B=+1$, $S=-2$, $I_3=-\frac{1}{2}$. Since these quantum numbers characterize the Ξ^- state, we therefore choose the mass m of this state to be approximately the mass of the Ξ^- particle. This same state also occurs in the 5×5 representation of the β_μ, with $\gamma_4\eta_4=+1$, $\eta_5=+1$. In order that this should describe the same particle, it is then necessary to choose $\epsilon=-1$. This choice automatically causes not only Q, but also B, S, and I_3 to assume the same values for Ξ^- as before. However, in this representation there are three other states, all neutral, with masses 1292 MeV, -959 MeV (spin $\frac{1}{2}$), and -1494 MeV (spin $\frac{3}{2}$) for the choice $m_0c^2=166$ MeV. Since ϵ has been fixed as -1 in this representation, it follows that, with $\eta_5=-1$, $S+B\doteqdot-1$ for all three states, the approximate value becoming an equality when mass differences are neglected (see below). Since $B=1$, -1, -1 for these three states respectively, it follows that $S\doteqdot-2$ for the first state and zero for the other two. The isospin component I_3 then assumes values appropriate to the particles Ξ^0, $\bar{n}$, $\bar{N}_{13}{}^{*0}$ with which these states are identified.

These same three states now appear in the 10×10 representation with $\gamma_4\eta_4=-1$. We must, therefore, choose $\epsilon=-1$ in this case so that the quantum numbers of each state will be the same as before. However, this representation also includes the charged states at ±1485 MeV (spin $\frac{3}{2}$) and ±939 MeV (spin $\frac{1}{2}$). Since ϵ has been already fixed as equal to -1 in this representation, it follows that $\lambda=+1$, so that $S=0$, $Q=B=2I_3$ for these states. These states of positive baryon numbers therefore have a positive charge, and those of negative baryon numbers a negative charge. In addition, the spins, masses, isospin, and strangeness of these states are appropriate for the description of the proton and the charged component of the $N_{13}{}^{*+}$ resonance, together with their antiparticles. States which appear as a charged particle together with its neutral counterpart in the same representation are characterized by $\tau=1$ [Eq. (2.2)] and states which appear with the other member of the isospin doublet in a different representation are characterized by $\tau=-1$.

The Dirac equation for the Ξ^- particle implies the existence of the $\bar{\Xi}^+$ particle, and no other particle states. In this generalized theory the same equation (since in the 1×1 representation the extra term in the Dirac equation is zero) implies the existence of a number of other states which have the correct values of B, S, I_3, and J and approximately the correct masses to describe the particles listed in Table I. The $\bar{\Xi}^+$ similarly leads to the corresponding antiparticles.

For neutral states ($\eta_5=-1$) we have, from (2.6),

$$B+S=-2I_3=\epsilon\int\psi^*\psi dV \tag{2.11}$$

so that $B+S$ has the same sign as ϵ. However, from Eq. (2.2), $B+S$ is not strictly equal to ±1 unless ψ is an eigenstate of η_4. Such is the case for the $N_{13}{}^{*0}$ states, but for the n and Ξ^0 states this is true only in the limit $m_0\to0$, i.e., in the limit in which mass differences of an isospin doublet are neglected. This is in agreement with the fact that in the current phenomenological description of isospin, such mass differences are in fact neglected, the integral values of $2I_3$ and, consequently, of S, being only approximations, valid in this limit. However, $S+2I_3$ is required to be strictly equal to the integer $2Q-B$, and this result follows from the definitions (2.5), (2.6), and (2.7). If such mass differences are neglected, the definitions (2.6) and (2.7) reduce to $I_3=\frac{1}{2}\eta_5Y$, with $Y=\epsilon$ for $\tau=1$, $Y=-\epsilon\eta_5$ for $\tau=-1$.

To examine the values of S and I_3 when mass differences are not neglected, we note that the neutron wave function was given in[5] Ref. 2 [IV, Eq. (3.19)] as

$$\psi=\frac{1}{(\sqrt{V})}\frac{1}{(3\beta^2-9)^{1/2}}\begin{bmatrix}\beta\\-i\beta\\-\beta\\3\end{bmatrix}, \tag{2.12}$$

$$\alpha\beta-2\beta+3=0\,,\quad \eta_5=-1\,,$$

$$\alpha=(m-m_n)/m_0\,,\quad \beta=-(m_n+m)/m_0\,.$$

For $m_0\ll m$, the first three components, for which $\eta_4=+1$, are large compared with the last component, for which $\eta_4=-1$. The multiplying factor normalizes ψ according to Eq. (2.2). However, since in this case $\epsilon=+1$ (see Table I) $S+B$ is now

$$Y=S+B=(\beta^2+3)/(\beta^2-3)\,,$$

the neutron mass being given by

$$m_n=-m_0+(m^2-2mm_0-2m_0^2)^{1/2}\,.$$

[5] m in this reference is here replaced by $\frac{1}{2}m$, and m_0 by $-\frac{1}{2}m_0$.

For the values of m, m_0 chosen in Table I, $(S+B)_n = 1.03$. To terms of order $(m_0/m)^2$, this may be written

$$(S+B)_n \doteq 1+3(m_n-m_p)/m .$$

However, mass differences are not accurately described at the present level of this classical field theory.

Similarly, for the Ξ^0, which appears in the same representation as the other solution of (2.12),

$$m_{\Xi^0} = -m_0 - (m^2-2mm_0-2m_0^2)^{1/2}, \quad (m=m_{\Xi^-})$$
$$\alpha = (m_{\Xi^-}+m_{\Xi^0})/m_0, \quad \beta = -(m_{\Xi^-}-m_{\Xi^0})/m_0,$$

it follows that $\beta^2 \ll 1$, and $S+B$, which has the same sign as ϵ, is now given by

$$Y = S+B = (3+\beta^2)/(3-\beta^2).$$

The fourth component of $\psi(\eta_4=-1)$ is now large compared with the other three. In this case

$$Y = S+B \doteq 1+(2/3m_0^2)(m_{\Xi^-}-m_{\Xi^0})^2, \qquad (2.13)$$

which again reduces to the correct integer when the $\Xi^- - \Xi^0$ mass difference is neglected. In Table I the values listed for S are in the limit in which such mass differences are neglected. We note how just the right states are picked out automatically as having a strangeness equal to ± 2.

As noted earlier, the third component I_3 of the isospin is given from (2.9) as $\frac{1}{2}(B+S)$ for charged states and $-\frac{1}{2}(B+S)$ for neutral states. For neutral states, it has the values $\pm\frac{1}{2}$ only in the limit in which mass differences are neglected. This consequence is borne out by noting that, in the 10×10 representation $(\gamma_4\eta_4=+1)$ for example, the proton state has six components, three large $(\eta_4=+1)$ and three small $(\eta_4=-1)$ [Ref. 2, IV Eq. (3.19)].

$$\psi = \begin{pmatrix} m_p+m \\ -i(m_p+m) \\ -(m_p+m) \\ i(m_p-m) \\ (m_p-m) \\ -i(m_p-m) \end{pmatrix},$$

$$\alpha\beta+2\alpha-2\beta=0, \quad \eta_5=+1, \quad m_p=[m(m-4m_0)]^{1/2},$$
$$\beta=-(m_p+m)/m_0, \quad \beta+2=2(m_p+m)/(m_p-m).$$

Again, the last three components vanish in the limit $m_0=0$. To complete the verification that I_3 is indeed the third component of the isospin, at least in this limit, we find that, if we neglect the three small components of ψ_p and the one small component of ψ_n it follows that, on examination of the components in this representation,

$$\beta_4\psi_p = -i\psi_n, \quad \beta_4\psi_n = i\psi_p, \quad \beta_4^2=1.$$

Since also

$$\eta_5\psi_p=\psi_p, \quad \eta_5\psi_n=-\psi_n,$$
$$\eta_5\beta_4+\beta_4\eta_5=0,$$

we may write the usual components of the isospin for the

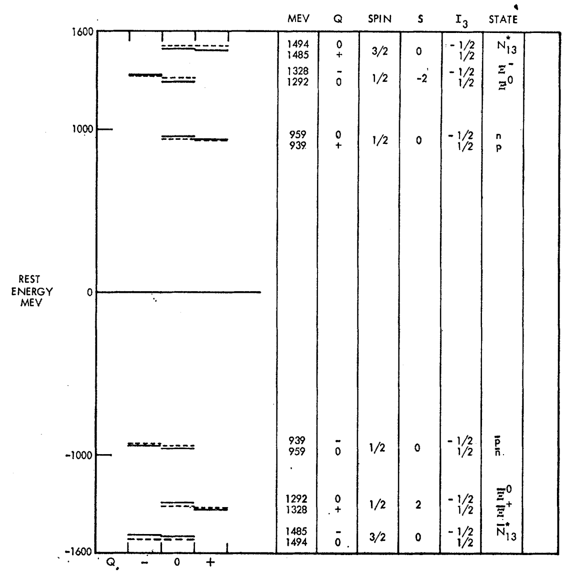

FIG. 2. Calculated properties of rest-energy states of Eq. (1.17) for $m=1328$ MeV, $m_0=166$ MeV. Experimental values for $I=\frac{1}{2}$, $J=\frac{1}{2}$, or $\frac{3}{2}$ are indicated by dashed lines. For negative energy states Q denotes the charge of the antiparticle.

proton and neutron thus: [cf. Eq. (2.9) with $B=1$, $S=0$]

$$I_1=\tfrac{1}{2}i\beta_4\eta_5, \quad I_2=-\tfrac{1}{2}\beta_4, \quad I_3=\tfrac{1}{2}\eta_5.$$

Thus, $I_1+iI_2=\frac{1}{2}i\beta_4(\eta_5-1)$, and hence gives zero when applied to the proton state, and similarly $(I_1-iI_2)\psi_n=0$.

As shown in Table I, it follows from the above analysis that the spins, charges, strangeness, baryon number, and isospin of every one of the states that occur as solutions of Eq. (1.17) are in agreement with experiment. Without any assumption about the values of the parameters m, m_0, the following relation between the masses of these states may be derived directly from the table.

$$2m_{\Xi^-}+m_{\Xi^0}=m_n+2m_{N_{13}^{*0}}. \qquad (2.14)$$

Experimentally, the left-hand side has the value 3958 MeV and the right-hand side the value 3974 MeV, an agreement of better than 0.5% accuracy. For the particular choice $mc^2=1328$ MeV, $m_0c^2=166$ MeV$=\frac{1}{8}mc^2$, the mass levels are as given in the table, and in Fig. 2.

The six mass levels are given by the formula

$$W/mc^2=\tfrac{1}{2}a(S_1-S_2) + \{[1+\tfrac{1}{2}a(S_1+S_2)]^2-a^2(\boldsymbol{\sigma}\cdot\boldsymbol{\lambda})^2\}^{1/2}, \qquad (2.15)$$

where S_1, S_2 are the eigenvalues of $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}$ given in Table II. The spin of a state is $\frac{1}{2}\hbar(\boldsymbol{\sigma}+2\boldsymbol{\Sigma})$ so that for spin-$\frac{1}{2}$ states $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}=0$ or -2, and for spin-$\frac{3}{2}$ states $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}=1$. The allowed eigenvalues of $(\boldsymbol{\sigma}\cdot\boldsymbol{\lambda})^2$ have been given in Ref. 2 II, Table I.

It was pointed out in Ref. 1 III that a generalization of Eq. (1.17) of the form

$$[i\gamma_\mu p_\mu+mc-\tfrac{1}{4}m_0c\gamma_{\mu\nu}(\beta_{\mu\nu}+\beta_{\mu\nu}')]\psi=0,$$

TABLE II. Eigenvalues of $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}$ and $(\boldsymbol{\sigma}\cdot\boldsymbol{\lambda})^2$.

Particle	S_1	S_2	$(\boldsymbol{\sigma}\cdot\boldsymbol{\lambda})^2$
Ξ^-	0	0	0
Ξ^0	0	-2	3
n	-2	0	3
p	-2	-2	4
N_{13}^{*0}	1	1	0
N_{13}^{*+}	1	1	1

where β_μ' are Kemmer operators that commute with the γ_μ and β_μ, includes all of the states discussed above, but also leads to other states of spin $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$. The highest state to which this leads has a spin $\frac{5}{2}$ and mass $mc^2(1+2a)$, i.e., with the choice $m_0c^2=amc^2=166$ MeV used in Table I this spin-$\frac{5}{2}$ state lies 166 MeV above the spin-$\frac{3}{2}$ N_{13}^{*0} state of Table I. There is, in fact, a spin-$\frac{5}{2}$ state N_{15}^* lying 166 MeV above the observed N_{13}^* resonance, although both calculated levels lie 23 MeV below those observed. Similar extension to include states of spin up to $\frac{7}{2}$ leads to a spin-$\frac{7}{2}$ state at $mc^2(1+3a)$, i.e., 166 MeV above the $\frac{5}{2}$ resonance. Experimentally, this energy difference is 217 MeV, although the broad width of the N_{37}^* resonance makes this uncertain. Extension of this analysis to higher spins would lead to a series of states of increasing spin, 166 MeV apart, i.e., a spin-$\frac{9}{2}$ resonance at 1992 MeV, a spin-11/2 resonance at 2158 MeV (see N_1^* at 2190±25 MeV), a spin-13/2 resonance at 2324 MeV (cf. N_3^* at 2360±25 MeV) etc. These states, and the N_{13}^{*0} state considered above have rest energies and spins

$$W=mc^2+(J_\Omega/\hbar)m_0c^2\,,$$
$$J=\tfrac{1}{2}\hbar+J_\Omega\,,$$

and correspond to the classical result (1.6). However, the isospins and strangeness of these states, and of lower ones that occur in this generalization, remain to be calculated.

In addition to electromagnetic interactions, which would occur in this theory from an interaction of the form

$$-(i\epsilon e_0/2c)\bar{\psi}\gamma_\mu(1+\eta_5)\psi A_\mu\,,$$

or, in the case of an explicit magnetic moment,

$$\bar{\psi}\gamma_{\mu\nu}\psi f_{\mu\nu}\,,$$

it is possible to construct invariant interactions of the form

$$ig\bar{\psi}\beta_\mu\psi B_\mu\,, \tag{2.16}$$

$$g'\bar{\psi}\beta_{\mu\nu}\psi B_{\mu\nu}\,,\ \text{etc.}, \tag{2.17}$$

which, if the B_i, B_{ij} are real and the B_4, B_{4i} are imaginary, do not destroy the conservation of s_μ [Eq. (2.1)]. From (2.16), for example, one obtains the wave equation

$$[\gamma_\mu\partial_\mu+(mc/\hbar)-(m_0c/4\hbar)\gamma_{\mu\nu}\beta_{\mu\nu}+ig\beta_\mu B_\mu]\psi=0\,.$$

A vector field B_μ coupled in this manner would cause transitions between charged and neutral states in the same representation (since β_μ anticommutes with η_5), but would not couple states with opposite signs of τ. In the 5×5 representation this interaction would couple the Ξ^- only to the Ξ^0, and in the 10×10 representation it would couple charged and neutral nucleon and N_{13}^* states. No interaction conserving s_μ and derived from the γ_μ and β_μ would couple the Ξ to the other states, since they occur in different representations.

3. BARYONS OF STRANGENESS ±1

It is found that Eq. (1.18) describes only three different mass levels, and that these states are neutral, with zero isospin.

According to Eq. (1.18), the conserved probability current is

$$s_\mu=ic\psi^*\eta_4\gamma_4\beta_\mu\psi\,, \tag{3.1}$$

leading to a probability density which is normalizable as in Eq. (2.2).

$$\tau=\frac{-i}{c}\int s_4dV=\int\psi^*\beta_4\gamma_4\psi dV=\pm1\,, \tag{3.2}$$

since $\eta_4\beta_4=\beta_4$. For a given state, the sign is unambiguously determined. As in the last section, states may be characterized by the eigenvalues ±1 of $\gamma_4\eta_4$. For either eigenvalue, and $m_0'<m'$, it is found that

$$B=\int\psi^*\beta_4\psi dV \tag{3.3}$$

has the same sign as the rest energy, and, if ψ is normalized according to (3.2), $|B|=1$. We may, therefore, interpret B as the baryon number.

In addition to (3.1) the four-vector

$$j_\mu=e_0c\psi^*\eta_4\gamma_4\gamma_5\beta_\mu\psi \tag{3.4}$$

is conserved, and we identify it with the charge-current density. The electric charge is then given in units of e_0 by

$$Q=-i\int\psi^*\eta_4\gamma_4\gamma_5\beta_4\psi dV\,. \tag{3.5}$$

For an eigenstate of $\gamma_4\eta_4$ this is zero, since $\gamma_4\eta_4$ and γ_5 anticommute. The states are therefore characterized by $\gamma_4\eta_4=\pm1$ and by two independently conserved quantities, Q and B, which assume the values 0 and ±1, respectively, the sign being that of the rest energy. If, as usual, one writes

$$Q=\tfrac{1}{2}(B+S)\,,$$

it follows that $S=-B$, $B=\pm1$, for all eigenstates of $\gamma_4\eta_4$.

In the rest system, Eq. (1.7) may be written thus:

$$[-\beta_4 E+\eta+\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}+\rho_1\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}]\psi=0\,, \tag{3.6}$$

where

$$\begin{aligned}&(\beta_{23},\beta_{31},\beta_{12})=i\boldsymbol{\Sigma}\,,\quad (\beta_{14},\beta_{24},\beta_{34})=i\boldsymbol{\lambda}\,,\\ &\gamma_i=\rho_2\boldsymbol{\sigma}\,,\quad \gamma_4=\rho_3\,,\quad \gamma_5=-\rho_1\,,\\ &E=W/m_0'c^2\,,\quad \eta=m'/m_0'.\end{aligned} \tag{3.7}$$

As before, spin-$\frac{3}{2}$ states are characterized by $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}=1$, and spin-$\frac{1}{2}$ states by $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}=0$ or -2.

Equation (3.6) has no solutions in the 1×1 representation of the β_μ. In the 5×5 representation, there is only one solution, of spin $\frac{1}{2}$,

$$\begin{bmatrix}\eta & 1 & -i & 1 & iE\\ 1 & \eta & i & -1 & 0\\ i & -i & \eta & -i & 0\\ 1 & -1 & i & \eta & 0\\ -iE & 0 & 0 & 0 & \eta\end{bmatrix}\begin{bmatrix}(2-\eta)E\\ E\\ iE\\ E\\ i(3-\eta)(\eta+1)\end{bmatrix}=0\,, \tag{3.8}$$

where

$$W=m_0'c^2E=\pm m'c^2[(1+b)(1-3b)/(1-2b)]^{1/2} \tag{3.9}$$

and $b=\eta^{-1}=m_0'/m'$.

This solution occurs both with $\gamma_4\eta_4=+1$, $\tau=+1$, and with $\gamma_4\eta_4=-1$, $\tau=-1$. In either case $B=W/|W|$. Similar solutions occur with the opposite value of J_z.

In the 10×10 representation there are two solutions

$$W=\pm m'c^2[(1+b)(1-3b)(1-4b)/(1-2b)]^{1/2}\,,\quad \text{spin } \tfrac{1}{2}\,, \tag{3.10}$$

and

$$W=\pm m'c^2(1+2b)^{1/2}\,,\quad \text{spin } \tfrac{3}{2}. \tag{3.11}$$

The six spin states, two signs of W, and two signs of $\gamma_4\eta_4$ correspond to the 24 times the energy occurs in the 40×40 matrix, which represents the operator of Eq. (3.6) in this representation. As before, for normalized solutions, $B=W/|W|$.

Explicitly, the spin-$\frac{1}{2}$ state is given by

$$\begin{bmatrix}\eta & -i & 1 & 0 & 1 & i & iE & 0 & 0 & 0\\ i & \eta & -i & -1 & 0 & 1 & 0 & iE & 0 & 0\\ 1 & i & \eta & i & -1 & 0 & 0 & 0 & iE & 0\\ 0 & -1 & -i & \eta & -i & 1 & 0 & 0 & 0 & 0\\ 1 & 0 & -1 & i & \eta & -i & 0 & 0 & 0 & 0\\ -i & 1 & 0 & 1 & i & \eta & 0 & 0 & 0 & 0\\ -iE & 0 & 0 & 0 & 0 & 0 & \eta & -i & 1 & 1\\ 0 & -iE & 0 & 0 & 0 & 0 & i & \eta & -i & -i\\ 0 & 0 & -iE & 0 & 0 & 0 & 1 & i & \eta & -1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & i & -1 & \eta\end{bmatrix}\begin{bmatrix}\psi\\ -i\psi\\ -\psi\\ \phi\\ -i\phi\\ -\phi\\ \chi\\ -i\chi\\ -\chi\\ \lambda\end{bmatrix}=0\,, \tag{3.12}$$

so that

$$\begin{aligned}(\eta-2)\psi-2i\phi+iE\chi&=0\,,\\ 2i\psi+(\eta-2)\phi&=0\,,\\ -iE\psi+(\eta-2)\chi+\lambda&=0\,,\\ 3\chi+\eta\lambda&=0\,,\end{aligned} \tag{3.13}$$

which leads to the eigenvalues (3.10). Spin-$\frac{3}{2}$ states ($J_z=\frac{3}{2}, -\frac{1}{2}$) are characterized by

$$\psi_{3/2,3/2}=\begin{vmatrix}\psi\\ i\psi\\ 0\\ \phi\\ i\phi\\ 0\\ \chi\\ i\chi\\ 0\\ 0\end{vmatrix}\qquad \psi_{3/2,-1/2}=\begin{vmatrix}\psi\\ -i\psi\\ 2\psi\\ \phi\\ -i\phi\\ 2\phi\\ \chi\\ -i\chi\\ 2\chi\\ 0\end{vmatrix}$$

with

$$\begin{aligned}(\eta+1)\psi+i\phi+iE\chi&=0\,,\\ -i\psi+(\eta+1)\phi&=0\,,\\ -iE\psi+(\eta+1)\chi&=0\,,\end{aligned} \tag{3.14}$$

which yields the eigenvalues (3.11).

The three lowest known states[4] with $B=1$, $S=-1$, $I=0$, $Q=0$ are Λ^0 (1115-MeV, spin $\frac{1}{2}$) $Y_0{}^*$ (1405 MeV, spin ?) $Y_{03}{}^*$ (1520 MeV, spin $\frac{3}{2}$). If the spin of the $Y_0{}^*$ is $\frac{1}{2}$, the masses and spins of these states are adequately described by the choice $m'c^2=1428$ MeV, $m_0'c^2=132$ MeV, which fit the Λ^0 and $Y_0{}^*$, and lead to a spin-$\frac{3}{2}$ state at 1554 MeV. A prediction of this theory, then, is that the $Y_0{}^*$, which occurs in the 5×5 representation [Eq. (3.9)] has a spin of $\frac{1}{2}$.

The generalization of Eq. (1.7) to include states of spin up to $\frac{5}{2}$ is

$$[i\beta_\mu p_\mu+m'c-\tfrac{1}{4}m_0'c\beta_{\mu\nu}(\gamma_{\mu\nu}+4\beta_{\mu\nu}')]\psi=0. \tag{3.15}$$

The conserved probability current is now

$$s_\mu=-ic\psi^*\eta_4\gamma_4\eta_4'\beta_\mu\psi\,, \tag{3.16}$$

leading to a probability density which is normalizable to ±1 as before.

$$\tau=-\frac{i}{c}\int s_4 dV=-\int\psi^*\gamma_4\eta_4'\beta_4\psi dV=\pm1. \tag{3.17}$$

States are now characterized by the eigenvalues ±1 of $\gamma_4\eta_4\eta_4'$, and by the eigenvalues ±1 of η_5'. We now find that

$$j_\mu = ie_0 c\psi^* \eta_4 \gamma_4 \eta_4' \{i\gamma_5 + \tfrac{1}{2}(1-\eta_5')\}\beta_\mu \psi$$

is conserved, so that

$$Q = \tfrac{1}{2}(B+S) + I_3,$$

where

$$B+S = 2i\int \psi^* \eta_4 \gamma_4 \eta_4' \gamma_5 \beta_4 \psi dV, \tag{3.18}$$

$$I_3 = \tfrac{1}{2}\int \psi^* \eta_4 \gamma_4 \eta_4' (1-\eta_5')\beta_4 \psi dV. \tag{3.19}$$

For $\beta_\mu'=0$ (so that $\eta_4'=-1$, $\eta_5'=+1$) Eq. (3.18) reduces to Eq. (3.5), $I_3=0$ and the solutions of Eq. (3.15) reduce to those already discussed. However, for wave functions normalized according to (3.17), we have in general $I_3=\pm 1$ or 0, with $B+S=0$ as before. Thus, Eq. (3.15) describes states of spin $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and of isospin 0 or 1. Detailed properties of these solutions have not yet been investigated, but the highest energy level is easily seen to be characterized by $\Sigma^2 = \Sigma'^2 = 2$, $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma} = \boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}' = \boldsymbol{\Sigma}\cdot\boldsymbol{\Sigma}' = 1$, and to have a spin $\frac{5}{2}$. To first order in b, its rest energy is $m'c^2(1+2b) = 1690$ MeV for the same values of m' and b as before. The next highest observed level Y_0^{**} is, in fact, at 1680 MeV, but its spin is unknown at present. Since Eq. (3.15) yields rest-energy levels not yet investigated, this identification is not certain. If the Σ particle appears at all in this theory, the simplest equation which could describe it is Eq. (3.15).

4. *K* PARTICLES

Bosons of strangeness ± 1 are described in this theory by Eq. (1.19), which in the rest system may be written thus:

$$E\psi = \{\rho_3 + \tfrac{1}{2}a(\rho_3 + i\rho_2\rho_1')\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'\}\psi, \tag{4.1}$$

where we have written

$$W = m''c^2E, \quad m_0'' = am'',$$

$$\gamma_i = \rho_2\boldsymbol{\sigma}, \quad \gamma_i' = \rho_2'\boldsymbol{\sigma}'; \quad i=1,2,3.$$

$$\gamma_4 = \rho_3, \quad \gamma_5 = -\rho_1, \quad \gamma_4' = \rho_3', \quad \gamma_5' = -\rho_1'.$$

On squaring Eq. (4.1) we obtain

$$E^2\psi = (1 + a\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')\psi,$$

the spin being $\frac{1}{2}\hbar(\boldsymbol{\sigma}+\boldsymbol{\sigma}')$. Hence,

$$\begin{aligned} W &= \pm m''c^2(1+a)^{1/2} \quad (\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'=1, \quad \text{spin } 1), \\ W &= \pm m''c^2(1-3a)^{1/2} \quad (\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'=-3, \quad \text{spin } 0). \end{aligned} \tag{4.2}$$

We postulate that the charge-current density associated with Eq. (1.19) is

$$j_\mu = ice_0\epsilon\psi^*\gamma_4\gamma_4'\gamma_\mu\psi, \tag{4.3}$$

which is conserved. The charge in units of e_0 is then

$$Q = \epsilon\int \psi^*\gamma_4'\psi dV. \tag{4.4}$$

The states may be characterized by the eigenvalues ± 1 of $\gamma_4\gamma_4'$ or by the eigenvalues ± 1 of γ_5'. We define

$$S = \int \psi^*\gamma_4\psi dV = \pm 1, \tag{4.5}$$

which is found to be normalizable to $+1$ for positive energy states and to -1 for negative energy states. We choose ϵ in Eq. (4.4) so that, if ψ is an eigenstate of $\gamma_4\gamma_4'$, $\epsilon\gamma_4\gamma_4' = +1$. This is necessary in order to ensure that the electric charge associated with a given state has the same sign in the two subspaces $\gamma_4\gamma_4' = +1$ and $\gamma_4\gamma_4' = -1$ in which this state occurs. Hence, for either eigenvalue of $\gamma_4\gamma_4'$ we have $Q=S$. If, however, ψ is an eigenstate of γ_5', we have $S=\pm 1$ as before, since γ_4, γ_5' commute, but $Q=0$, since γ_4', γ_5' anticommute. Thus, if

$$Q = \tfrac{1}{2}S + I_3,$$

we have

$$\begin{aligned} I_3 &= \tfrac{1}{2}S(\epsilon\gamma_4\gamma_4'\psi = +\psi) \\ &= -\tfrac{1}{2}S(\gamma_5'\psi = \pm\psi). \end{aligned}$$

It is not difficult to choose the two parameters $m''c^2 = 808$ MeV, $m_0''c^2 = 166$ MeV to fit the masses of the K and K^* mesons! However, it may be significant that this value of m_0'', chosen to give the best fit to these experimental masses, is indistinguishable from the value of m_0, used in the other extended Dirac equation (1.6). In addition, the strangeness, spins and conjugate doublet structure of the K, K^* particles are seen to emerge from this analysis. The states $S=1$, $I_3=\frac{1}{2}$, $Q=1$, and the states $S=-1$, $I_3=-\frac{1}{2}$, $Q=-1$ have also the correct spins and masses for the K^+, K_1^{*+}, and K^-, K_1^{*-} states, respectively, and the states $S=1$, $I_3=-\frac{1}{2}$, and $S=-1$, $I_3=\frac{1}{2}$ correctly describe the $\bar{K}^0$, $\bar{K}_1^{*0}$. In the classical field theory, the twelve K^* states of spin 1 ($J_z=\hbar, 0, -\hbar$) and the four K states of spin 0 correspond to the sixteen components of ψ in Eq. (4.1).

5. BOSONS OF ZERO STRANGENESS

We complete this analysis by examining the eigenfunctions and rest-energy eigenvalues of Eq. (1.20). With the notation

$$(\beta_{23}, \beta_{31}, \beta_{12}) = i\boldsymbol{\Sigma}, \quad (\beta_{14}, \beta_{24}, \beta_{34}) = i\boldsymbol{\lambda}$$

and similarly for $\beta_{\mu\nu}'$, the equation in the rest system may be written

$$[\beta_4E - \eta - (\boldsymbol{\Sigma}\cdot\boldsymbol{\Sigma}' + \boldsymbol{\lambda}\cdot\boldsymbol{\lambda}')]\psi = 0, \tag{5.1}$$

where

$$W = 2m_0'''c^2E, \quad \eta = m'''/2m_0'''.$$

The conserved probability current is now

$$s_\mu = ic\psi^* \eta_4\eta_4'\beta_\mu\psi\,,$$

and the electric charge-current density is

$$j_\mu = \tfrac{1}{2} i e_0 c\psi^* \eta_4\eta_4'(1+\eta_5')\beta_\mu\psi\,, \tag{5.2}$$

with

$$\eta_\mu = 2\beta_\mu^2 - 1\,,\quad \eta_\mu' = 2\beta_\mu'^2 - 1\,,\quad \eta_5' = \eta_1'\eta_2'\eta_3'\eta_4'\,.$$

The electric charge in units of e_0 is then

$$Q = \frac{1}{2}\int \psi^* \eta_4'(1+\eta_5')\beta_4\psi dV\,. \tag{5.3}$$

The solutions are simultaneously characterized by the eigenvalues ± 1 of the commuting operators $\eta_4\eta_4'$ and η_5'. As in Sec. 2, charged states correspond to $\eta_5' = +1$ and neutral states to $\eta_5' = -1$.

In each representation, each solution occurs with both signs of the energy and with the same eigenvalues of $\eta_4\eta_4'$ and of η_5'. In what follows, we restrict our attention to the positive energy solutions, normalized according to

$$\int \psi^* \beta_4 \psi dV = 1\,, \tag{5.4}$$

so that $\eta_4\eta_4' = +1$, $\eta_5' = +1$ denotes states of charge $+e_0$, and $\eta_4\eta_4' = -1$, $\eta_5' = +1$ states of charge $-e_0$.

We may define an operator β_5', such that, as with the other β',

$$\beta_5'\eta_5' = \eta_5'\beta_5' = \beta_5' \quad \eta_5' = 2\beta_5'^2 - 1\,,\ \eta_4'\beta_5' + \beta_5'\eta_4' = 0\,. \tag{5.5}$$

In the 10×10 representation, as defined by Kemmer,

$$\beta_5' = \left|\begin{array}{cccccccccc} \cdot & \cdot & \cdot & 1 & & & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & & 1 & & & & & \\ \cdot & & & & & 1 & & & & \\ 1 & & & & & & & & & \\ & 1 & & & & & & & & \\ & & 1 & & & & & & & \\ \cdot & & & & & & & & & \\ \cdot & & & & & & & & & \\ \cdot & & & & & & & & & \\ \cdot & & & & & & & & & \end{array}\right|\,.$$

We find that, for any two states ψ_+, ψ_- which occur in the same representation with the same rest-energy eigenvalue, but with opposite signs of the charge,

$$\beta_5'\psi_+ = \psi_-\,,\quad \beta_5'\psi_- = \psi_+\,. \tag{5.6}$$

Such states are always accompanied by a neutral state $\psi_0(\eta_5' = -1,\ \eta_4\eta_4' = +1)$ such that, if small components are neglected as in Sec. 2,

$$\beta_4'\psi_+ = i\psi_0\,,\quad \beta_4'\psi_0 = -i\psi_+\,. \tag{5.7}$$

Hence

$$\beta_4'\beta_5'\psi_- = i\psi_0\,,\quad \beta_5'\beta_4'\psi_0 = -i\psi_-\,. \tag{5.8}$$

The masses of the neutral and charged states which correspond in this manner always differ by terms of order $(m_0'''/m''')^2$.

We now define the operators

$$\begin{aligned} I_1 &= (i/\sqrt{2})[\eta_5'\beta_4' - \beta_4'\beta_5' + \beta_5'\beta_4']\,,\\ I_2 &= (1/\sqrt{2})[\beta_4' - \beta_4'\beta_5' - \beta_5'\beta_4']\,,\\ I_3 &= \tfrac{1}{2}\eta_4'(1+\eta_5') = \eta_4'\beta_5'^2\,. \end{aligned} \tag{5.9}$$

Using (5.5) and other properties of the β algebra it may be shown that

$$\mathbf{I}\times\mathbf{I} = i\mathbf{I}\,, \tag{5.10}$$

i.e., that the operators (5.9) satisfy the commutation relations for angular momentum. However, they are not related to the spin; for wave functions normalized according to (5.4) the charge Q [Eq. (5.3)] is given by

$$Q = I_3\,, \tag{5.11}$$

where I_3 has the eigenvalues 0, ± 1. To further verify that $\mathbf{I}$ is indeed the isospin we note that, in the space of the wave functions

$$\begin{bmatrix} \psi_+ \\ \psi_0 \\ \psi_- \end{bmatrix}$$

as related by Eqs. (5.6), (5.7), and (5.8), we have

$$\beta_4' = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \beta_5' = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\eta_4' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad \eta_5' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that, from (5.9),

$$I_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix};\quad I_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix};$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

the familiar matrices which represent isospin unity.

Quantum numbers which characterize the eigenstates of Eq. (5.1) in the rest system are shown in Table III. States of isospin unity, the components of which are linked together by Eqs. (5.7) and (5.8), occur only in the 10×10 representation of the β_μ' [three times (with spin 0, 1, 2) in the 10×10 representation of the β_μ, and once (with spin 1) in the 5×5 representation of the β_μ]. In each case the neutral component is slightly heavier than the charged components. Each neutral component of a state with $I=1$ is accompanied in another representation by a state $I=0$ with the same mass and spin but with the opposite sign of $\eta_4\eta_4'$. At least in the case of

TABLE III. Eigenstates of Eq. (5.1) for bosons with $S=0$. The bracketed levels are components of isospin triplets according to Eqs. (5.7) and (5.8). The parameters m''', $m_0'''=am'''$ have been chosen as 1041 and 106 MeV, respectively, i.e., $a \doteqdot 0.1$.

Representations β_μ	β_μ'	η_5'	$\eta_4\eta_4'$	J	Q	$(W/m'''c^2)^2$	W (MeV)	Possible identification	W (MeV) experiment[a]
	1×1	1	−1	0	—	1	1041		
	5×5	1	1	0	+	1	1041		
		−1	−1	1	0	$1-4a^2$	1019	ϕ	1019
		−1	1	0	0	$(1-6a)(1+2a)(1-4a)^{-1}$	926	x	922
	10×10	1	1	1	+	$(1-4a)(1+2a)(1-2a)^{-1}$	986 }		
		−1	1	1	0	$1-4a^2$	1019 }	x	1000
		1	−1	1	—	$(1-4a)(1+2a)(1-2a)^{-1}$	986 }		
		−1	−1	0	0	$(1-6a)(1+2a)(1-4a)^{-1}$	926	x	922
10×10	1×1	1	−1	1	—	1	1041		
	5×5	−1	−1	2	0	$(1+2a)^2$	1253	f	1253
		1	1	1	+	1	1041		
		−1	1	1	0	$(1+2a)^2(1-4a)$	965	x	958
		−1	−1	1	0	$(1-4a)(1-4a^2)$	785	ω	783
		−1	−1	0	0	$(1-4a)(1-6a)(1+2a)$	548	η	548
	10×10	1	1	2	+	$1+4a$	1235 }		
		−1	1	2	0	$(1+2a)^2$	1253 }	A,B	1220
		1	−1	2	—	$1+4a$	1235 }		
		−1	−1	1	0	$(1+2a)^2(1-4a)$	965	x	958
		1	1	1	+	$(1-4a)^2(1+2a)(1-2a)^{-1}$	759 }		757
		−1	1	1	0	$(1-4a)(1-4a^2)$	785 }	ρ	754, 770
		1	−1	1	—	$(1-4a)^2(1+2a)(1-2a)^{-1}$	759 }		
		1	1	0	+	$1-8a$	449 }		
		−1	1	0	0	$(1-4a)(1-6a)(1+2a)$	549 }		
		1	−1	0	—	$1-8a$	449 }		

[a] Data from M. Roos, Phys. Letters 8, 1 (1964) and recent experiments.

(ρ,ω), and with less certainty in the cases (ζ,η), (χ_1,ϕ) and (B,f) there is some evidence to support this conclusion. In Table III and Fig. 3, the bare masses predicted by the theory for the various states are shown for the choice $m=1040$ MeV, $a\equiv m_0'''/m'''=0.1$. The agreement with observation is much better than one would expect, not only with respect to the structure of the states, the various spin values and the automatic grouping of most of the levels into states with $I=0$ and with $I=1$, but also with respect to the mass levels themselves.

A state of $J=2$, $S=0$, $I=1$ is predicted around 1.24 BeV (the B $\pi-\omega$ resonance ?) in addition to the observed $I=0$ f state at this energy. A $J=0$, $S=0$, $I=1$ state also appears around 450 MeV, together with $I=0$ states at 965 MeV (spin 1) and 926 MeV (spin 0). (ϕ_3 ?) Each of these latter resonances exists in two states with $\eta_4\eta_4'=\pm1$ according to this picture. There also exists in this theory charged resonances of spin 0 and 1 around 1 BeV, which do not possess neutral counterparts in the sense of Eqs. (5.7) and (5.8).

TABLE IV. Values of S, I, J allowed by Eq. (1.11), and values of m, m_0 used in the text.

Equation number	$\epsilon_{\mu\nu}$	$\lambda_{\mu\nu}$	S	I	J	mc^2 (MeV)	m_0c^2 (MeV)
(1.17)	$\frac{1}{4}\gamma_{\mu\nu}$	$\beta_{\mu\nu}$	$0,\pm2$	$\frac{1}{2}$	$\frac{1}{2},\frac{3}{2}$	1328	166
(1.18)	$\beta_{\mu\nu}$	$\frac{1}{4}\gamma_{\mu\nu}$	±1	0	$\frac{1}{2},\frac{3}{2}$	1428	132
(1.19)	$\frac{1}{4}\gamma_{\mu\nu}$	$\frac{1}{4}\gamma_{\mu\nu}'$	±1	$\frac{1}{2}$	0, 1	808	166
(1.20)	$\beta_{\mu\nu}$	$\beta_{\mu\nu}'$	0	0, 1	0, 1, 2	1041	106

To terms of first order in $a=m_0'''/m'''$ we note from Table III that

$$m_\omega \doteqdot m_\rho, \quad m_f+m_\omega \doteqdot 2m_\phi, \quad m_\phi+m_\eta \doteqdot 2m_\omega.$$

Experimentally, the last two equations are accurate to better than 0.3%, since these states form part of the equally spaced $I=0$ series f, ϕ, ω, η, ω_{ABC}.

6. SUMMARY

The values of S, I, J to which the four equations (1.17), (1.18), (1.19), and (1.20) give rise are summarized in Table IV. In each case $S+2I+2J$ is an even integer, a result which is valid for every particle and resonance state so far observed. The values of m and m_0 chosen in the text to fit the experimental mass values are also listed in Table IV. It is a weakness of this theory that these values differ from one equation to the next.

Each equation gives rise to two independently conserved currents, and the particular linear combination of these which is identified with the electric current is arbitrary. The electric charge is related to B, S, and I_3 in the usual way. The baryon number B for the fermion states [Eqs. (2.5) and (3.3)] is always conserved for eigenstates of $\gamma_4\eta_4$ and is equal to $+1$ for positive-energy states and -1 for negative-energy states. The strangeness and isospin are separately conserved only in the

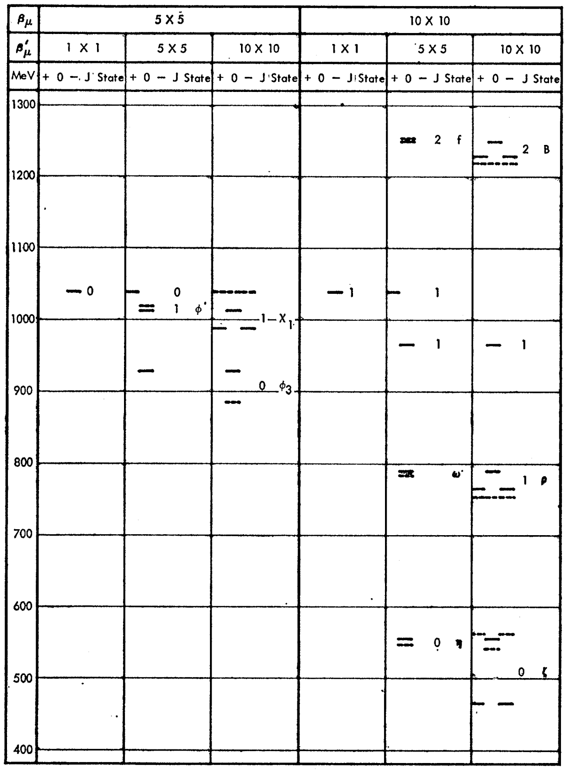

FIG. 3. Mass, spin, and charge eigenstates in different representations of the generalized Kemmer equation for bosons of zero strangeness (full lines) and experimental values (dashed lines). The π and ABC states do not appear in this analysis. Data from Ref. 4.

limit in which mass differences are neglected, although $\frac{1}{2}S+I_3$ is exactly conserved. The values of S, I_3, J, B, which are derived for the various states described by the four equations are all correct with the possible exceptions of Y_0^* (spin $\frac{1}{2}$) and the $\pi-\omega$ resonance ($J=2$), and χ_1 ($J=1$). Extensions of these equations to describe higher values of S, I, and J have so far led to the correct mass and spin for the N_{15}^* resonance, and to the correct mass for the Y_0^{**}, with a prediction that the spin of this state is $\frac{5}{2}$.

The spins of the individual states appearing in this analysis are given in Eq. (1.15) as the sum of two operators, one of which is the spin operator of the unmodified Dirac or Kemmer equation. Thus, the states described by Eq. (1.17) and analyzed in Sec. 2 may be denoted by $(\frac{1}{2},1)$ or $(\frac{1}{2},0)$ the first number referring to the usual spin, and the second to the additional spin component which is the basis of this theory. We notice that particles of even strangeness are described when this additional spin component is integral, and particles of odd strangeness when it is half-integral. While it is generally required in an interaction that the total spin should be conserved, reactions in which the sum of the components of each spin should be separately conserved would allow particles of type $(\frac{1}{2},1)$ or $(\frac{1}{2},0)$ to transform into states of type $(1,\frac{1}{2})$ or $(0,\frac{1}{2})$ only by the emission or absorption of states of type $(\frac{1}{2},\frac{1}{2})$. Similarly, states of type $(\frac{1}{2},1)$ etc., could then transform into themselves, or other states of the same type, only by the emission or absorption of states of type (1,1) etc., as described by Eq. (1.20). When supplemented by the requirements of charge and baryon number conservation, this approximate rule would lead to the conservation of strangeness in the production of the bosons described by Eqs. (1.19) and (1.20) by interactions between the fermions described by Eqs. (1.17) and (1.18).

There is ample evidence in the literature that it is relatively easy to invent schemes which yield the correct values for the quantum numbers and mass values for a number of elementary particle states. The theory described in this paper differs from most of these attempts in that it is related through the correspondence principle to the classical equations of motion of a spinning point-particle, equations which one has no hesitation in using to describe the motion of an elementary particle under conditions in which quantum effects may be neglected. In addition, the existence of quantum numbers which may possibly be identified with isospin and strangeness is *deduced* from the field equations, these dynamical variables assuming their correct integral or half-integral values in the limit in which mass differences are neglected. However, it has yet to be shown that S and I as they emerge from these equations are in fact conserved when interactions which are known experimentally to conserve them are introduced into the field equations, or that these encouraging consequences of a classical field theory will stand the test of second quantization.