Symmetry Energy and the Isotopic Spin Dependence of the Single-Particle Potential in Nuclear Matter

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Nuclear matter with a given neutron excess is treated within the frame of the K-matrix theory. General expressions for the symmetry energy $\epsilon_{\rm sym}$ and the single-particle potential are obtained with the help of the reaction matrix which depends on two different Fermi momenta for neutrons and protons. In particular, an expression for the isotopic spin-dependent part U_1 of the single-particle potential is obtained, and then specialized for single particles at the Fermi surface. With suitable approximations numerical values of $\epsilon_{\rm sym}$ and U_1 at the Fermi surface are obtained with the help of the Bruckner-Gammel solution for nuclear matter. The results are: $\epsilon_{\rm sym}=64$ MeV, $U_1(k_F)=126$ MeV.

I. INTRODUCTION

In the case of nuclear matter with a given neutron excess, characterized by the parameter $\alpha=(N-Z)/A$, the energy per nucleon can be written in the following form:

$$E/A=-\epsilon_{\rm vol}+\tfrac{1}{2}\epsilon_{\rm sym}\alpha^2. \quad (1)$$

There have been many attempts to calculate the volume energy $\epsilon_{\rm vol}$ and the symmetry energy $\epsilon_{\rm sym}$ as well as the equilibrium density of nuclear matter, starting with the nucleon forces determined in the free nucleon-scattering experiments. Because of the singular character of the nucleon-nucleon forces, the problem could not have been solved before the K-matrix theory was formulated. The most complete calculation[1] based on the K-matrix theory produced a remarkable agreement between the calculated and empirically determined values of the parameters of nuclear matter.

This agreement includes also the symmetry energy, although the situation here is not quite simple, as some of the empirical estimates of this quantity give different results. One can only say that the value of $\epsilon_{\rm sym}$ obtained in BG in the most realistic case of the Gammel-Thaler nuclear forces[2,3] is well within the range of the different empirical estimates of this parameter.

The symmetry energy has been calculated in BG in an approximate way, namely, with the help of the reaction matrix K determined in the case of $N=Z=\frac{1}{2}A$. This means the intrinsic dependence of K on the neutron excess has not been taken into account. However, in a similar calculation[4] of the spin symmetry energy of liquid He^3 at low temperature, it was found that the "rearrangement" term, i.e., the term resulting from the dependence of the K matrix on the spin excess, was very big and essential to achieving the spin stability of the system.

In the present paper the rearrangement contribution to the symmetry energy of nuclear matter is calculated. The general formulas of the symmetry energy are derived in Sec. IIA.

The symmetry energy is closely related to the isotopic spin dependence of the single-particle potential. The interest in the isotopic spin dependence of the single-particle potential has increased considerably since the observation of the excitation of the analog state in the (p,n) reaction by Anderson *et al.*[5]

In Sec. IIB of the present paper, general formulas for the isotopic spin-dependent part of the single-particle potential $U_1(m)$ are derived for an arbitrary value of the single-particle momentum m.

In Sec. IIC, the general formulas for the single-particle potential are used to get the value of the single-particle potential for particles at the top of the Fermi sea. The expressions thus obtained show the connection between $U_1(k_F)$ and the symmetry energy.

Similar approximations to those which have been used in Ref. 4 are applied in Sec. III to calculate the rearrangement contribution to the symmetry energy and the isotopic spin-dependent part of the single-particle potential.

In Sec. IV numerical results are presented, based on the BG solution of the nuclear-matter problem. The results show that the rearrangement terms increase the symmetry energy by an appreciable amount, which, however, can be considered to be a correction only when compared to the other terms calculated in BG. At the same time the rearrangement terms produce the most important part of U_1. Only by including the rearrangement terms in the calculation of U_1 does one get a satisfactory agreement with the experimental estimates.

II. GENERAL FORMULAS

A. The Symmetry Energy

We consider here the case of nuclear matter with a given neutron excess. On the other hand, we assume that there is no spin excess, i.e., every momentum state

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[1] K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958); hereafter referred to as BG.

[2] J. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957).

[3] J. Gammel and R. M. Thaler, Phys. Rev. **107**, 1337 (1957).

[4] K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1040 (1958); **121**, 1863 (E) (1961).

[5] J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. **126**, 2170 (1962).

is occupied by two neutrons with spin up and down, and/or by two protons with spin up and down, or otherwise the momentum state is empty.

To calculate the symmetry energy $\epsilon_{\rm sym}$, we have to expand the total energy $E=E(\alpha)$ of nuclear matter in power series of α, neglecting terms of higher order than α^2.

Let us divide the total energy E into the kinetic and potential part.

$$E(\alpha)=E_{\rm kin}(\alpha)+E_{\rm pot}(\alpha)\,. \tag{2}$$

The expansion of $E_{\rm kin}$ gives the well-known expression

$$E_{\rm kin}(\alpha)=E_{\rm kin}(0)+\tfrac{1}{2}\epsilon_{\rm sym}{}^{\rm kin}\alpha^2 A\,, \tag{4}$$

where

$$\epsilon_{\rm sym}{}^{\rm kin}=\tfrac{2}{3}(\hbar k_F)^2/2M\,. \tag{4}$$

The potential energy is in the K-matrix theory given by the expression

$$E_{\rm pot}(\alpha)=\tfrac{1}{2}\sum_{s_3}\sum_{t_3}\sum_{\rm m}{}^{(t_3)}V(\mathbf{m}s_3t_3)\,, \tag{5}$$

where

$$V(\mathbf{m}s_3t_3)=\sum_{s_3'}\sum_{t_3'}\sum_{\mathbf{m}'}{}^{(t_3')}\times(\mathbf{m}s_3t_3\mathbf{m}'s_3't_3'\,|\,K(k_F{}^+k_F{}^-)\,|\,\mathbf{m}s_3t_3\mathbf{m}'s_3't_3')-\text{exchange}\,. \tag{6}$$

By t_3 we denote the third component of the isotopic spin of the nucleons. We use the convention $t_3=\frac{1}{2}(-\frac{1}{2})$ for neutrons (protons). The third component of the ordinary spin of the nucleons is denoted by s_3. By $\sum^{(t_3)}$ we denote the sum over all momenta states occupied by nucleons with the third component of the isotopic spin equal to t_3. That means we have

$$\sum_{\rm m}{}^{(t_3)}=\sum_{\rm m}{}^{\pm}\,, \tag{7}$$

where the plus (minus) sign is to be used for $t_3=\frac{1}{2}(-\frac{1}{2})$ and means that $|\mathbf{m}|\leq k_F{}^+(k_F{}^-)$. By $k_F{}^+(k_F{}^-)$ we denote the Fermi momentum of neutrons (protons)

$$k_F{}^{\pm}=k_F(1\pm\alpha)^{1/3}\,, \tag{8}$$

where

$$\tfrac{2}{3}\pi^{-2}k_F{}^3=A/\Omega=(\tfrac{4}{3}\pi r_0{}^3)^{-1}\,. \tag{9}$$

In Eq. (6) we have indicated the dependence of the K matrix on the two Fermi momenta by writing explicitly $K=K(k_F{}^+k_F{}^-)$. The first argument of K indicates the Fermi momentum of neutrons, and the second one the Fermi momentum of protons. Thus, e.g., $K(k_F{}^-k_F{}^+)$ is the K matrix for neutron excess "$-\alpha$."

Now we must change the spin and isotopic spin wave functions in Eq. (6) with the help of the known relations

$$|s_3=\pm\tfrac{1}{2}\;s_3'=\pm\tfrac{1}{2})=|s=1\;m_s=\pm1)\,,$$
$$|s_3=\pm\tfrac{1}{2}\;s_3'=\mp\tfrac{1}{2})=2^{-1/2}\{|s=1\;m_s=0)\pm|s=0\;m_s=0)\}\,, \tag{10a}$$
$$|t_3=\pm\tfrac{1}{2}\;t_3'=\pm\tfrac{1}{2})=|T=1\;T_3=\pm1)\,,$$
$$|t_3=\pm\tfrac{1}{2}\;t_3'=\mp\tfrac{1}{2})=2^{-1/2}\{|T=1\;T_3=0)\pm|T=0\;T_3=0)\}\,. \tag{10b}$$

Let us notice that K does not depend on the sign of m_s and is diagonal in m_s for zero-angle scattering. Furthermore, K is diagonal in s, T, and T_3. The exchange term contributes a factor 2 in the T, s representation.

Because of the charge independence of the intersection, we obviously have

$$K(sm_sTT_3;\,k_F{}^+k_F{}^-)=K(sm_sT-T_3;\,k_F{}^-k_F{}^+)\,. \tag{11}$$

With the help of Eqs. (10) and (11) we get for V, Eq. (6), the expressions

$$V(m+)=\sum_{\mathbf{m}'}{}^{+}(\mathbf{m}\mathbf{m}'\,|\,K_1(k_F{}^+k_F{}^-)\,|\,\mathbf{m}\mathbf{m}')+\sum_{\mathbf{m}'}{}^{-}(\mathbf{m}\mathbf{m}'\,|\,K_0(k_F{}^+k_F{}^-)\,|\,\mathbf{m}\mathbf{m}')\,, \tag{12a}$$
$$V(m-)=\sum_{\mathbf{m}}{}^{-}(\mathbf{m}\mathbf{m}'\,|\,K_1(k_F{}^-k_F{}^+)\,|\,\mathbf{m}\mathbf{m}')+\sum_{\mathbf{m}'}{}^{+}(\mathbf{m}\mathbf{m}'\,|\,K_0(k_F{}^-k_F{}^+)\,|\,\mathbf{m}\mathbf{m}')\,, \tag{12b}$$

where $V(m\pm)=V(\mathbf{m}s_3t_3=\pm\frac{1}{2})$. Notice that V does not depend either on s_3 or on the direction of $\mathbf{m}$. In Eq. (12) the following notation has been introduced:

$$K_1(k_F{}^+k_F{}^-)=\sum_{sm_sT}TK(sm_sTT_3=1;\,k_F{}^+k_F{}^-)\,,\qquad K_0(k_F{}^+k_F{}^-)=\tfrac{1}{2}\sum_{sm_sT}K(sm_sTT_3=0;\,k_F{}^+k_F{}^-)\,. \tag{13}$$

Let us notice that Eq. (11) implies that

$$K_0(k_F{}^+k_F{}^-)=K_0(k_F{}^-k_F{}^+)\,. \tag{14}$$

Inserting V of Eq. (12) into Eq. (5) we get for $E_{\rm pot}$

$$E_{\rm pot}(\alpha)=\sum_{\rm m}{}^{+}\sum_{\mathbf{m}'}{}^{+}(\mathbf{m}\mathbf{m}'\,|\,K_1(k_F{}^+k_F{}^-)\,|\,\mathbf{m}\mathbf{m}')+\sum_{\rm m}{}^{-}\sum_{\mathbf{m}'}{}^{-}(\mathbf{m}\mathbf{m}'\,|\,K_1(k_F{}^-k_F{}^+)\,|\,\mathbf{m}\mathbf{m}')+2\sum_{\rm m}{}^{+}\sum_{\mathbf{m}'}{}^{-}(\mathbf{m}\mathbf{m}'\,|\,K_0(k_F{}^+k_F{}^-)\,|\,\mathbf{m}\mathbf{m}')\,. \tag{15}$$

Now we have to calculate the second derivative of $E_{\rm pot}(\alpha)$ with respect to α, for $\alpha=0$, taking into account the dependence of $k_F{}^{\pm}$ on α as given by Eq. (8). (The first derivative vanishes.)

The second derivative consists of two parts.

The first part comes from the dependence on α of the limits of the sums in Eq. (15). Its contribution to $\epsilon_{\rm sym}{}^{\rm pot}$, which we denote by $\epsilon_{\rm sym}{}^{(0)\rm pot}$, can be easily calculated, and the result is

$$\epsilon_{\rm sym}{}^{(0)\rm pot}=\tfrac{1}{3}k_F[\partial V_0(m)/\partial m]_{m=k_F}+2S_{k_F}\,, \tag{16}$$

where $V_0(m)=V(mt_3)$ in the case $N=Z=A/2$ and does not depend on t_3. The general definition of S_k is

$$S_k=\tfrac{1}{8}A(4\pi)^{-1}\int d\hat{k}_F(\mathbf{k}\mathbf{k}_F\,|\,\textstyle\sum_T(T-\frac{1}{2})K(T)\,|\,\mathbf{k}\mathbf{k}_F)\,, \tag{17}$$

where

$$K(T)=\sum_{sm_s}K(sm_sT;\,k_F)\,. \tag{18}$$

Let us notice that $K(T)$ is defined for $N=Z=A/2$ and does not depend on T_3.

The second part of $[\partial^2E_{\rm pot}(\alpha)/\partial\alpha^2]_{\alpha=0}$ consists of terms which result from the intrinsic dependence of K on $k_F{}^+$ and $k_F{}^-$. The contribution of this second part to $\epsilon_{\rm sym}{}^{\rm pot}$ we shall denote by $\Delta\epsilon_{\rm sym}$ and shall call it the

rearrangement part of the symmetry energy. This rearrangement part is given by the following expression:

$$\Delta\epsilon=\Delta_0\epsilon+\Delta_1\epsilon, \tag{19}$$

where

$$\Delta_0\epsilon=-\tfrac{1}{4}A\int_m\int_n\left(\mathbf{mn}\left|k_F\left(\frac{\partial}{\partial\kappa}+\frac{\partial}{\partial\lambda}\right)\times[K_1(\kappa\lambda)+K_0(\kappa\lambda)]\right|\mathbf{mn}\right), \tag{20}$$

$$\Delta_1\epsilon=\tfrac{1}{2}A\left\{\int_m\left(\mathbf{k}_F\mathbf{m}\left|k_F\left(\frac{\partial}{\partial\kappa}-\frac{\partial}{\partial\lambda}\right)K_1(\kappa\lambda)\right|\mathbf{k}_F\mathbf{m}\right)+\frac{1}{4}\int_m\int_n\left(\mathbf{mn}\left|k_F{}^2\left(\frac{\partial^2}{\partial\kappa^2}+\frac{\partial^2}{\partial\lambda^2}-2\frac{\partial^2}{\partial\kappa\partial\lambda}\right)\times[K_1(\kappa\lambda)+K_0(\kappa\lambda)]\right|\mathbf{mn}\right)\right\}, \tag{21}$$

where all the derivatives with respect to λ and κ should be calculated at the point $\kappa=\lambda=k_F$, and where we have applied the notation

$$\int_m\equiv(4\pi k_F{}^3)^{-1}\int\int\int d\mathbf{m}=\tfrac{4}{3}A\sum_{\mathbf{m}}. \tag{22}$$

Now with the help of the identity

$$\left(\frac{\partial}{\partial\kappa}+\frac{\partial}{\partial\lambda}\right)[K_1(\kappa\lambda)+K_0(\kappa\lambda)]=\frac{d}{dk_F}[K_1(k_Fk_F)+K_0(k_Fk_F)]=\frac{d}{dk_F}\sum_T\left(\frac{2T+1}{2}\right)K(T), \tag{23}$$

we can rewrite Eq. (20) in the simpler form

$$\Delta_0\epsilon=-\tfrac{1}{4}A\int_m\int_n\left(\mathbf{mn}\left|k_F\frac{d}{dk_F}\times\sum_T\left(\frac{2T+1}{2}\right)K(T)\right|\mathbf{mn}\right). \tag{24}$$

Collecting all the terms together we get the following expression for the symmetry energy:

$$\epsilon_{\rm sym}=\epsilon_{\rm sym}{}^{\rm kin}+\tfrac{1}{3}k_F[\partial V_0(m)/\partial m]_{m=k_F}+2S_{k_F}+\Delta_0\epsilon+\Delta_1\epsilon. \tag{25}$$

The first three terms in Eq. (25) are those which have been used in BG for computation of the symmetry energy.

B. Isotopic-Spin Dependence of the Single-Particle Potential

In the case of a nucleus with a given neutron excess the single-particle potential U of a nucleon of momentum $\mathbf{m}$ depends on α, and in the linear approximation in α can be written in the form

$$U(m\pm)=U_0(m)\pm\tfrac{1}{4}\alpha U_1(m), \tag{26}$$

where, as before, $+(-)$ refers to the case of neutron (proton). We have introduced the factor $\frac{1}{4}$ to be consistent with the form $U_0+U_1A^{-1}\mathbf{t}\cdot\mathbf{T}$ of the optical-model potential suggested by Lane.[6] ($\mathbf{t}$, $\mathbf{T}$ are the isotopic spin operators of the scattered nucleon and the target nucleus, respectively.)

It should be stressed that U is the potential energy of a real hole in the nomenclature of Brueckner and Goldman,[7] and is defined together with the kinetic energy, $\epsilon(m)=\hbar^2m^2/2M$, as the energy required to remove a particle from the system leaving a hole in the state $(\mathbf{m}\pm)$.

$$\epsilon(m)+U(m)=\partial E/\partial n(\mathbf{m}\pm), \tag{27}$$

where $n(\mathbf{m}\pm)$ is the occupation number of the state $\mathbf{m}\pm$.[8] Or equivalently we can write

$$U(m+)=\partial E_{\rm pot}/\partial n(\mathbf{m}+)=E_{\rm pot}(NZ)-E_{\rm pot}(N-1_{\rm m}Z), \tag{28}$$

$$U(m-)=\partial E_{\rm pot}/\partial n(\mathbf{m}-)=E_{\rm pot}(NZ)-E_{\rm pot}(NZ-1_{\rm m}). \tag{29}$$

Here, instead of indicating the dependence of $E_{\rm pot}$ on the neutron and proton Fermi momenta, $k_F{}^+$ and $k_F{}^-$, we have indicated the dependence of $E_{\rm pot}$ on Z and N. This notation is more convenient for our present considerations. Of course we can always switch from one dependence to the other by means of the relations

$$\tfrac{1}{3}\pi^{-2}(k_F{}^+)^3=N/\Omega,\quad \tfrac{1}{3}\pi^{-2}(k_F{}^-)^3=Z/\Omega. \tag{30}$$

One should, however, keep in mind that the potential energy (as well as other quantities like the K matrix) do depend on all the occupation numbers. Only when all the occupation numbers are equal one below the Fermi level and zero otherwise this dependence on the occupation numbers is reduced to the dependence on the Fermi momentum or the number of particles. This is not the case after one particle has been removed from the system. Thus, e.g., $E_{\rm pot}(N-1_{\rm m}Z)$ of Eq. (28), which denotes the potential energy of the system after one neutron with momentum $\mathbf{m}$ has been removed, depends on N, Z, and $\mathbf{m}$.

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[6] A. M. Lane, Nucl. Phys. **35**, 676 (1962).

[7] K. A. Brueckner and D. T. Goldman, Phys. Rev. **117**, 207 (1960).

[8] The reader who is interested only in the value of U_1 at the Fermi surface is advised to proceed from here straight to Eq. (56). The definition of U_0 appearing in Eq. (56) is given in Eq. (33).

Here we consider the case $m<k_F$. Eventually we shall put $m=k_F$ and thus obtain the value of the single-particle potential at the top of the Fermi sea. This can be compared directly with the value of the optical-model potential at the Fermi surface, since it has been shown in Ref. 9 that U defined by Eq. (27) for a particle added to the system ($m>k_F$) is the properly defined optical-model potential.

The potential U as defined in Eqs. (28) and (29) differs from V of Eq. (12) by the rearrangement potential V_R.

$$U(m\pm)=V(m\pm)+V_R(m\pm). \tag{31}$$

With the help of the expression (15) for $E_{\rm pot}(NZ)$, one can easily calculate V_R. For instance, to get $V_R(m_0+)$ one has to write first an expression for $E_{\rm pot}(N-1_{\rm m_0}Z)$, which can be obtained from Eq. (15) by including a factor $(1-\frac{1}{2}\delta_{\rm mm_0})$ in each of the $\sum_{\rm m}{}^{+}$ summations,[10] and by changing properly ($N\rightarrow N-1_{\rm m_0}$) the arguments of the K matrices. By subtracting $E_{\rm pot}(N-1_{\rm m_0}Z)$ from $E_{\rm pot}(NZ)$, one gets

$$\begin{aligned}V_R(m+)=&\textstyle\sum_{\rm m_1}{}^{+}\sum_{\rm m_2}{}^{+}(\mathbf{m}_1\mathbf{m}_2|K_1(NZ)\\&-K_1(N-1_{\rm m}Z)|\mathbf{m}_1\mathbf{m}_2)\\&+\textstyle\sum_{\rm m_1}{}^{-}\sum_{\rm m_2}{}^{-}(\mathbf{m}_1\mathbf{m}_2|K_1(ZN)\\&-K_1(ZN-1_{\rm m})|\mathbf{m}_1\mathbf{m}_2)\\&+2\textstyle\sum_{\rm m_1}{}^{+}\sum_{\rm m_2}{}^{-}(\mathbf{m}_1\mathbf{m}_2|K_0(NZ)\\&-K_0(N-1_{\rm m}Z)|\mathbf{m}_1\mathbf{m}_2).\end{aligned} \tag{32}$$

The expression for $V_R(m-)$ can be obtained from Eq. (32) by changing

$$\begin{aligned}&K_1(N-1_{\rm m}Z)\rightarrow K_1(NZ-1_{\rm m}),\\&K_1(ZN-1_{\rm m})\rightarrow K_1(Z-1_{\rm m}N),\\&K_0(N-1_{\rm m}Z)\rightarrow K_0(NZ-1_{\rm m}).\end{aligned}$$

With the help of the decomposition (31) of $U(m+)$, we get, according to Eq. (26),

$$U_0(m)=V_0(m)+V_{0R}(m), \tag{33}$$

$$U_1(m)=V_1(m)+V_{1R}(m), \tag{34}$$

where $V_0(m)$ and $V_{0R}(m)$ are the single-particle "model" and rearrangement potentials in the case $N=Z=\frac{1}{2}A$. In our notation they have the forms [which can be obtained by putting $N=Z=\frac{1}{2}A$ in Eqs. (12) and (32)]

$$V_0(m)=\textstyle\sum_{\rm m_1}(\mathbf{m}\mathbf{m}_1|\sum_T\frac{1}{2}(2T+1)K(T)|\mathbf{m}\mathbf{m}_1), \tag{35}$$

$$\begin{aligned}&V_{0R}(m)\\&=\textstyle\sum_{m_1}\sum_{\rm m_2}(\mathbf{m}_1\mathbf{m}_2|[G(NZ)-G(N-1_{\rm m}Z)]_{\alpha=0}|\mathbf{m}_1\mathbf{m}_2),\end{aligned} \tag{36}$$

where

$$G(NZ)=K_1(NZ)+K_1(ZN)+2K_0(NZ). \tag{37}$$

In Eqs. (35), (36), as well as in the remaining part of our considerations, sums over $\mathbf{m}$, $\mathbf{m}_i$, $\mathbf{m}'$, $\mathbf{n}$, etc., run over $m<k_F$ if no special indication (like $+$, $-$) is attached to the corresponding $\sum$ symbol.

The definitions of V_1 and V_{1R} are

$$\begin{aligned}V_1(m)=&4[\partial V(m+)/\partial\alpha]_{\alpha=0}\\=&-4[\partial V(m-)/\partial\alpha]_{\alpha=0},\end{aligned} \tag{38}$$

$$\begin{aligned}V_{1R}(m)=&4[\partial V_R(m+)/\partial\alpha]_{\alpha=0}\\=&-4[\partial V_R(m-)/\partial\alpha]_{\alpha=0}.\end{aligned} \tag{39}$$

The dependence on α of V and V_R enters through the dependence of V and V_R on $k_F{}^{+}$ and $k_F{}^{-}$. There are two kinds of dependence of V and V_R on $k_F{}^{\pm}$: the first one through the limits of the sums in Eqs. (12) and (32), and the second one through the intrinsic dependence of the K matrices on $k_F{}^{\pm}$. Taking both of them into account, one easily can calculate V_1 and V_{1R} according to Eqs. (38) and (39). To simplify the equations we again use the notation: $\kappa=k_F{}^{+}$, $\lambda=k_F{}^{-}$. The result is

$$\begin{aligned}V_1(m)=4\Big\{&2S_m+\tfrac{1}{4}A\int_n\Big(\mathbf{m}\mathbf{n}\Big|k_F\Big(\frac{\partial}{\partial\kappa}-\frac{\partial}{\partial\lambda}\Big)\\&\times K_1(\kappa\lambda)\Big|\mathbf{m}\mathbf{n}\Big)\Big\},\end{aligned} \tag{40}$$

$$\begin{aligned}V_{1R}(m)=\tfrac{3}{2}A^2\Big\{&\int_n\Big(\mathbf{k}_F\mathbf{n}\Big|[K_1(NZ)-K_1(N-1_{\rm m}Z)]\\&-[K_1(ZN)-K_1(ZN-1_{\rm m})]|\mathbf{k}_F\mathbf{n}\Big)\\&+\frac{1}{2}\int_{m_1}\int_{m_2}\Big(\mathbf{m}_1\mathbf{m}_2\Big|k_F\Big(\frac{\partial}{\partial\kappa}-\frac{\partial}{\partial\lambda}\Big)\\&\times[G(NZ)-G(N-1_{\rm m}Z)]\Big|\mathbf{m}_1\mathbf{m}_2\Big)\Big\}.\end{aligned} \tag{41}$$

The right-hand side of Eqs. (40) and (41) is to be calculated for $\alpha=0$. This means that all the derivatives are to be evaluated at the point $\kappa=\lambda=k_F$. It also means that, e.g., $[K_1(NZ)-K_1(N-1_{\rm m}Z)]$ in the first part of Eq. (41) is the change in $K_1(A)$ caused by the removal from the system of one neutron of momentum $\mathbf{m}$. Hence, we could have written instead of $[K_1(NZ)-K_1(N-1_{\rm m}Z)]$ simply $\partial K_1(A)/\partial n(\mathbf{m}+)$.[11] However, the notation applied in Eq. (40) is more suitable for the discussion which follows.

The form of V_1 and V_{1R} given by Eqs. (40) and (41), although a bit involved, can be readily evaluated. The change in the K matrices caused by the removal of one

[9] K. A. Brueckner, Phys. Rev. **110**, 597 (1958).

[10] The factor $\frac{1}{2}$ has to be included because there are two neutrons (with spin up and down) in each momentum state.

[11] Similarly, the second term in the first part of Eq. (40) is simply $\partial K_1(A)/\partial n(\mathbf{m}-)$, and is obviously different from $\partial K_1(A)/\partial n(\mathbf{m}+)$. Namely, K_1, defined in Eq. (13), describes the scattering of two neutrons ($T_3=1$). And for the scattering of two neutrons, it makes a difference whether we remove from the system a neutron or proton, although they have the same momentum $\mathbf{m}$ and in the initial system the numbers of neutrons and protons are equal.

nucleon has been calculated by Brueckner and Goldman[7] in a very good approximation. The expressions of Ref. 7 can be easily generalized for the case of $N \neq Z$. After differentiating these generalized expressions with respect to κ and λ one gets formulas which seem to be capable of being computed or estimated.

In the present paper, however, we shall restrict ourselves to investigating the values of V_1 and V_{1R} at the Fermi surface where Eqs. (40) and (41) simplify essentially.

C. Connection between U at the Fermi Surface and ε_{sym}

To get the value of $U(k_F)$ we have to put $m=k_F$ in the equations obtained in B. This is trivial in the case of $V_0(m)$, Eq. (35), and we get simply

$$V_0(k_F)=\sum_{\mathbf{m}_1}\left(\mathbf{k}_F\mathbf{m}_1\left|\sum_T\left(\frac{2T+1}{2}\right)K(T)\right|\mathbf{k}_F\mathbf{m}_1\right). \quad (42)$$

To calculate $V_{0R}(k_F)$ from Eq. (36) for $V_{0R}(m)$, let us notice[12] that

$$[G(NZ)-G(N-1_{\mathbf{m}}Z)]_{\mathbf{m}=\kappa}$$
$$=G(NZ)-G(N-1Z)=\frac{\partial}{\partial N}G(NZ)=\frac{\partial\kappa}{\partial N}\frac{\partial G(\kappa\lambda)}{\partial\kappa}$$
$$=\frac{\partial\kappa}{\partial N}\frac{\partial}{\partial\kappa}[K_1(\kappa\lambda)+K_1(\lambda\kappa)+2K_0(\kappa\lambda)]. \quad (43)$$

Equation (43) has to be evaluated at the point $\alpha=0$ ($\kappa=\lambda=k_F$), where $\partial K_1(\lambda\kappa)/\partial\kappa\equiv\partial K_1(\kappa\lambda)/\partial\lambda$. With the help of Eq. (14), we then get

$$[G(NZ)-G(N-1_{\mathbf{m}}Z)]_{m=\kappa,\alpha=0}$$
$$=\frac{\partial\kappa}{\partial N}\left(\frac{\partial}{\partial\kappa}+\frac{\partial}{\partial\lambda}\right)[K_1(\kappa\lambda)+K_0(\kappa\lambda)]. \quad (44)$$

By using the identity (23) and the value

$$\partial\kappa/\partial N=\tfrac{1}{3}\kappa/N \quad (45)$$

[which follows from Eq. (30)] we finally get the following known expression for the rearrangement potential at the top of the Fermi sea:

$$V_{0R}(k_F)=\tfrac{3}{8}A\int_m\int_n\left(\mathbf{mn}\left|k_F\frac{d}{dk_F}\right.\right.$$
$$\left.\left.\times\sum_T\left(\frac{2T+1}{2}\right)K(T)\right|\mathbf{mn}\right). \quad (46)$$

Here and in several other places of our considerations, we take the liberty of switching from the NZ variables to the $\kappa\lambda$ variables. This point has been discussed before [after Eq. (29)].

The same procedure can be applied in calculating the first part of $V_{1R}(k_F)$, i.e., the part of the right-hand side of Eq. (41) with the single integral $\int_n$. We get

$$\{[K_1(NZ)-K_1(N-1_{\mathbf{m}}Z)]$$
$$-[K_1(ZN)-K_1(ZN-1_{\mathbf{m}})]\}_{m=\kappa,\alpha=0}$$
$$=\tfrac{2}{3}A^{-1}k_F\left(\frac{\partial}{\partial\kappa}-\frac{\partial}{\partial\lambda}\right)K_1(\kappa\lambda). \quad (47)$$

In the second part of V_{1R}, i.e., in the part with the double integral $\int_{m_1}\int_{m_2}$ there are two terms: the one containing the derivative with respect to κ and the one containing the derivative with respect to λ. There is no problem in calculating the latter term, and by applying the procedure just described, one gets

$$\left\{-k_F\frac{\partial}{\partial\lambda}[G(NZ)-G(N-1_{\mathbf{m}}Z)]\right\}_{m=\kappa,\alpha=0}$$
$$=\tfrac{2}{3}A^{-1}\left(-2k_F{}^2\frac{\partial^2}{\partial\kappa\partial\lambda}\right)[K_1(\kappa\lambda)+K_0(\kappa\lambda)]. \quad (48)$$

In calculating the κ derivative we shall use the following identity:

$$\frac{\partial}{\partial\kappa}[G(NZ)-G(N-1_\kappa Z)]$$
$$=\left\{\frac{\partial}{\partial\kappa}[G(NZ)-G(N-1_{\mathbf{m}}Z)]\right\}_{m=\kappa}$$
$$+\left\{\frac{\partial}{\partial m}[G(NZ)-G(N-1_{\mathbf{m}}Z)]\right\}_{m=\kappa}. \quad (49)$$

This is simply the standard rule for calculating a derivative. To get the physical meaning of Eq. (49), let us notice that $G(NZ)$ defined in Eq. (37) represents an effective two-body interaction in the system of N neutrons and Z protons. $G(NZ)-G(N-1_\kappa Z)$ is the change in this effective interaction caused by the removal of a neutron from the Fermi surface, i.e., by creating a hole in the neutron Fermi surface. On the left-hand side of Eq. (49) we shift the neutron Fermi surface together with the hole in it, and ask how the shift effects this change in the effective interaction. The right-hand side of Eq. (49) shows that this shift is equivalent to a sum of two shifts: the first one in which the neutron Fermi surface is shifted with the hole being fixed, and the second one in which the hole is shifted with the Fermi surface being fixed.

[12] Compare the discussion which follows after Eq. (30).

Now the left-hand side of Eq. (49) is simply

$$\frac{\partial}{\partial\kappa}[G(NZ)-G(N-1Z)]=\frac{\partial}{\partial\kappa}\frac{\partial G(NZ)}{\partial N}=\frac{\partial}{\partial\kappa}\left[\frac{\partial\kappa}{\partial N}\frac{\partial G(\kappa\lambda)}{\partial\kappa}\right]$$
$$=\frac{\partial\kappa}{\partial N}\left[\frac{\partial^2 G(\kappa\lambda)}{\partial\kappa^2}-\frac{2}{\kappa}\frac{\partial G(\kappa\lambda)}{\partial\kappa}\right], \quad (50)$$

where we have used the value

$$(\partial/\partial\kappa)(\partial\kappa/\partial N)=-(2/\kappa)(\partial\kappa/\partial N),$$

which follows from Eq. (45).

With the help of Eqs. (49), (50), and by applying the procedure described previously, one gets

$$\left\{k_F\frac{\partial}{\partial\kappa}[G(NZ)-G(N-1_mZ)]\right\}_{m=\kappa,\alpha=0}$$
$$=\tfrac{2}{3}A^{-1}\left\{k_F^2\left(\frac{\partial^2}{\partial\kappa^2}+\frac{\partial^2}{\partial\lambda^2}-2\frac{\partial^2}{\partial\kappa\partial\lambda}\right)[K_1(\kappa\lambda)+K_0(\kappa\lambda)]\right.$$
$$\left.-2k_F\frac{d}{dk_F}\sum_T\left(\frac{2T+1}{2}\right)K(T)\right\}$$
$$-k_F\left\{\frac{\partial}{\partial m}[G(NZ)-G(N-1_mZ)]\right\}_{m=\kappa,\alpha=0}. \quad (51)$$

Inserting Eqs. (47), (48), (51) into Eq. (41) and making use of Eqs. (36), (46) we finally get

$$V_{1R}=A\int_n\left(\mathbf{k}_F\mathbf{n}\left|k_F\left(\frac{\partial}{\partial\kappa}-\frac{\partial}{\partial\lambda}\right)K_1(\kappa\lambda)\right|\mathbf{k}_F\mathbf{n}\right)$$
$$+\tfrac{1}{2}A\int_{m_1}\int_{m_2}\left(\mathbf{m}_1\mathbf{m}_2\left|k_F^2\left(\frac{\partial^2}{\partial\kappa^2}+\frac{\partial^2}{\partial\lambda^2}-2\frac{\partial^2}{\partial\kappa\partial\lambda}\right)\right.\right.$$
$$\left.\left.\times[K_1(\kappa\lambda)+K_0(\kappa\lambda)]\right|\mathbf{m}_1\mathbf{m}_2\right)-\frac{8}{3}V_{0R}(k_F)$$
$$-\tfrac{4}{3}k_F[\partial V_{0R}(m)/\partial m]_{m=k_F}. \quad (52)$$

The only remaining part of $U(k_F)$, namely $V_1(k_F)$, presents no problem, and Eq. (40) gives

$$V_1(k_F)=8S_{k_F}+A\int_n\left(\mathbf{k}_F\mathbf{n}\left|k_F\left(\frac{\partial}{\partial\kappa}-\frac{\partial}{\partial\lambda}\right)K_1(\kappa\lambda)\right|\mathbf{k}_F\mathbf{n}\right). \quad (53)$$

Comparing Eqs. (52), (53) with Eqs. (19), (21), (23) we notice that

$$U_1(k_F)=V_1(k_F)+V_{1R}(k_F)$$
$$=4\{2S_{k_F}+\Delta\epsilon-\tfrac{1}{3}k_F[\partial V_{0R}(m)/\partial m]_{m=k_F}\}. \quad (54)$$

Let us notice that Eq. (24), when compared with Eq. (46), gives

$$\Delta_0\epsilon=-\tfrac{2}{3}V_{0R}(k_F). \quad (55)$$

With the help of Eq. (25) we can write Eq. (54) in the form

$$\epsilon_{\rm sym}=\epsilon_{\rm sym}^{\rm kin}+\tfrac{1}{3}k_f[\partial U_0(m)/\partial m]_{m=k_F}+\tfrac{1}{4}U_1(k_F). \quad (56)$$

Equation (56) presents the familiar form of the relation between the symmetry energy and the single-particle energy at the Fermi surface. This relation can be obtained directly from the definition of U, Eq. (27), which for the state $(m,\pm)=(k_F^{\pm},\pm)$ can be written in the form

$$\epsilon(k_F^+)+U(k_F^+,+)=\partial E/\partial N, \quad (57a)$$

$$\epsilon(k_F^-)+U(k_F^-,-)=\partial E/\partial Z. \quad (57b)$$

If we subtract Eq. (57b) from Eq. (57a) and calculate $[\partial E/\partial N-\partial E/\partial Z]$ from Eq. (1), we get

$$2\epsilon_{\rm sym}\alpha=\epsilon(k_F^+)-\epsilon(k_F^-)+U(k_F^+,+)-U(k_F^-,-). \quad (58)$$

If we now expand the right-hand side of Eq. (58) in a power series of α and keep only terms linear in α, and use the form (26) for $U(k_F^{\pm},\pm)$ we get our Eq. (56).

Hence, if one is interested only in the value of $U_1(k_F)$, one does not have to follow all the considerations of Secs. IIB, C, as $\epsilon_{\rm sym}$ calculated in Sec. IIA, and the relation (56) supply one directly with the value of $U_1(k_F)$. All the considerations of Secs. IIB, C are, however, essential for any calculation of $U_1(m)$ for $m\neq k_F$, a quantity of considerable importance in problems of nuclear structure. Furthermore, the considerations of Secs. IIB, C, enable us to see the origin of the different parts of U_1, even if we are interested only in $U_1(m)$ for $m=k_F$.

In the remaining part of this paper we shall present an actual calculation of $\epsilon_{\rm sym}$ and $U_1(k_F)$, based on the BG solution of the nuclear matter problem.

All quantities necessary to calculate $\epsilon_{\rm sym}$, Eq. (25), except for $\Delta\epsilon$ have been calculated in BG. An approximate calculation of $\Delta_1\epsilon$ is presented in Sec. III. As far as $\Delta_0\epsilon$ is concerned, we can obtain its value directly from BG by applying Eq. (55). Namely, the rearrangement potential at the Fermi surface, $V_{0R}(k_F)$, is the difference between the single-particle "model" energy at the Fermi surface and the separation energy, both of them calculated in the case $N=Z=A/2$. The separation energy, however, is at equilibrium density equal to the mean energy per nucleon.[9] Both the mean energy per nucleon and the single-particle model energy have been calculated in BG. Hence, one knows from BG the numerical value of $V_{0R}(k_F)$ and thus of $\Delta_0\epsilon$.

To calculate $U_1(k_F)$, after we already know the value of $\epsilon_{\rm sym}$, we shall apply Eq. (56). However, to do it we still have to know $[\partial V_{0R}(m)/\partial m]_{m=k_F}$. We shall evaluate this quantity from the values of $V_{0R}(m)$ calculated by Brueckner *et al.*[13]

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[13] K. A. Brueckner, J. L. Gammel, and J. T. Kubis, Phys. Rev. **118**, 1438 (1960).

III. AN APPROXIMATE CALCULATION OF $\Delta_1\epsilon$

We shall follow here an approximate procedure applied previously[4] in the calculation of the spin symmetry energy of liquid He^3. Namely, we introduce two simplifying assumptions:

$$\text{(i)}\quad \begin{aligned} K_1(k_F^+ k_F^-) &\approx K_1(k_F^+)\,, \\ K_1(k_F^- k_F^+) &\approx K_1(k_F^-)\,, \end{aligned} \tag{59}$$

$$\text{(ii)}\quad K_0(k_F^+ k_F^-) \approx K_0(k_F')\,, \tag{60}$$

with

$$k_F' = 2^{-1/2}[(k_F^+)^2+(k_F^-)^2]^{1/2}, \tag{61}$$

where the K matrices on the right-hand side of Eqs. (59) and (60) are calculated in the case of equal number of protons and neutrons with the Fermi momentum equal k_F^+, k_F^- and k_F', respectively.

The assumption (i) says that the effect of the neutron excess on the scattering of two neutrons (or protons) is determined by the shift of the Fermi momentum of neutrons (or protons). This assumption seems to be physically plausible and it corresponds exactly to the way in which the action of the exclusion principle is altered by the neutron excess.

The assumption (ii) applies to the more complicated case of neutron-proton scattering. One is led to this assumption by inspecting the way in which the action of the exclusion principle on neutron-proton scattering is altered by the neutron excess. Furthermore this assumption satisfies Eq. (14). Let us also mention that the assumption (ii) satisfies the K_0 part of the identity (23).

We shall come back to this point in Sec. IV.

If we apply the Eqs. (59) and (60) to calculate the derivatives appearing in Eq. (21), we get

$$\begin{aligned} \Delta_1\epsilon = \tfrac{1}{2}A\Bigg\{ &\int_m \left(\mathbf{k}_F\mathbf{m}\left|k_F\frac{d}{dk_F}K(T=1)\right|\mathbf{k}_F\mathbf{m}\right) \\ &+\frac{1}{8}\int_m\int_n\left(\mathbf{mn}\left|k_F\frac{d}{dk_F}\textstyle\sum_T K(T)\right|\mathbf{mn}\right) \\ &+\frac{1}{4}\int_m\int_n\left(\mathbf{mn}\left|k_F^2\frac{d^2}{dk_F^2}K(T=1)\right|\mathbf{mn}\right)\Bigg\}\,. \end{aligned} \tag{62}$$

The expression (62) can be evaluated if we make use of the density (or Fermi momentum) dependence of the K matrix as determined by Brueckner, Gammel, and Weitzner[14] (compare also Ref. 15). There it was shown that the variation of K with density could, to an excellent approximation, be entirely included in the core repulsion part of the K matrix. According to Refs. 14 and 15,

$$\begin{aligned} \langle \mathbf{r}_{12}|K(sm_sT;k_F)|\mathbf{r}_{12}'\rangle = &\langle \mathbf{r}_{12}|K(sm_sT)|\mathbf{r}_{12}'\rangle_{\text{attractive}} \\ &+\langle \mathbf{r}_{12}|K(s;k_F)|\mathbf{r}_{12}'\rangle_{\text{core}}\,, \end{aligned} \tag{63}$$

where $K_{\text{attractive}}$ was independent of k_F and

$$\begin{aligned} &\langle \mathbf{r}_{12}|K(s;k_F)|\mathbf{r}_{12}'\rangle \\ &\quad = [A_s(r_0)/4\pi r_c^2]\delta(r_{12}-r_c)\delta(r_{12}'-r_c)\,, \end{aligned} \tag{64}$$

where r_c is the hard-core radius of the nucleon-nucleon interaction ($r_c=0.4$F for the Gammel-Thaler interaction) and r_0 is connected with k_F by Eq. (9), which implies:

$$k_F r_0 = (9\pi/8)^{1/3} = 1.52\,. \tag{65}$$

Since K_{core} acts only in the $l=0$ state of the relative motion, we do not have to indicate explicitly the value of T. We only notice that for $T=0$: $s=1$ (spin triplet), and for $T=1$: $s=0$ (spin singlet).

The short range of K_{core} in the configuration space makes the Fourier transofrms, which occur in Eq. (62), practically independent of $\mathbf{m}$, $\mathbf{m}'$. It is then easy to calculate all the terms of Eq. (62), and one gets the following results:

$$A\int_m\left(\mathbf{k}_F\mathbf{m}\left|k_F\frac{dK(1)}{dk_F}\right|\mathbf{k}_F\mathbf{m}\right) = \left(\frac{r_c}{r_0}\right)^2\left\{-\frac{d}{dr_0}A_{\text{singl.}}\right\}, \tag{66}$$

$$\begin{aligned} A\int_m\int_{m'}&\left(\mathbf{mm}'\left|k_F\frac{d}{dk_F}\textstyle\sum_T K(T)\right|\mathbf{mm}'\right) \\ &= \left(\frac{r_c}{r_0}\right)^2\left\{-\frac{d}{dr_0}(A_{\text{tripl.}}+\tfrac{1}{3}A_{\text{singl.}})\right\}, \end{aligned} \tag{67}$$

$$\begin{aligned} A\int_m\int_{m'}&\left(\mathbf{mm}'\left|k_F^2\frac{d^2}{dk_F^2}K(1)\right|\mathbf{mm}'\right) \\ &= \frac{1}{3}\left(\frac{r_c}{r_0}\right)^2\frac{d}{dr_0}\left(r_0\frac{d}{dr_0}+1\right)A_{\text{singl.}}. \end{aligned} \tag{68}$$

Notice that because of the relation (65) we have

$$k_F d/dk_F = -r_0 d/dr_0$$

and

$$k_F^2 d^2/dk_F^2 = r_0(d/dr_0)[r_0(d/dr_0)+1]\,.$$

For the sake of completeness, let us also write an equation for the quantity which appears in Eq. (24) for $\Delta_0\epsilon$ and in Eq. (46) for $V_{0R}(k_F)$.

$$\begin{aligned} A\int_m\int_{m'}&\left(\mathbf{mm}'\left|k_F\frac{d}{dk_F}\textstyle\sum_T\left(\frac{2T+1}{2}\right)K(T)\right|\mathbf{mm}'\right) \\ &= \left(\frac{r_c}{r_0}\right)^2\left\{-\frac{d}{dr_0}\tfrac{1}{2}(A_{\text{singl.}}+A_{\text{tripl.}})\right\}. \end{aligned} \tag{69}$$

[14] K. A. Brueckner, J. L. Gammel, and H. Weitzner, Phys. Rev. **110**, 431 (1958).

[15] K. A. Brueckner and D. T. Goldman, Phys. Rev. **116**, 424 (1959).

However, in accordance with the remarks made in Sec. IIIC [after Eq. (58)] we shall not need Eq. (69) as the value of $\Delta_0\epsilon$ can be directly obtained from the BG separation energy.

IV. NUMERICAL RESULTS AND DISCUSSION

In the numerical calculations we have used the following values of the nuclear matter parameters obtained in BG with the Gammel-Thaler[2,3] nuclear forces:

$$\begin{aligned} r_0&=1.02\text{ F},\\ \epsilon_{\rm sym}{}^{\rm kin}&=30.7\text{ MeV},\\ 2S_{k_F}&=9.8\text{ MeV},\\ k_F[\partial V_0(m)/\partial m]_{m=k_F}&=34.1\text{ MeV},\\ V_{0R}(k_F)&=12.3\text{ MeV}. \end{aligned} \tag{70}$$

In applying the approximate expressions of Sec. III, we have used the form of the functions $A_s(r_0)$ given in Ref. 15.

From the calculated values of $V_{0R}(m)$ of Ref. 13 we have obtained the numerical estimate

$$k_F[\partial V_{0R}(m)/\partial m]_{m=k_F}\approx -28.1\text{ MeV}. \tag{71}$$

The results of our calculation are

$$\begin{aligned} \epsilon_{\rm sym}&=64\text{ MeV},\\ V_1(k_F)&=70\text{ MeV},\\ V_{1R}(k_F)&=56\text{ MeV},\\ U_1(k_F)&=V_1(k_F)+V_{1R}(k_F)=126\text{ MeV}. \end{aligned} \tag{72}$$

To see the importance of the rearrangement contributions, resulting from the intrinsic dependence of the effective interaction $\bar{K}$ on the neutron and proton densities, let us write the results obtained without taking the rearrangement contributions into account.

$$\begin{aligned} [\epsilon_{\rm sym}]_{\rm no\ rear.}&=52\text{ MeV},\\ [V_1(k_F)]_{\rm no\ rear.}&=[U_1(k_F)]_{\rm no\ rear.}=39\text{ MeV}. \end{aligned} \tag{73}$$

Let us notice that the value of $[U_1(k_F)]_{\rm no\ rear.}$ coincides very well with the value of $U_1(k_F)$ obtained in Ref. 16 by applying the impluse approximation.

Our results show that the rearrangement contribution to the symmetry energy, $\Delta\epsilon=12$ MeV, is an important correction. And in the case of the isotopic spin-dependent part of the single-particle potential U_1, the rearrangement contribution constitutes its major part and approximately triples its no-rearrangement value.

The empirical estimates of the symmetry energy do not supply us with a unique value of $\epsilon_{\rm sym}$ (see ,e.g., the discussion in Ref. 17). By assuming a pure volume symmetry energy Green[18] finds $\epsilon_{\rm sym}=47$ MeV. By assuming that there is also a surface part of the symmetry energy, Green[18] finds $\epsilon_{\rm sym}=61$ MeV and Cameron[19] finds $\epsilon_{\rm sym}=63$ MeV.

The situation with the empirical determination of U_1 is much worse. A review of the situation is given in Refs. 6 and 20 (compare also Ref. 21, where the most recent estimates of U_1 are quoted). One can summarize the results of all these estimates by saying that they indicate a value of $U_1(k_F)=100\pm50$ MeV.

In view of the uncertainty of the empirical estimates of $\epsilon_{\rm sym}$, U_1, and of the approximations in our calculations, which we shall discuss in a moment, it is difficult to make a precise comparison between our results and the experiment. However, one sees that the calculated values of $\epsilon_{\rm sym}$ and U_1 given in Eq. (70) are in the range of the empirical estimates.

Let us now discuss the approximations of our calculation. Among the approximations of Sec. III, the approximation (ii), Eqs. (60) and (61), seems to be not as well justified as the approximation (i), Eq. (59). However, the approximation (ii) has been used in calculating only a small part of $\epsilon_{\rm sym}$ [4%] and U_1 [9%]. Hence the possible corrections to the approximation (ii) would have only a small effect on the calculated values of $\epsilon_{\rm sym}$ and U_1.

Only after more precise experimental values of $\epsilon_{\rm sym}$ and U_1 are available would a more accurate calculation of these quantities be desirable. Such a calculation would require the knowledge of the exact dependence of the K matrices on both the Fermi momenta $k_F{}^+$ and $k_F{}^-$. For this purpose one would have to solve the equations of the K-matrix theory with two different Fermi momenta $k_F{}^+$ and $k_F{}^-$. Or at least one should calculate the first and second derivatives of K with respect to $k_F{}^+$ and $k_F{}^-$, which enter into the expressions for $\epsilon_{\rm sym}$ and U_1. This can be done approximately by applying the procedure of Brueckner *et al.*[7,13] in their calculation of the rearrangement energy.

Much more important for our results is the proper value of $k_F[\partial V_{0R}(m)/\partial m]_{m=k_F}$ which, however, enters only into the calculations of U_1, and in our calculation constitutes about 30% of U_1. To get the value of $k_F[\partial V_{0R}(m)/\partial m]_{m=k_F}$ given in Eq. (71) we have used the values of $V_{0R}(m)$ for $m=k_F$ and $m=0.1k_F$ calculated in Ref. 13. This, of course, is only an estimate of $k_F[\partial V_{0R}(m)/\partial m]_{m=k_F}$, and to get a more accurate result for U_1, one should calculate $k_F[\partial V_{0R}(m)/\partial m]_{m=k_F}$ more precisely.

It should be stressed that the calculated values of $\epsilon_{\rm sym}$ and especially of U_1 are sensitive to the density dependence of the effective interaction K. Hence, a better knowledge of the empirical values of $\epsilon_{\rm sym}$ and U_1 could be helpful in determining the density dependence of K.

We finally make one comment concerning the effective mass. The effective mass $M_0{}^*$ calculated in BG is

[16] J. Dąbrowski and A. Sobiczewski, Phys. Letters **5**, 87 (1963).
[17] D. S. Falk and L. Wilets, Phys. Rev. **124**, 1887 (1961).
[18] A. E. S. Green, Rev. Mod. Phys. **30**, 569 (1958); Phys. Rev. **95**, 1006 (1954).
[19] A. G. W. Cameron, Can. J. Phys. **35**, 1021 (1957).
[20] P. E. Hodgson, Phys. Letters **3**, 352 (1963).
[21] J. Dąbrowski, Phys. Letters **8**, 90 (1964).

defined by the equation

$$k\partial V_0(k)/\partial k = 2\epsilon(k)\{[M/M_0^*(k)]-1\}, \quad (74)$$

where $\epsilon(k)=\hbar^2k^2/2M$. In problems, where one is interested in the real single-particle potential defined in Eq. (27), more important than the "model" effective M_0^* is the "real" effective mass M^* defined by the equation

$$k\partial U_0(k)/\partial k = k\partial[V_0(k)+V_{0R}(k)]/\partial k = 2\epsilon(k)\{[M/M^*(k)]-1\}. \quad (75)$$

From Eqs. (74) and (75) one gets

$$[M/M_0^*(k)]-[M/M^*(k)] = -\tfrac{1}{2}k\epsilon(k)^{-1}\partial V_{0R}(k)/\partial k. \quad (76)$$

If we insert into Eq. (76) the numerical values given in Eqs. (70) and (71), we get $M^*(k_F)/M=0.94$ compared to the BG value of $M_0^*(k_F)/M=0.73$. This is only a rough estimate based on the approximate value of $k_F[\partial V_{0R}(m)/\partial m]_{m=k_F}$ given in Eq. (71). However, an appreciable increase in the effective-mass results from the rearrangement effects.

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Decay of Cd^{117}

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The Cd^{117} activity produced by (n,γ) reaction on Cd^{116} was found to decay with a half-life of ~3 h indicating that the possible isomer also has a half-life almost equal to that of the ground state. The presence of such an isomer was established by beta-gamma coincidence measurements. The highest energy beta group as studied in the intermediate image beta-ray spectrometer showed an end point of 2250 keV. The singles gamma spectrum was complex in nature and extended up to 2450 keV. Beta-gamma and gamma-gamma coincidence studies revealed 29 gamma transitions and eight beta groups belonging to the decay of both the activities of Cd^{117}. Based on these and the results of sum coincidence and total absorption studies a decay scheme with levels in In^{117} at 310, 590, 660, 750, 880, 1070, 1410, 1700, 1890, 1980, 2120, 2320, and 2450 keV has been proposed. Possible spins and parities for these levels have been discussed.

I. INTRODUCTION

THE decay of Cd^{117} was first investigated by Cork and Lawson.[1] They used (d,p) reaction on Cd and identified the Cd^{117} activity of 3.75-h half-life from its daughter In^{117}. Coryell and co-workers[2,3] ascertained the genetic relationship of Cd^{117} by milking it from 1.1-min Ag^{117} produced in the fission of uranium. They reported that the ground state of Cd^{117} decays with a half-life of 50-min and that it has an isomer with a half-life of 3-h. Gleit[4] used (n,γ) reaction on enriched Cd^{116} to produce Cd^{117}. In his study, he observed various gamma rays of energies between 90 and 2000 keV. These were ascribed to 3-h Cd^{117m} decaying to 1-h In^{117}. He has indicated that about 20% of Cd^{117m} decays to the 50-min ground state by a 440-keV isomeric transition. According to him, the 50-min ground state of Cd^{117} shows little gamma activity and it mainly decays by a beta transition of end-point energy 2300 keV to the 1.9-h isomer of In^{117}. The 1.9-h isomer of In^{117} decays[5] partly by an isomeric transition of energy 310 keV to the 1-h ground state and partly by beta emission to various energy levels of Sn^{117}. The maximum beta energy in this decay is 1770 keV. The ground state of In^{117} mainly decays by beta emission to the 726-keV level of Sn^{117}. It has been suggested[4] that the 3-h activity of Cd^{117} is the $h_{11/2}$ state and the 50-min activity of Cd^{117} is the $s_{1/2}$ ground state. The energy levels of In^{117} have not been established so far. Recently, Tang and Coryell[6] have reported that the 50-min activity of Cd^{117} is not produced appreciably by (n,γ) reaction on enriched Cd^{116}.

In the present work a systematic study has been carried out to establish the energy levels of In^{117}. Efforts have been made to search for the reported isomer of Cd^{117}.

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[1] J. M. Cork and J. L. Lawson, Phys. Rev. **56**, 291 (1939).

[2] C. D. Coryell, P. Leveque, and H. G. Richter, Phys. Rev. **89**, 903 (A) (1953).

[3] U. Schindewolf, J. M. Alexander, and C. D. Coryell, Phys. Rev. **111**, 228 (1958).

[4] C. E. Gleit, MIT Laboratory for Nuclear Science, Annual Progress Report, 1957, p. 35 (unpublished).

[5] C. L. McGinnis, Phys. Rev. **97**, 93 (1955).

[6] C. W. Tang and C. D. Coryell, MIT Laboratory for Nuclear Science, Progress Report No. NYO-10062, p. 14, 1962 (unpublished).