Thus

$$\mathrm{Im}\left[\phi^*\frac{d\phi}{d(kr)}\right]=1, \quad \text{for all } r. \tag{A6}$$

Now, from Eq. (4.14) we write $S=\eta e^{2i\delta}$ in terms of $f_{\mathrm{eff}}\equiv f_r+if_i$

$$S=-\left(\frac{f_{\mathrm{eff}}\phi^*-\beta\phi^{*\prime}}{f_{\mathrm{eff}}\phi-\beta\phi'}\right), \tag{A7}$$

where the prime denotes differentiation with respect to kr, $\beta\equiv kr_0$ and ϕ, ϕ', ϕ^*, $\phi^{*\prime}$ are evaluated at $kr=\beta$.

$$\therefore S=\frac{A^*+i\phi^*f_i}{A+i\phi f_i}, \tag{A8}$$

where $A\equiv f_r\phi-\beta\phi'$.

$$\therefore |S|^2=\frac{|A|^2+f_i^2|\phi|^2-2f_i\,\mathrm{Im}A\phi^*}{|A|^2+f_i^2|\phi|^2+2f_i\,\mathrm{Im}A\phi^*}. \tag{A9}$$

But

$$A\phi^*=f_r|\phi|^2-\beta\phi'\phi^*.$$

$$\therefore \mathrm{Im}A\phi^*=-\beta\,\mathrm{Im}\phi^*\frac{d\phi}{d(kr_0)}$$

$$=-\beta \quad [\text{from (A4)}]. \tag{A10}$$

Thus, from (A9) and (A10),

$$|S|^2=\frac{|A|^2+f_i^2|\phi|^2+2\beta f_i}{|A|^2+f_i^2|\phi|^2-2\beta f_i}$$

$$\leqslant 1, \text{ if and only if, } \mathrm{Im}f_{\mathrm{eff}}\leq 0; \tag{A11}$$

we know that $|S|^2\leq 1$, if and only if, f is Hermitian. Thus, f is Hermitian, if and only if,

$$\mathrm{Im}f_{\mathrm{eff}}\leq 0. \tag{A12}$$

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Departures from the Eightfold Way: Theory of Strong Interaction Symmetry Breakdown*

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We consider the three kinds of departure from exact unitary symmetry: medium-strong interactions which leave only isospin and hypercharge as good symmetries, electromagnetism, and weak interactions. We postulate the existence of an octet of scalar mesons that give the possibility of symmetry-breaking tadpole diagrams. Our fundamental dynamical assumption—that symmetry-violating processes are dominated by symmetry-breaking tadpole diagrams—gives an immediate explanation of the success of two empirical laws: the Gell-Mann–Okubo mass formulas and the nonleptonic $\Delta I=\frac{1}{2}$ rules. Moreover, including tadpole diagrams and some other electromagnetic corrections, we calculate the six electromagnetic mass splittings of mesons and baryons in terms of a single unknown parameter correctly to within 0.5 MeV.

I. INTRODUCTION

WE assume that the fundamental interactions of elementary particles fall into the following classes, arranged in order of diminishing strength (we omit gravity):

1. Very-strong interactions, invariant under the transformations of "the eightfold way"—the symmetry scheme of Gell-Mann[1] and Ne'eman,[2] based on the group SU(3).
2. Medium-strong symmetry-breaking interactions, invariant under only the isospin-hypercharge subgroup of SU(3). Whether these interactions are introduced at the beginning, or whether they arise by some kind of spontaneous symmetry breakdown is immaterial to our discussion.
3. Electromagnetism.
4. Weak interactions.

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[1] M. Gell-Mann, California Institute of Technology Synchrotron Report CTSL-20, 1961 (unpublished); Phys. Rev. **125**, 1067 (1962).

[2] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

We call interactions of the last three classes "symmetry-breaking interactions." One may consider departures from exact unitary symmetry caused by each of the three classes of symmetry-breaking interactions alone, in the absence of the other two classes. This is natural and reasonable for the medium-strong interactions, since they are far stronger than either electromagnetism or the weak interactions. Neglect of the medium-strong interactions is certainly less well justified, but the relations that are thus obtained for electromagnetism[3] and for the structure of leptonic weak interactions[4] appear to be in good agreement with experiment.[5]

When we consider the symmetry-breaking interactions in this light, a certain curious regularity appears:

1. Masses of the elementary particles within unitary multiplets satisfy a sum rule, the Gell-Mann–Okubo mass formula. This is true for all known multiplets: the pseudoscalar octet, the vector octet and singlet, the baryon octet, and the $j=\frac{3}{2}$ decuplet of meson-baryon resonances. The mass formula is equivalent to the statement that the effective-mass Lagrangian giving rise to departures from exact unitary symmetry (and hence, degeneracy in mass) transforms like the neutral ($I=0$, $Y=0$) member of a unitary octet. This property of the effective-mass Lagrangian is by no means a consequence of unitary symmetry; it is easy to construct models of broken unitary symmetry in which the baryon octet possesses effective-mass terms transforming, in part, like the neutral component of a unitary 27-plet. In fact, such terms are virtually absent.

Conventionally,[1,6,7] the mass formula is attributed to the transformation properties of the medium-strong symmetry-breaking interaction: One assumes that the symmetry-breaking Lagrangian itself transforms like the neutral member of an octet, and that it contributes linearly to mass splittings. (Sakurai's ϕ-ω mixing[7] is an illustrative example.) Then lowest-order contributions to mass splittings will also have octet transformation properties, and thus satisfy the Gell-Mann–Okubo formula.

2. Electromagnetic corrections to baryon masses satisfy an approximate $\Delta I=1$ rule; that is to say, the masses of Σ^+, Σ^0, and Σ^- are approximately equally spaced. (With current observed masses, the $\Delta I=2$ term is about 10% of the $\Delta I=1$.) This rule also applies, though not so well, to meson electromagnetic mass splittings when expressed in mass squared: the $\Delta I=1$ kaon splitting is three times greater than the $\Delta I=2$ pion splitting. We observe that such a selection rule is obtained if the effective-mass Lagrangian describing the electromagnetic departure from exact unitary symmetry transforms like a member (with $I=1$, $I_3=1$, $Y=0$) of a unitary octet. Once again, unitary symmetry allows the presence both of octet and 27-plet contributions to electromagnetic masses, but the octet contribution dominates.

3. Nonleptonic weak interactions satisfy an approximate $\Delta I=\frac{1}{2}$ rule. Such a rule is obtained immediately if the effective Lagrangian for weak decays transforms like a member (with $I=\frac{1}{2}$, $Y=\pm 1$) of a unitary octet. For a third time, one might expect the appearance of many other representations of SU(3) involving higher isotopic spins, but again the octet contributions dominate.[8]

Conventionally, the appearance of a $\Delta I=\frac{1}{2}$ rule for nonleptonic weak interactions is attributed to the transformation properties of these interactions: One assumes the existence of at least four weak-interaction currents (Lee and Yang's schizon model[9] is an example involving two oppositely charged currents and two neutral currents) whose self-couplings transform like an isospinor and thus give an exact $\Delta I=\frac{1}{2}$ rule.

These three phenomena are similar in that they all involve a mysterious dominance of unitary octets. Nevertheless, the explanations they have elicited in the literature are quite different. The conventional explanations of the Gell-Mann–Okubo formula and the nonleptonic $\Delta I=\frac{1}{2}$ rule involve drastic assumptions about the structure of the SU(3)-breaking interactions and of the weak interactions, respectively. In both cases, a selection rule observed in nature is attributed to a residual symmetry of the symmetry-breaking interaction. Explanations of the electromagnetic $\Delta I=1$ rule are less common, perhaps because the form of the electromagnetic interaction is known and its symmetry properties cannot be as easily adjusted to produce the desired result.

We propose a theory of symmetry-breaking interactions that differs radically from the theories mentioned above. Our fundamental assumptions are dynamical in nature; we assume that symmetry breaking processes are dominated by a certain class of Feynman diagrams. We obtain a unified explanation of the three phenomena discussed above that is independent of the transformation properties of the symmetry-breaking Lagrangians. In particular, the Gell-Mann–Okubo formula comes about whatever the

[3] S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961).

[4] N. Cabbibo, Phys. Rev. Letters 10, 531 (1963).

[5] Our evaluation of strange baryon magnetic moments (and form factors) in terms of those of nucleons has not been decisively checked. Our sum rule for electromagnetic mass splittings (Ref. 3) is well satisfied. Cabbibo (Ref. 4) assumes that the charged weak-interaction currents, like the electric current, have octet transformation properties; he obtains good agreement with experiment for leptonic weak decays.

[6] S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[7] J. J. Sakurai, Phys. Rev. 132, 434 (1963). For an earlier discussion of ϕ-ω mixing see J. J. Sakurai, Phys. Rev. Letters 9, 472 (1962).

[8] The parallel between Gell-Mann's mass formula and the nonleptonic weak interactions (that both involve minimal, or octet, violations of unitary symmetry) was noted by M. Baker and S. L. Glashow, Nuovo Cimento 26, 803 (1962).

[9] T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960).

nature of the SU(3)-breaking interactions, and an approximate nonleptonic $\Delta I=\frac{1}{2}$ rule follows even if there is only a single current-current weak interaction.

We conjecture that there exists a unitary octet of scalar mesons.[7,10] These mesons are an isotopic singlet, with hypercharge zero, which we call η'; an isotopic triplet with hypercharge zero, which we call π'; and two isotopic doublets, with hypercharges one and minus one, which we call K' and $\bar{K}'$. In Sec. II below, we examine the scanty experimental evidence for the existence of this octet. If the scalar meson octet does exist, there is the possibility of a class of Feynman diagrams that vanish for every other kind of particle. These are the scalar tadpoles, diagrams with only one external line. In the limit of exact unitary symmetry, all scalar tadpoles vanish; but as we turn on the various symmetry-breaking interactions, the scalar tadpoles acquire nonzero values. The SU(3)-breaking interactions can make an η' tadpole; the electromagnetic interactions can make a $\pi^{0\prime}$ tadpole; and the weak interactions can make a $K'_{(1)}$ tadpole (since they violate parity, they can also make a $K_{(1)}$ tadpole). Figure 1 shows a typical electromagnetic contribution to the $\pi^{0\prime}$ tadpole.

There is a class of Feynman diagrams that contribute to symmetry-violating processes, which we call "symmetry-breaking tadpole diagrams." These are diagrams that may be broken into two parts, connected only by a scalar meson line, such that one part is a tadpole and the other part involves only the SU(3)-invariant very-strong interactions. Figure 2 shows four symmetry-breaking tadpole diagrams contributing, respectively, to mass differences between isotopic multiplets within a unitary supermultiplet, to electromagnetic mass splittings within isotopic multiplets, to parity-conserving nonleptonic weak decays, and to parity-violating nonleptonic weak decays. Our fundamental dynamical assumption is that *symmetry-violating processes are dominated by symmetry-breaking tadpole diagrams*. For brevity, we shall refer to this assumption as "tadpole dominance."

This assumption immediately explains the three instances of octet dominance we have cited above.[11]

FIG. 1. (a) The general scalar meson tadpole diagram. (b) A typical $\pi^{0\prime}$ tadpole, due to electromagnetism.

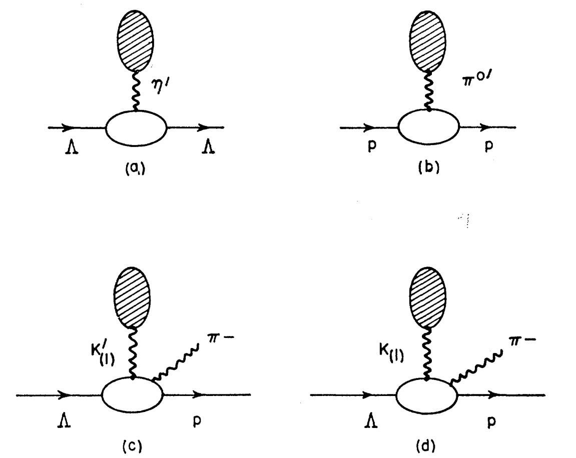

FIG. 2. Tadpole contributions to (a) mass splittings between baryon multiplets, (b) baryon electromagnetic mass splittings, (c) parity-conserving nonleptonic weak decays, and (d) parity-violating nonleptonic weak decays.

Octet dominance occurs not because of any simple transformation properties of the symmetry-breaking Lagrangians, but because the scalar mesons form a unitary octet. (If they formed a 27-plet, we would obtain 27-plet dominance.)

In Sec. III we calculate the values of the meson and baryon masses, assuming tadpole dominance. We find seven relations among the thirteen different masses. In Sec. IV we exploit out knowledge of the form of the electromagnetic interactions of mesons and baryons, to estimate the leading *nontadpole* contributions to electromagnetic mass splittings. Including these corrections, we obtain better agreement of our formulas with experiment. We are able to fit all six electromagnetic mass splittings to within 0.5 MeV with only one free parameter (the value of the $\pi^{0\prime}$ tadpole). In Sec. V we discuss the nonleptonic $\Delta I=\frac{1}{2}$ rules. Section VI contains a brief summary of our results and discusses some unanswered questions.

II. SCALAR MESONS

To obtain tadpole diagrams, we need an enhancement in the scalar-octet channel at zero four-momentum transfer. Most simply, this is obtained by positing the existence of an octet of scalar mesons. We now give a brief discussion of the properties of these conjectured particles. However, we cannot exclude the possibility of obtaining enhancement without the appearance of the physical particles: our explanation of symmetry-breaking phenomena suggests, but does not require, the existence of scalar mesons.

[10] S. L. Glashow, in *Istanbul Summer School on Group-Theoretic Methods in Elementary Particle Physics, 1962* (Gordon and Breach Publishers, New York, to be published); S. L. Glashow, in *Proceedings of Athens Conference on Resonant Particles* (Ohio State University Press, Columbus, Ohio, 1963), p. 25.

[11] Tadpole diagrams were first suggested as an explanation of mass splittings by J. Schwinger, Ann. Phys. 2, 407 (1957). J. J. Sakurai (Ref. 7) first suggested obtaining the Gell-Mann–Okubo formula by means of η' tadpoles. That K and K' tadpoles could be responsible for the $\Delta I=\frac{1}{2}$ rule was first recognized by A. Salam and J. Ward, Phys. Rev. Letters **5**, 390 (1960). See also the earlier discussion of kaon tadpoles by J. Schwinger, Phys. Rev. **104**, 1164 (1954) and of scalar kaons by M. Gell-Mann, in *Proceedings of the 1960 International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tinect, and A. C. Melissions (Interscience Publishers, Inc., New York, 1960), p. 510.

The K' may possibly be identified with the κ(730 MeV).[12,13] The absence of the decay mode $K^*(885) \to \kappa(730)+\pi$ suggests the spin-parity assignment $j^P = 1^+$.[13,14]

The scalar pion π' has odd G parity, and couples to no fewer than five pions (or, $\eta\pi$). If it is heavier than 700 MeV, it can decay into $\eta\pi$ by strong interactions; if it is lighter than 700 MeV it can only decay electrodynamically—into $2\pi+\gamma$ to order α, or into 2π, 2γ, $\pi+2\gamma$ to order α^2. A possible candidate is the ζ, which has been seen as a peak in the 2π mass spectrum at 570 MeV. The evidence for the existence of the ζ is not convincing,[15] but its most probable spin-parity-G-parity assignment is that of the π', $j^{PG}=0^{+-}$.

The remaining member of the scalar octet is η' with $j^{PG}=0^{++}$. It should show up as a $I=0$, s-wave pion-pion resonance. The possible existence of such a resonance with a mass near that of the ρ^0 has been reported.[16] Moreover, the observations[17] of the electromagnetic mode $\omega \to 2\pi$ may alternatively be interpreted as due to the decay of η' with a mass near that of ρ^0.

Consider the following striking regularity among the masses of the pseudoscalar mesons and the masses of the vector mesons[18]:

$$\rho - \pi = K^* - K\,,$$

which holds to an accuracy limited only by the experimental uncertainty of the ρ mass. This formula cannot be immediately applied to the $I=0$, $Y=0$ members of both octets because of the mixing between ω and ϕ.[7] Assuming equal mixing, however, we obtain:

$$\rho - \pi = K^* - K = \hat{\omega} - \eta\,,$$

where $\hat{\omega}$ denotes the mean square mass of ω and ϕ. Alternatively we can say that the only difference between the spectra (in mass squared) of the pseudoscalar and vector octets is an over-all displacement. It is amusing to apply this "rule" to the masses of the conjectured scalar mesons. Starting from the assignment of K' at 730 MeV, we obtain η' at 770 MeV and π' at 560 MeV—in excellent agreement with the assignments we have discussed.

The scalar octet is assumed to have very strong interactions invariant under SU(3). Unitary symmetry allows two invariant Yukawa couplings of scalar mesons to the baryon octet, and one invariant trilinear coupling of scalar mesons to pseudoscalar mesons. If we use the standard notation[3] in which the baryon fields are arranged into a traceless 3×3 matrix Ψ, the pseudoscalar mesons into a matrix $\boldsymbol{\phi}$, and the scalar mesons into a matrix $\boldsymbol{\phi}'$, then we may write the invariant interaction Lagrangian involving scalar mesons as

$$\mathfrak{L}' = d\,\mathrm{Tr}\{\bar{\Psi},\Psi\}\boldsymbol{\phi}' + f\,\mathrm{Tr}[\bar{\Psi},\Psi]\boldsymbol{\phi}' + g\,\mathrm{Tr}\boldsymbol{\phi}\boldsymbol{\phi}\boldsymbol{\phi}' + \cdots. \quad (1)$$

Note that the interaction of scalar mesons with pseudoscalar mesons is D-type. In addition, there will be couplings of $\boldsymbol{\phi}'$ with other multiplets: a D-type coupling of scalar mesons bilinear in the vector fields; an F-type coupling of vector mesons bilinear in the scalar fields; a D-type self-coupling trilinear in $\boldsymbol{\phi}'$; one Yukawa coupling of $\boldsymbol{\phi}'$ to the baryon decuplet, etc.

Under our assumption of tadpole dominance, in which all intermultiplet mass splittings arise from η' tadpoles, the equality of the differences in mass between corresponding members of different octets is equivalent to the assertion that the coupling constants of the scalar mesons to pseudoscalar mesons, to vector mesons, and to themselves are equal. We know how to guarantee universal couplings of vector mesons, but we know of no way to insure the apparent universality of scalar-meson couplings.[19]

III. MASSES OF MESONS AND BARYONS

In this section we calculate the contributions to the masses of mesons and baryons from the symmetry-breaking tadpole diagrams. These contributions depend upon the values of the two scalar tadpoles, which we call $\langle\eta'\rangle$ and $\langle\pi^{0\prime}\rangle$, and upon the couplings of η' and $\pi^{0\prime}$ to the baryons and mesons. These couplings are implicit in Eq. (1), but it is convenient to display those terms involving $\pi^{0\prime}$ and η';

$$\begin{aligned}\mathfrak{L}' = \pi^{0\prime}\{ & d(\bar{p}p - \bar{n}n + 2/\sqrt{3}\bar{\Sigma}^0\Lambda + 2/\sqrt{3}\bar{\Lambda}\Sigma^0 - \bar{\Xi}^0\Xi^0 + \bar{\Xi}^-\Xi^-) \\ & + f(\bar{p}p - \bar{n}n + 2\bar{\Sigma}^+\Sigma^+ - 2\bar{\Sigma}^-\Sigma^- + \bar{\Xi}^0\Xi^0 - \bar{\Xi}^-\Xi^-) \\ & + 2g(K^+K^- - \bar{K}^0K^0 + 2/\sqrt{3}\pi^0\eta)\} \\ & + 1/\sqrt{3}\eta'\{d(-\bar{p}p - \bar{n}n - 2\bar{\Lambda}\Lambda + 2\bar{\Sigma}^+\Sigma^+ \\ & + 2\bar{\Sigma}^-\Sigma^- + 2\bar{\Sigma}^0\Sigma^0 - \bar{\Xi}^0\Xi^0 - \bar{\Xi}^-\Xi^-) \\ & + 3f(\bar{p}p + \bar{n}n - \bar{\Xi}^0\Xi^0 - \bar{\Xi}^-\Xi^-) \\ & \quad + 2g(-K^+K^- - \bar{K}^0K^0 - \eta^2 + 2\pi^+\pi^- + \pi^0\pi^0)\}\,. \end{aligned} \quad (2)$$

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[12] G. Alexander, G. R. Kalbfleish, D. H. Miller, and G. A. Smith, Phys. Rev. Letters **8**, 447 (1962); D. H. Miller, G. Alexander, O. Dahl, L. Jacobs, G. R. Kalbfleisch, and G. A. Smith, Phys. Letters **5**, 279 (1963).

[13] S. G. Wojcicki, G. R. Kalbfleisch, and M. H. Alston, Phys. Letters **5**, 283 (1963).

[14] S. L. Glashow, in *Proceedings of Athens Conference on Resonant Particles* (Ohio State University Press, Columbus, Ohio, 1963), p. 25; S. Goldhaber, *ibid.*, p. 92.

[15] For a review of the evidence on the ζ, see D. B. Lichtenberg, Stanford Linear Accelerator Report No. 13, 1963 (unpublished), p. 53.

[16] V. Hagopian and W. Selove, Phys. Rev. Letters **10**, 533 (1963); Z. Guiragossian, *ibid.* **11**, 85 (1963).

[17] W. J. Flickinger, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters **10**, 457 (1963). References to earlier relevant experiments are given in this work.

[18] Throughout this work, we denote by the name of a meson the square of its mass (e.g., $\pi = 0.02$ GeV^2) and by the name of a baryon its mass (e.g., $p = 939$ MeV).

[19] The universality extends to the coupling of the scalar mesons to baryons and to the $j=\frac{3}{2}^+$ decuplet. For the baryons, we have (in MeV)

$$m = \Lambda - 190Y + 38[I(I+1) - Y^2/4].$$

For the decuplet, $I(I+1) - Y^2/4 = 2 + 3Y/2$; thus, assuming universality, we obtain an equal spacing in the decuplet with intervals of 133 MeV. The observed spacing is 145 MeV; universality is good to within 10%.

Applying our assumption of tadpole dominance to the baryon and meson self-energy operators, we obtain the following formulas for the masses of the baryons and the squares of the masses of the mesons[20]:

$$p=m_0+(3f-d)\langle\eta'\rangle/\sqrt{3}+(d+f)\langle\pi^{0\prime}\rangle, \quad (3a)$$

$$n=m_0+(3f-d)\langle\eta'\rangle/\sqrt{3}-(d+f)\langle\pi^{0\prime}\rangle, \quad (3b)$$

$$\Xi^-=m_0-(3f+d)\langle\eta'\rangle/\sqrt{3}+(d-f)\langle\pi^{0\prime}\rangle, \quad (3c)$$

$$\Xi^0=m_0-(3f+d)\langle\eta'\rangle/\sqrt{3}-(d-f)\langle\pi^{0\prime}\rangle, \quad (3d)$$

$$\Sigma^+=m_0+2d\langle\eta'\rangle/\sqrt{3}+2f\langle\pi^{0\prime}\rangle, \quad (3e)$$

$$\Sigma^0=m_0+2d\langle\eta'\rangle/\sqrt{3}, \quad (3f)$$

$$\Sigma^-=m_0+2d\langle\eta'\rangle/\sqrt{3}-2f\langle\pi^{0\prime}\rangle, \quad (3g)$$

$$\Lambda=m_0-2d\langle\eta'\rangle/\sqrt{3}, \quad (3h)$$

$$K^+=\mu_0^2-g\langle\eta'\rangle/\sqrt{3}+g\langle\pi^{0\prime}\rangle, \quad (3i)$$

$$K^0=\mu_0^2-g\langle\eta'\rangle/\sqrt{3}-g\langle\pi^{0\prime}\rangle, \quad (3j)$$

$$\pi^+=\mu_0^2+2g\langle\eta'\rangle/\sqrt{3}, \quad (3k)$$

$$\pi^0=\mu_0^2+2g\langle\eta'\rangle/\sqrt{3}, \quad (3l)$$

$$\eta=\mu_0^2-2g\langle\eta'\rangle/\sqrt{3}, \quad (3m)$$

where m_0 is the common baryon mass in the absence of symmetry-breaking interactions, and μ_0 is the common meson mass in the absence of symmetry-breaking interactions.

Equations (3) express thirteen masses in terms of six independent unknown parameters; thus, we may obtain seven sum rules. We arrange these rules into three classes: intermultiplet rules, intramultiplet rules, and hybrid rules. In writing these rules we take advantage of the fact that $\langle\pi^{0\prime}\rangle$ is much smaller than $\langle\eta'\rangle$; whenever a formula involves the difference in mass of two members of different isotopic multiplets, we substitute the mean mass of the multiplet. This simplifies some of our results at a negligible loss in accuracy.

Intermultiplet Sum Rules

$$\tfrac{1}{2}(N+\Xi)=\tfrac{1}{4}(3\Lambda+\Sigma). \quad (4)$$

$$K=\tfrac{1}{4}(3\eta+\pi). \quad (5)$$

These are Gell-Mann's mass formulas.[1] Note that we automatically obtain the formulas in terms of masses for the baryons and in terms of squares of masses for the bosons.[20]

We emphasize that our derivation of these formulas does not depend on the transformation properties of the symmetry-breaking Lagrangian. It does not even depend on the existence of a symmetry-breaking Lagrangian; tadpole dominance can also occur in a field theory with spontaneously broken symmetry. (The original example of Goldstone[21] is a field theory of this kind.)

For the baryons, Eq. (4) is accurate to 0.5%; for the mesons, Eq. (5) is accurate to 5% in mass squared.

Intramultiplet Sum Rules

$$\Xi^- - \Xi^0=\Sigma^- - \Sigma^+ + p - n. \quad (6)$$

$$\Sigma^0=\tfrac{1}{2}(\Sigma^+ + \Sigma^-). \quad (7)$$

$$\pi^0=\pi^+. \quad (8)$$

Equation (6) is our formula of 1961[3]; it agrees with experiment within the limits of experimental error. Equations (7) and (8) are the $\Delta I=1$ rules for baryons and for mesons; we have discussed their approximate validity in the Introduction.

Hybrid Sum Rules

$$\frac{\Sigma^- - \Sigma^+}{\Xi - N}=\frac{2}{3}\,\frac{\Xi^- - \Xi^0 + p - n}{\Sigma - \Lambda}. \quad (9)$$

$$\frac{K^0-K^+}{K-\pi}=\frac{\tfrac{1}{2}(n+\Xi^0)-\tfrac{1}{2}(p+\Xi^-)}{\tfrac{1}{2}(\Xi+N)-\Sigma}. \quad (10)$$

These are perhaps the most surprising results of our analysis. There is nothing in our intuition about the expected signs or magnitudes of mass differences that would keep the left-hand side of either one of these equations from being an order of magnitude different from the right-hand side. In fact, the left-hand side of Eq. (9) is 0.021; the right-hand side is 0.038. The left-hand side of Eq. (10) is 0.017; the right-hand side is 0.038.

We may also consider the electromagnetic mass splittings within the decuplet of $j=\frac{3}{2}^+$ meson-baryon resonances[22]: Δ_δ (1238), Σ_δ (1385), Ξ_δ (1530), and Ω_δ (unobserved). There is only a single coupling constant of scalar mesons to the decuplet: $\pi^{0\prime}$ is coupled to the decuplet neutral isospin current and η' is coupled with the same strength to $(3/4)^{1/2}$ times their hypercharge current. We obtain immediately an "equal-splitting rule":

$$\Delta_\delta^{++}-\Delta_\delta^{+}=\Delta_\delta^{+}-\Delta_\delta^{0}=\Delta_\delta^{0}-\Delta_\delta^{-}$$
$$=\Sigma_\delta^{+}-\Sigma_\delta^{0}=\Sigma_\delta^{0}-\Sigma_\delta^{-}=\Xi_\delta^{0}-\Xi_\delta^{-}. \quad (11)$$

Comparing decuplet splittings with baryon octet splittings, we may deduce another hybrid mass formula:

$$\frac{\Delta_\delta^{++}-\Delta_\delta^{+}}{\Delta_\delta-\Sigma_\delta}=\frac{\Sigma^+-\Sigma^-}{N-\Xi}. \quad (12)$$

This yields $\Delta_\delta^{++}-\Delta_\delta^{+}=-3.1$ MeV; the remaining mass splittings are determined by Eq. (11). (It should

[20] Tadpoles add a constant to the inverse propagator: $\gamma_\mu p^\mu+m$ for Fermions, but $k^2+\mu^2$ for bosons. Equivalently, we may say the tadpoles represent a constant addition to $\langle\phi'\rangle$, the vacuum expectation values of the fields ϕ'. Rewriting the Lagrangian (Ref. 21) in terms of well-behaved fields with vanishing-vacuum expectation values, $\phi'-\langle\phi'\rangle$, we obtain effective-mass terms of the form $(m_0+f\langle\phi'\rangle)\bar{\Psi}\Psi$ for Fermions, but of the form $(\mu_0^2+g\langle\phi'\rangle)\phi^2$ for bosons.

[21] J. Goldstone, Nuovo Cimento **19**, 154 (1961).

[22] S. L. Glashow and A. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

TABLE I. Electromagnetic mass splittings of baryons.

	Nontadpole contribution[a] (MeV)	Tadpole contribution[b] (MeV)	Calculated splitting (MeV)	Observed splitting[c] (MeV)
$p-n$	0.9	-2.6	-1.7	-1.3
$\Sigma^+-\Sigma^0$	0.7	-3.9	-3.2	-3.6 ± 0.5
$\Sigma^--\Sigma^0$	0.0	3.9	$+3.9$	4.5 ± 0.4
$\Xi^--\Xi^0$	0.0	5.2	5.2	5.6 ± 1.4

[a] According to H. Schnitzer (preliminary results).
[b] Tadpole contributions involve a single parameter which is chosen to give best over-all fit.
[c] Experimental masses from A. Rosenfeld, Wallet Card No. 1 [University of California, Lawrence Radiation Laboratory Report UCRL-8030 Revised, 1963 (unpublished)]; $\Xi^--\Xi^0$ splitting from H. Ticho at Brookhaven Conference on Weak Interactions, 1963 (unpublished).

be said that the nontadpole contribution to Δ_δ^{++} is likely to be considerable because it is doubly charged.)

IV. SOME ELECTROMAGNETIC MASS CORRECTIONS

Until now, we have neglected all mass-splitting diagrams other than tadpole diagrams. In the case of the medium-strong symmetry-breaking interactions, this is a necessary consequence of our ignorance; but for electromagnetism, it is a needless handicap. If our assumption of tadpole dominance is to be consistent, the magnitude of the nontadpole mass diagrams must be much less than that of the tadpole diagrams, but this does not mean they are negligible. As we shall see, for electromagnetism, the nontadpole diagrams are about 20% of the size of the tadpole diagrams.

What we call nontadpole contributions are conventionally regarded as the only contributions to electromagnetic mass splittings. The first estimate of these contributions to the $n-p$ mass splitting was due to Feynman and Speisman.[23,24] More recently, there has been an attempt of Coleman and Schnitzer[25] to calculate the nontadpole contributions to all the baryon mass splittings, within the framework of the eightfold way.

The leading nontadpole contributions to the electromagnetic self-masses of the baryons come from intermediate states containing one baryon and one photon. Inclusion of these states leads to an expression for the self-mass in terms of electromagnetic form factors. Experimental form factors are used for the nucleon and form factors of the strange baryons are obtained from these by unitary symmetry. Allowance is made for the breakdown of unitary symmetry by the use of the physical masses in the intermediate states. The results of these calculations are presented in the first column of Table I. The next contribution comes from the intermediate states containing one photon and one spin-$\frac{3}{2}$ resonance; this is known to be small.[24]

[23] R. P. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954).
[24] The formula of Feynman and Speisman was rederived on the basis of dispersion relations by M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters **2**, 7 (1959).
[25] S. Coleman and H. Schnitzer (unpublished).

TABLE II. Electromagnetic mass splittings of mesons.

	Nontadpole contribution[a] (MeV)^2	Tadpole contribution[b] (MeV)^2	Calculated splitting (MeV)	Observed splitting[c] (MeV)
K^+-K^0	2280	-4880	-2.6	-3.9 ± 0.6
$\pi^+-\pi^0$	1140	0	4.2	4.6

[a] According to R. Socolow.
[b] Tadpole contributions involve a single parameter which is chosen to be the same as that of Table I.
[c] Experimental masses from A. Rosenfeld, Wallet Card No. 1 [University of California, Lawrence Radiation Laboratory Report UCRL-8030 Revised, 1963 (unpublished)].

The leading nontadpole contributions to the electromagnetic self-masses of the pseudoscalar mesons comes from intermediate states containing one photon and one meson.[26] There is also a smaller, but still significant, contribution from the next intermediate states, those containing one photon and one vector meson. These mass corrections have been calculated by Socolow.[27] Little is known about the experimental form factors, so one-pole expressions are used. As above, account is taken of the physical masses. These results are shown in the first column of Table II.

Of all the conventional or nontadpole calculations of electromagnetic mass splittings, only the $\pi^+-\pi^0$ result (to which there is no tadpole contribution) is in agreement with experiment.

We can use these results to calculate "corrected" mass differences, defined as the differences between the experimental mass differences and the calculated nontadpole electromagnetic differences. These corrected differences should be almost entirely due to tadpoles. Therefore, if we insert them in Eqs. (6)–(10) we should much improve the agreement of these formulas with experiment. The reader will easily verify that Eqs. (6), (7), and (8) are indeed in perfect agreement with experiment. Equations (9) and (10) are improved to an accuracy of 20%. This should not be surprising; these equations are derived by neglecting nontadpole diagrams for both electromagnetism and the medium-strong symmetry-breaking interactions. We have corrected for the first source of error but not for the second.

We may present our results in another way. We use Eqs. (6)–(10) to express the tadpole contributions to the electromagnetic mass differences in terms of one free parameter and then choose this parameter to produce the best fit to the experimental differences.

In the second columns of Tables I and II we present the tadpole contributions to the electromagnetic splittings of baryons and mesons. These are determined, except for scale, by Eqs. (6)–(10). The scale is chosen so that the sum of tadpole and nontadpole contributions (shown in the third columns) is in best over-all agree-

[26] The relevant formula was again first proposed by Feynman and Speisman (Ref. 23), and later rederived from dispersion relations by Riazuddin, Phys. Rev. **114**, 1184 (1959).
[27] R. Socolow (private communication).

ment with the observed mass splittings (shown in the fourth columns). Our calculated values for the six mass splittings are in agreement with experiment to within 0.5 MeV.

Our knowledge of the form of the electromagnetic interaction enables us to justify the approximations of this section in terms of dispersion relations. We discuss baryon self-masses only; a similar argument may be constructed for mesons. Let $\bar{u}'M(p,k;p'k')u$ be the matrix element for scattering of photons off baryons, where p is the incoming-baryon four momentum, k the incoming-photon four momentum, u the incoming-baryon spinor, and p', k', and u' the corresponding quantities for the outgoing particles. The baryon four momentum is on the mass shell, but the photon four momentum need not be. Then it is easy to see that the electromagnetic self-mass of the baryon, to order α^2, is given by the formula[24]

$$\bar{u}\delta m u=\bar{u}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}M(p,k;p,k)u\,, \tag{13}$$

where the integration runs over all Euclidean four momenta.

Thus, every approximation for the scattering of (unphysical) photons off baryons induces an approximation for the self-mass. Let us describe this process in terms of the usual Mandelstam invariants, s, t, and u, and approximate the scattering amplitude by only including the pole terms. There are poles in the s and u variables due to one-baryon intermediate states [Figs. 3(a) and 3(b)]. These lead to terms in the self-mass dependent on the electromagnetic form factors of the baryons—our leading nontadpole contributions. There are many poles in the t variable, but only those with even parity and zero angular momentum can survive the integration over all Euclidean k and give a contribution to the self-mass. The only such pole is that due to the one-scalar meson intermediate state [Fig. 3(c)]—it leads to our tadpole contribution.

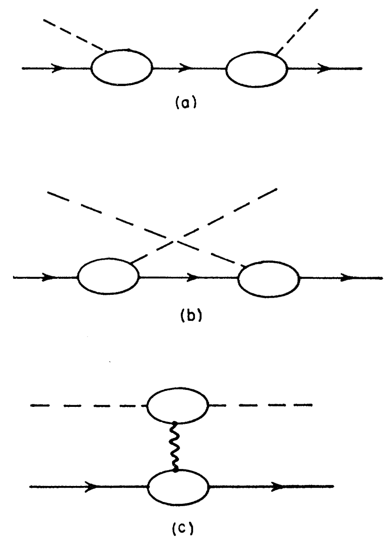

FIG. 3. The poles in the scattering of unphysical photons off baryons. Dashed lines represent photons, wiggly lines scalar mesons, directed lines baryons. We only include those poles that survive the integration over all Euclidean photon four momenta to give a contribution to the baryon self-mass.

V. NONLEPTONIC WEAK INTERACTIONS

This section is divided into four parts. In part A we discuss the consequences of tadpole dominance for the nonleptonic weak decays of hyperons, assuming unitary symmetry for the very-strong interactions. In part B we combine this with the tadpole dominance theory of the medium-strong interactions, discussed in the preceding sections. In part C we consider certain specific assumptions about the transformation properties of the weak-interaction Lagrangian. In part D we discuss the consequences of the possible existence of octets of scalar and pseudoscalar mesons with abnormal charge conjugation properties.

A.

A number of weak-interaction models have been suggested[9,28] that possess a built-in nonleptonic $\Delta I=\frac{1}{2}$ rule, that is, for which the relevant part of the Lagrangian transforms like an isospinor. All these models are complicated and involve at least four weak currents. They are subject to two criticisms:

1. They require the introduction of some currents which are mysteriously not coupled to leptons. (They may be neutral or doubly charged.) On the other hand, leptonic processes are well understood with only a single current-current interaction.

2. These models give an exact nonleptonic $\Delta I=\frac{1}{2}$ rule, while in nature the rule is approximate and holds to about 10% in amplitude. Electromagnetic corrections are, in general, too small to account for such large departures from an exact rule.

Other suggestions keep the attractive notion of a single self-coupled singly-charged current (or of a pair of intermediate vector mesons, $Z^{\pm}$). It is then necessary to find a dynamical mechanism that enhances the $\Delta I=\frac{1}{2}$ channel for all nonleptonic decays.[29] Salam and Ward[11] recognized that the K meson (and the conjectured scalar K' meson) tadpole diagrams have this effect. In our framework, however, the dominance of this kind of diagram is justified by the success of the exactly analogous assumption for η' tadpoles (giving the Gell-Mann–Okubo formula) and for $\pi^{0\prime}$ tadpoles (giving the electromagnetic mass differences). Departures form the nonleptonic $\Delta I=\frac{1}{2}$ rule are due to nontadpole diagrams. These give a contribution of 10% to $\Delta I\neq\frac{1}{2}$ amplitudes.

It is important to emphasize that our deduction of the nonleptonic $\Delta I=\frac{1}{2}$ rule is completely independent of whether or not there is a leptonic $\Delta I=\frac{1}{2}$ rule. Our arguments are unaffected by the presence of $\Delta I=\frac{3}{2}$

[28] For example, B. d'Espagnat, Nuovo Cimento 18, 287 (1960); T. D. Lee, Phys. Rev. Letters 9, 319 (1962).

[29] There have been attempts to obtain dynamical enhancement in the $\Delta I=\frac{1}{2}$ channel without recourse to tadpoles. See, for example, S. Oneda, J. C. Pati, and B. Sakita, Phys. Rev. 119, 482 (1960). M. Gell-Mann has often stressed the desirability of a dynamic origin to the nonleptonic weak-selection rules (private communication).

currents; octet dominance is a property of the strong interactions (the existence of an octet of pseudoscalar mesons, and maybe one of scalar mesons), not of the weak.[30]

It might be thought that we could obtain an equation similar to Eq. (3) for the nonleptonic weak decays and from this, sum rules corresponding to Eqs. (4)–(10). This is not so. There are only a small number of parameters in Eq. (3) because the processes shown in Figs. 2(a) and 2(b) involve only the symmetric three-point function. The processes shown in Figs. 2(c) and 2(d) involve the symmetric four-point functions for $K'_{(1)}+\text{baryon}\rightarrow$ pseudoscalar meson+baryon, and for $K_{(1)}+\text{baryon}\rightarrow$ pseudoscalar meson+baryon. There are five independent amplitudes for each of these processes; these make four linearly independent contributions to the nonleptonic decays of hyperons. On the other hand, given the $\Delta I=\frac{1}{2}$ rule, there are only four independent observable amplitudes for these decays. The tadpole-dominance hypothesis thus predicts no relationship among the (s wave or p wave) observable hyperon decays not predicted by the $\Delta I=\frac{1}{2}$ rule.

Tadpole dominance connects the ratios of parity-violating nonleptonic decays with the ratios of amplitudes for the process

$$\text{pion}+\text{baryon}\rightarrow\text{baryon}+\text{kaon}.$$

Of the two decay modes $\Sigma^{\pm}\rightarrow n+\pi^{\pm}$, one is almost pure s wave, the other is almost pure p wave. It follows that the s-wave contribution to one of the processes, $\pi^{\pm}+n\rightarrow\Sigma^{\pm}+K^0$, should be greatly suppressed compared to the other if it is permissible to extrapolate the physical scattering amplitude to zero-kaon four momentum.[31]

B.

We now combine our tadpole-dominance theory of nonleptonic hyperon decays with the tadpole-dominance theory of medium-strong symmetry breakdown, discussed in Sec. III. We find that a cancellation takes place that strongly suppresses the effects of the $K'_{(1)}$ tadpoles.[32]

Let us write the Lagrangian

$$L=L_0+L_s+L_t+L_{nt}, \tag{14}$$

where L_0 is the free Lagrangian, L_s describes the symmetric very strong interactions. L_t is the tadpole Lagrangian (obtained from L_s by substituting for ϕ', the 3×3 matrix representing the octet of scalar meson fields, $\langle\phi'\rangle$, the 3×3 matrix made up of their vacuum expectation values), and L_{nt} is the nontadpole symmetry breaking Lagrangian (defined as the difference of the symmetry-breaking Lagrangian and L_t). As a consequence of its definition, L_{nt} makes no contribution to the symmetry-breaking tadpole diagrams. We write L_{nt} as the sum of a medium-strong part and a weak part,

$$L_{nt}=L_{nt}^{ms}+L_{nt}^{w}. \tag{15}$$

(We neglect electromagnetic corrections.)

There exists an element of SU(3) that diagonalizes $\langle\phi'\rangle$. Let U be the unitary operator in the Hilbert space of the states of the system that corresponds to this element. To lowest order in the weak interactions

$$U=I+i\langle K'_{(1)}\rangle F_6/\langle\eta'\rangle, \tag{16}$$

where F_6 is the $Q=0$, $Y=1$, $C=-1$ Hermitian generator of unitary symmetry transformations. Let us transform all the baryon and meson fields by U. L_0+L_s commutes with U and is unchanged. L_t now commutes with the newly defined hypercharge and makes no contribution to nonleptonic decays. Thus, to lowest order in the weak interactions the only terms in L that contribute to nonleptonic weak decays are

$$i\langle K'_{(1)}\rangle[F_6,L_{nt}^{ms}]\langle\eta'\rangle^{-1}+L_{nt}^{w}. \tag{17}$$

The first of these preserves the $\Delta I=\frac{1}{2}$ rule; the second, in general, does not. The ratio of their contributions to any decay may be written as

$$r=-i\frac{\langle K'_{(1)}\rangle^{-1}\langle f|L_{nt}^{w}|i\rangle}{\langle\eta'\rangle^{-1}\langle f|[F_6,L_{nt}^{ms}]|i\rangle}. \tag{18}$$

Tadpole dominance tells us that both the numerator and the denominator of this fraction are small, but it tells us nothing about their ratio. Thus, tadpole dominance *per se* gives us no explanation of the $\Delta I=\frac{1}{2}$ rule for parity-conserving decays. Of course, it might be that tadpole dominance is a better approximation for weak interactions that for medium-strong interactions. With this as an unjustified additional assumption, r is small, and we regain the parity-conserving $\Delta I=\frac{1}{2}$ rule.

For parity-violating decays, the proof given above does not hold, and our derivation of the $\Delta I=\frac{1}{2}$ rule is still valid.

C.

We have emphasized that our explanation of the $\Delta I=\frac{1}{2}$ rule is independent of the transformation properties of the weak interaction Lagrangian; tadpole dominance, if it occurs, is a property of the very strong interactions rather than of the weak. However, this statement is not strictly true: Certain weak-interaction Lagrangians may lead to a selection rule forbidding the

[30] B. Lee [Phys. Rev. Letters **12**, 83 (1964)] independently suggests that the nonleptonic weak interactions should have octet transformation properties under SU(3). In addition, he assumes R symmetry and from this obtains relations among nonleptonic decay amplitudes. No attempt is made to justify either assumption. One of us (S. L. G.) acknowledges several valuable discussions with Professor Lee.

[31] The total cross sections for these processes are comparable at a pion kinetic energy of 1100 MeV. [R. Kraemer, M. Nussbaum, L. Madansky, and A. Pevsner, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962).]

[32] H. J. Lipkin has independently observed that in the limit of complete tadpole dominance the parity-conserving weak hyperon decays vanish (private communication).

weak interactions from contributing to the tadpole we need. In this case, of course, tadpole dominance will not obtain, no matter how much the strong interactions want to enhance tadpoles. We investigate here whether this possibility arises with a currently popular weak-interaction Lagrangian.

We assume the weak-interaction Lagrange density is of the form $j_\mu{}^\dagger j^\mu$, where j is some quantity bilinear in the fundamental fields that transforms like a four vector under the action of the connected Lorentz group and that changes the electric charge by one. For simplicity, we assume that j is made up only of baryon fields; inclusion of boson terms does not change our results. Further, we will follow Cabibbo[33] and Gell-Mann,[34] and assume that j is the sum of objects that transform under the action of SU(3) like the components of a unitary octet. There are four octet currents that may be constructed from the baryon fields: an F-type vector, a D-type vector, an F-type axial vector, and a D-type axial vector. Each of these has both a strangeness-conserving and a strangeness-violating part. Thus there are eight linearly independent terms which may be used to construct j. It is possible to show, as a consequence of CP invariance, that the coefficients of these terms must be real (with the phases of the octet currents appropriately defined).

Let ϕ be the matrix of fields corresponding to an octet of spinless mesons which is transformed into itself by the action of CP. Then there are two possibilities; either

$$\mathrm{CP}:\quad \phi(\mathbf{x},t)\rightarrow\phi^T(-\mathbf{x},\,t)\,, \tag{19a}$$

in which case we say the octet is even; or

$$\mathrm{CP}:\quad \phi(\mathbf{x},t)\rightarrow-\phi^T(-\mathbf{x},\,t)\,, \tag{19b}$$

in which case we say the octet is odd. The normal case is for scalar mesons to be even and pseudoscalar mesons to be odd. The mesons we have been discussing have this property. However, there is nothing that prevents objects arising which have the opposite (abnormal) transformation properties. Such mesons cannot be constructed out of baryon-antibaryon pairs, but they could be made from baryon-antibaryon-normal meson triplets.

Now let us return to our weak-interaction Lagrangian. Straightforward calculation establishes the following *theorem*: If two of the four currents that occur in j have the same ratio of strangeness-conserving part to strangeness-violating part, then their product contributes only to even octets. If they have opposite ratios, then the strangeness-changing part of their product contributes only to odd octets. If neither of these conditions applies, then both even and odd octets occur.

Let us apply this theorem to some suggested candidates for j. Cabibbo[33] has proposed that all four contributions to j point in the same direction in unitary space; that is to say, that the four strangeness-changing/strangeness-conserving ratios are equal. This implies that all the octets that occur in the weak-interaction Lagrangian are even; this is the normal case for scalars, but the abnormal case for pseudoscalars. Thus, if we adopt this suggestion, we are unable to obtain tadpole dominance for the parity-violating hyperon decays because the $K_{(1)}$ tadpole must vanish.

A weakened version of the Cabibbo proposal would be to require the vector current and the axial vector current each to point in a definite direction in unitary space, but not the same direction. Then normal tadpoles (both scalar and pseudoscalar) may contribute to weak interactions and our explanation of the parity-violating $\Delta I=\frac{1}{2}$ rule is undamaged.

An interesting special case of the above occurs if we choose the axial vector ratios to be opposite to the vector ratios. Then the weak interactions contribute only to normal octets (scalar and pseudoscalar). The current constructed in this way makes the same predictions for leptonic decay rates as that of Cabibbo.[4]

D.

We conclude our analysis of weak decays by examining the consequences of the existence of scalar and pseudoscalar octets of spinless mesons with *abnormal* charge conjugation properties. The empirical evidence about abnormal octets of spinless mesons is uncertain: Such objects do not couple to pairs of normal particles, so there are no pole terms in their production cross sections, which should make them difficult to produce. Also, their decay modes are unusually difficult ones to observe.

If such octets exist, we would expect abnormal tadpoles to dominate those channels in which they occur. They make no contribution to the masses of the baryons and mesons, but they do contribute to the nonleptonic decays of hyperons. For both s-wave and p-wave decays, these each make three independent contributions to the decay amplitudes. Thus, if the abnormal tadpoles dominated not only the nontadpole diagrams in their channels, but also the normal tadpoles, we would obtain one sum rule beyond that given by the $\Delta I=\frac{1}{2}$ rule. We will now display a model in which this dominance occurs.

We completely neglect nontadpole diagrams; that is to say, we will set L_{nt} in Eq. (14) equal to zero. Then, as explained in part B above, the effects of the normal scalar tadpoles may be completely transformed away. If the weak-interaction Lagrangian satisfies the conditions discussed in part C, it has zero amplitude for making abnormal scalar tadpoles. However, departures from exact unitary symmetry (due to η' tadpoles) can make such objects, and thus the abnormal octet dominates the parity-conserving decays.

Now let us further assume that the weak interaction current is of the type proposed by Cabibbo. Then there is zero amplitude for the creation of normal pseudoscalar

[33] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964).
[34] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).

tadpoles. Of course, as above, departures from unitary symmetry create such objects, but they are suppressed relative to abnormal tadpoles.

Thus, for both s-wave and p-wave decays we have dominance of the abnormal octet, and hence a sum rule for the four amplitudes describing nonleptonic decays of hyperons. It is

$$2\Xi+\Lambda=\sqrt{3}\Sigma\,, \tag{20}$$

where Λ, Ξ, Σ denote the amplitudes for $\Lambda\to p\pi^-$, $\Xi^-\to\Xi^0\pi^-$, $\Sigma^+\to p\pi^0$ expressed as vectors in the s–p plane. This sum rule was originally derived by Lee,[30] assuming R symmetry in addition to octet transformation behavior for the nonleptonic weak interactions. We have obtained it on quite different grounds, which do not require that the very-strong interactions be R symmetric.[35] As Lee has shown, Eq. (20) is in agreement with experiment.

VI. CONCLUSIONS

In the two years since it was first proposed, the successes of the eightfold way have been considerable. Most striking, perhaps, has been the correlation of mesons, baryons, and meson-baryon resonances into unitary multiplets.[22] In this paper, we have proposed a dynamical mechanism—tadpole dominance—for departures from exact unitary symmetry. These are our results:

1. We have derived the Gell-Mann–Okubo mass formula, in terms of masses for the baryons and in terms of squares of masses for the bosons. This formula is accurate to within 5% for the bosons and to within 0.5% for the baryon octet and for the decuplet of meson-baryon resonances.

2. We have derived a set of five formulas that enable us to fit all the meson and baryon electromagnetic mass splittings with one free parameter, to within 0.5 MeV.

3. We have derived the approximate $\Delta I=\frac{1}{2}$ rule for parity-violating nonleptonic weak decays. This rule is obeyed to within 10% in amplitude.

4. In addition, with the aid of one additional assumption about the relative magnitude of certain tadpoles, we have derived the $\Delta I=\frac{1}{2}$ rule for parity conserving nonleptonic decays. This rule is also obeyed to within 10% in amplitude.

5. Finally, we have shown that if there exist octets of scalar mesons with abnormal charge conjugation properties, and if we assume a weak-interaction Lagrangian of the form suggested by Gell-Mann[34] and Cabibbo,[33] then we obtain the sum rule first derived by Lee.[30]

All of our results have been approximate, simply because, although tadpole diagrams dominate symmetry breakdown, they are not the only diagrams contributing to symmetry breakdown. Thus the deviations from the $\Delta I=\frac{1}{2}$ rule for nonleptonic decays are not to be ascribed to anomalously large electromagnetic corrections, but simply to the effects of nontadpole diagrams.

Nevertheless, we are not without unanswered questions. Here are a few:

1. Does there really exist an octet of scalar mesons? If so, is their existence a dynamical consequence of the already known particles? If not, can we obtain the results of tadpole dominance without the existence of a scalar octet?

2. What is the explanation of the apparent universality of scalar-meson coupling discussed at the end of Sec. II? Is there some way, within the framework of field theory, of guaranteeing universality for scalar fields?

3. Why is the Gell-Mann–Okubo formula so accurate? The derivation of this formula differs from our other results in that in order to obtain it, we must neglect not only nontadpole diagrams, but many-tadpole diagrams. For electromagnetism and the weak interactions, the many-tadpole diagrams are of higher order in the (very small) coupling constant, and may legitimately be ignored, but for the medium-strong symmetry-breaking interactions, they should make a significant contribution to the self-masses. (When we calculate deviations from exact unitary symmetry using physical masses for intermediate states, we are including many-tadpole diagrams.)

Perhaps some light is cast on this question by recent calculations[7,36] that show a remarkable stability of the mass formula. If we calculate the baryon self-masses using physical meson masses, we find the mass formula is preserved. Perhaps a large contribution to the self-masses comes from many-tadpole diagrams that, nevertheless, preserve the mass formula. Some evidence for this is offered by the fact that the insertion of corrected masses in Sec. IV improved the intramultiplet sum rules to perfect agreement with experiment, while the hybrid rules were only improved to agreement to within 20%. This is what we would expect if there were a significant many-tadpole contribution to the masses. However, this is only an attempt at an explanation; much remains to be done here.

4. Finally, what are the medium-strong symmetry-breaking interactions? Or, are the departures from the eightfold way due only to spontaneous symmetry breakdown? If so, why are there no massless scalar bosons? It is curious that we know so much about the effects of these mysterious interactions, and so little about their structure.[37]

[35] M. Gell-Mann (Ref. 34) deduces the s-wave part of Lee's sum rule by showing that the Cabibbo currents give only normal scalar octets and abnormal pseudoscalar octets.

[36] R. E. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963); S. L. Glashow, Phys. Rev. **130**, 2132 (1963).

[37] R. E. Cutkosky and P. Tarjanne [Phys. Rev. **132**, 1355 (1963)] describe a model in which strong interaction symmetries are dynamically determined. Their analysis indicates the existence of self-supporting mass splittings of the octet type. They examine the structure of the SU(3)-symmetric nonlinear matrix equation expressing particle masses self-consistently. Allowed dissym-

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APPENDIX: REMARKS ADDED IN PROOF

1. It has been suggested[38] that the possible resonance[39] in the two-pion system near 400 MeV may be η'. If we retain the identification of K^* (730 MeV) as K', the mass formula puts π' at 1300 MeV, in an as yet unexplored region. With this assignment we lose the apparent universality of scalar meson couplings.

2. Tadpole dominance enables us to calculate the $\rho^0-\omega$ electromagnetic transition amplitude.[40] We assume the vector mesons comprise an unitary singlet and octet which mix, and we include $\pi^{0\prime}$ tadpole diagrams connecting ρ^0 to both the unitary singlet part and the unitary octet part of the physical ω. The ratio $\langle\pi^{0\prime}\rangle/\langle\eta'\rangle = 0.02$ is determined from the masses of the pseudoscalar mesons, and permits us to express the $\rho^0-\omega$ amplitude in terms of the mass differences among vector meson multiplets. We find for the branching ratio of the decay mode $\omega \to 2\pi$ the value of 4%.

3. A preliminary calculation of nontadpole contributions to electromagnetic splittings within the decuplet indicates that they tend to decrease the splitting among the members of the Δ_δ multiplet but increase it between the two Ξ_δ's. Because of this, and also because of the narrow width of the states involved, the latter splitting might be the easiest to measure.[41]

4. It might be thought that there would be a large second-order tadpole contribution to the $K_{(1)}-K_{(2)}$ mass splitting. However, the arguments of Sec. V, part B, may be used to show that this contribution vanishes, for intermediate states consisting of one spinless meson.[42]

5. Finally, we would like to list some alternative mechanisms which might yield the same results as tadpole dominance without necessarily requiring the existence of a scalar octet:

(a) We have already discussed the spontaneous breakdown model of Cutkosky and Tarjanne.[37]

(b) In a recent calculation[43] of a vector-meson bootstrap (beginning with a pseudoscalar octet), it was found that self-consistency demanded the existence of a scalar octet. This offers some verification for our final remarks in Ref. 37.

(c) J. Schwinger[44] has proposed a theory in which the observed octets are bound pairs of fundamental triplets. By replacing the products of two triplet field operators, in certain expressions for octet masses, by their vacuum expectation values, he is able to obtain many of our results. The structures which we call tadpole diagrams become, in his theory, objects analogous to the Hartree diagrams of ordinary statistical mechanics.

(d) A. Pignotti[45] has suggested that the tadpoles might be associated with an octet of even Regge trajectories that pass through negative t at $j=0$. The residues must vanish at this point, so these objects produce no bound states, but in many other ways they act like scalar tadpoles.

(e) Lastly, there is a possibility that many of the effects of scalar tadpole may be duplicated by "antibound states," poles in the s-wave scattering amplitude that lie below threshold but on the second sheet. A recent calculation[46] using some very crude approximations has found such an object in pion-pion scattering with isospin zero. This may be part of a unitary octet.

metries, either "spontaneous" or electromagnetic, are determined by the eigenspectrum of this matrix. They thus deduce the Gell-Mann–Okubo mass formula and the enhancement of $I=1$ electrodynamic mass splittings. Our hybrid mass formulas are also probably implicit in their work. Cutkosky and Tarjanne establish octet dominance by a method apparently unrelated to ours; however, should self-consistency require the existence of a scalar octet, the two approaches could be complementary. We are extremely grateful to Professor Cutkosky for copies of this work prior to publication, and for a very interesting discussion.

[38] N. Samios (private communication); S. Meshkov (private communication); L. Brown and P. Singer (unpublished).

[39] N. P. Samios, A. H. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shephard, Phys. Rev. Letters **9**, 139 (1962); C. Richardson, R. Kraemer, M. Meer, M. Nussbaum, A. Peosner, R. Strand, T. Toohig, and M. Block, *Proceedings of the 1962 Annual International Conference on High-Energy Physics, at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 96; J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. **130**, 2481 (1963).

[40] We are indebted to J. Schwinger for suggesting this calculation.

[41] These arguments are due to R. Socolow (private communication).

[42] Thus, the calculation of Biswas and Bose, Phys. Rev. Letters **12**, 176 (1964), which is identical to that discussed above, should give the value zero. Biswas and Bose obtain a nonzero value because they use empirical masses, instead of fitting the masses to the Gell-Mann–Okubo formula. However, because the formula is so nearly satisfied, they find (as they remark) a result 2 orders of magnitude less than they expect. We are indebted to B. Lee for an analysis of this paper.

[43] H. Chan, P. DeCelles, and J. Paton (unpublished).

[44] J. Schwinger, Phys. Rev. Letters **12**, 237 (1964).

[45] A. Pignotti (private communication).

[46] D. Atkinson (unpublished).