Radiative Pion-Nucleon Scattering*

P. CARRUTHERS†
Laboratory of Nuclear Studies and Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York

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The cross section for bremsstrahlung in pion-nucleon scattering is calculated from the results of the static nucleon theory. The result agrees well with a recent measurement of the reaction $\pi^+ + p \to \pi^+ + p + \gamma$ at an average pion laboratory kinetic energy of about 300 MeV. The theoretical result is 0.25 mb, compared to the experimental value 0.22 ± 0.05 mb. The energy distribution of the final pion is computed at laboratory energies of 200, 300, and 400 MeV. The 3-3 isobar shows up clearly at the latter two energies, in agreement with experiment. Experiments are suggested whereby the bremsstrahlung can be used as a probe to obtain information about the higher resonances in pion-nucleon scattering.

I. INTRODUCTION

THE reaction $\pi^+ + p \to \pi^+ + p + \gamma$ was measured recently by a CERN group[1] for an interval of pion lab momenta between 350 and 500 MeV/c, with a resultant cross section of 0.22 ± 0.05 mb for photons of energy greater than 50 MeV. It will be observed that the matrix element for this reaction can be obtained by crossing, from that of the reaction $\gamma + p \to p + \pi^+ + \pi^-$. In a previous paper,[2] it was shown that the latter process is described adequately (at energies not too far above threshold) by the static theory.[3–5] That conclusion is made more firm by the present result, that the same approximations give a good description of the bremsstrahlung reaction.

The relevant formulas, based on previous work by Cutkosky,[4,5] are given in Sec. II. In Sec. III the numerical results are presented. In Sec. IV experiments are suggested that should provide valuable information about the pion-nucleon interaction near the higher resonances.

II. CROSS SECTIONS FOR PHOTON EMISSION

Consider the reaction in which a positive pion of momentum $\mathbf{q}$ hits a (static) proton giving a final pion of momentum $\mathbf{p}$, a proton, and a photon of momentum $\mathbf{k}$. From the work of Cutkosky[4] one can derive the production amplitude for various charge states. The meson current and interaction current give rise to the processes of Fig. 1. We ignore the (fairly small) rescattering contributions.[5] The meson current gives rise to the usual dk/k photon spectrum at long wavelengths. In Figs. 1(a) and (b) the meson shakes off the photon before or after scattering in the 3-3 resonance state. The interaction current is especially interesting. In Fig. 1(c) we see an s-wave meson converted to an electric dipole photon along with the $P_{3/2}$ resonance (N*). Figure 1(d) illustrates the de-excitation of N* via decay into an s-wave pion and an electric dipole photon. The interaction current dominates the high-energy end of the gamma-ray spectrum, the meson current the low-frequency end.

For positive pions and protons the amplitude is[4]

$$T = -\frac{12\pi e}{(8k\omega_p\omega_q)^{1/2}}\left\{\frac{h(\omega_p)\mathbf{q}\cdot\boldsymbol{\varepsilon}}{k\omega_q(1-\beta_q\cos\theta_q)}\times[\mathbf{p}\cdot(\mathbf{k}-\mathbf{q})-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{p}\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})] + \frac{h(\omega_q)\mathbf{p}\cdot\boldsymbol{\varepsilon}}{k\omega_p(1-\beta_p\cos\theta_p)}[(\mathbf{p}+\mathbf{k})\cdot\mathbf{q}-\tfrac{1}{3}\boldsymbol{\sigma}\cdot(\mathbf{p}+\mathbf{k})\boldsymbol{\sigma}\cdot\mathbf{q}] + h(\omega_p)(\mathbf{p}\cdot\boldsymbol{\varepsilon}-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{p}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}) + h(\omega_q)(\mathbf{q}\cdot\boldsymbol{\varepsilon}-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\boldsymbol{\sigma}\cdot\mathbf{q})\right\}. \quad (1)$$

In Eq. (1) the terms represent successively the contributions of Figs. 1(a) to (d). $h(\omega_p)$ is equal to $e^{i\delta_p}\sin\delta_p/p^3$, where δ_p is the phase shift in the pion-nucleon 3-3 resonant state. $\boldsymbol{\varepsilon}$ is the polarization vector of the emitted photon, $\beta_p = p/\omega_p$, $\cos\theta_p = \mathbf{p}\cdot\mathbf{k}/pk$, and e is the rationalized electric charge, $e^2/4\pi \equiv \alpha = 1/137$.

In order to compare (1) with experiment we suppose it to be applicable to the c.m. system. The energy available to the emitted photon and pion is taken as $W-M$, the total kinetic energy of the initial state plus

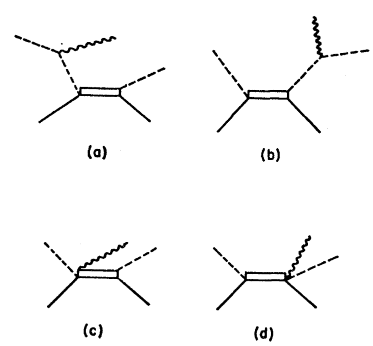

FIG. 1. The dashed lines denote mesons, wiggly lines photons and the double lines the 3-3 isobar. (a) and (b) show photon emission from the meson current; (c) and (d) by the interaction current.

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† Alfred P. Sloan Fellow.

[1] V. E. Barnes, D. V. Bugg, I. Derado, A. Minguzzi, L. Montanet, R. T. Van de Walle, R. Carrara, M. Cresti, A. Grigoletto, A. Loria, L. Peruzzo, and R. Santangelo, CERN Report 63-27, Track Chamber Division, 22 July 1963 (unpublished).

[2] P. Carruthers and How-Sen Wong, Phys. Rev. **128**, 2382 (1962).

[3] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

[4] R. E. Cutkosky, Phys. Rev. **109**, 209 (1958).

[5] R. E. Cutkosky, Phys. Rev. **113**, 727 (1959).

a pion rest mass. We do not correct the phase space but do use the correct relativistic flux factor (this reduces the "true static" cross section by the ratio E/W, when E is the total c.m. energy of the initial nucleon and W the total c.m. energy). Then the energy distribution of the final pion is given (in units of $1/\mu^2=20$ mb, μ=pion rest mass) by

$$\frac{d\sigma}{d\omega_p}=\frac{8\alpha kpE}{qW}\left\{\frac{\sin^2\delta_p}{p^4}[G(q,k)+H_1(p,q,k)]+\frac{\sin^2\delta_q}{q^4}[G(p,k)+H_2(p,q,k)]+\frac{2\sin\delta_p}{p^2}\frac{\sin\delta_q}{q^2}\cos(\delta_p-\delta_q)H_3(p,q,k)\right\}, \quad (2)$$

where the functions G and H_j are defined by

$$G(p,k)=1-\frac{\omega_p}{k}\left[1-\frac{1}{p\omega_p}\ln(p+\omega_p)\right], \quad (3)$$

$$H_1(p,q,k)=\frac{1}{k^2}\left[(k\omega_q-p^2)+\left(\frac{p^2\omega_q-k}{q}\right)\ln(q+\omega_q)\right], \quad (4)$$

$$H_2(p,q,k)=\frac{1}{k^2}\left[\left(\frac{q^2\omega_p+k}{p}\right)\ln(p+\omega_p)-(k\omega_p+q^2)\right], \quad (5)$$

$$H_3(p,q,k)=-\frac{1}{4}\left\{\tfrac{1}{2}J_1(p)J_1(q)-\frac{3p}{2k}J_1(q)J_2(p)+\frac{3q}{2k}J_1(p)J_2(q)+\frac{3pq}{k^2}J_2(p)J_2(q)\right\}, \quad (6)$$

$$J_1(p)=\tfrac{2}{3}[Q_0(\omega/p)-Q_2(\omega/p)]=(\omega/p)[1-(p\omega)^{-1}\ln(p+\omega)], \quad (7)$$

$$J_2(p)=\tfrac{2}{5}[Q_1(\omega/p)-Q_3(\omega/p)]. \quad (8)$$

In (7) and (8) the Q_j are Legendre functions of the second kind of order j. Explicit forms may be found in Whittaker and Watson.[6]

The various terms in Eq. (2) have the following origin: The first term arises from the processes of Figs. 1(a) and (c), the second from the processes of Figs. 1(b) and (d). The third term arises from the interference of the meson current contributions. Fortunately, this complicated term is quite small in the photon energy range of interest. The function G contains two terms: Unity denotes the contribution of the interaction current while $(\omega/k)[1-\ln(\omega+p)/p\omega]$ arises from the interference of the interaction and meson currents. H_1 and H_2 arise from the meson current contributions of Figs. 1(a) and (b), respectively.

[6] E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge University Press, New York, 1952), 4th ed., p. 321.

We give some simple results for the total cross section due to the interaction current *alone*. For the kinematical conditions considered in this paper the interaction current cross section will generally be roughly half of the total. The partial cross section at the low-energy pion region (high-energy photons) will, however, be given accurately by this result. We denote by $\sigma_{\rm int}(s\to p)$ the contribution of Fig. 1(c) and $\sigma_{\rm int}(p\to s)$ that of Fig. 1(d). If we use the sharp resonance approximation

$$\sigma_{33}(\omega)\approx\tfrac{1}{3}(32\pi^2f^2)p\delta(\omega-\omega_r), \quad (9)$$

then the process $s\to p$ gives ($f^2=0.08$)

$$\sigma_{\rm int}(s\to p)\approx(32\pi\alpha f^2E/3qW)\times(W-M-\omega_r)\theta(W-M-\omega_r). \quad (10)$$

In these equations ω_r is the pion energy at the 3-3 resonance and θ is the step function, $\theta(x)=1$, $x>0$, $\theta(x)=0$, $x<0$. Of course, Eq. (10) is a very poor approximation at and below the threshold $W=M+\omega_r$. Above the threshold the approximation (9) consistently overestimates the cross section by roughly 30% (see Ref. 2). The other contribution is

$$\sigma_{\rm int}(p\to s)=[\alpha\sigma_{33}(\omega_q)/q^3]I(\omega_q); \quad (11)$$

$$I(\omega_q)=\tfrac{1}{2}\omega_q[q\omega_q-\ln(q+\omega_q)]-\tfrac{1}{3}q^3, \quad (12)$$

where $\sigma_{33}=8\pi\sin^2\delta_q/q^2$.

These equations display correctly the qualitative result that $\sigma_{\rm int}(p\to s)$ is important only at energies less than or about equal to the 3-3 resonance energy. Clearly $\sigma_{\rm int}(s\to p)$ is large only above the resonance energy. A similar result holds for the corresponding meson current terms. Thus, for incident energies appreciably above the 3-3 resonance energies we expect to see the 3-3 resonance stand out clearly in the final state, as is observed.[1]

In all calculations we have used Eq. (2) rather than more transparent but less accurate approximations inherent in Eqs. (10) and (11). The numerical values of the 3-3 phase shift have been taken from Gell-Mann and Watson.[7]

III. RESULTS

In Figs. 2–4 the differential cross sections $d\sigma/d\omega_p$ are shown for incident pion lab kinetic energies of 200, 300, and 400 MeV. In Fig. 2 we have shown explicitly the contribution of the interaction current terms in order to illustrate their behavior at low energy. The arrows in Figs. 3 and 4 indicate the positions of the cutoffs due to the experimental restriction $E_\gamma>50$ MeV. The area under the curve to the left of this point is then the observed cross section. At 300 MeV the 3-3 isobar is clearly discernible, while at 400 MeV its appearance is very striking. The energy variation of the total cross section is shown in Fig. 5. The threshold for 50-MeV gamma rays is 58.7-MeV pion lab kinetic energy.

[7] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).

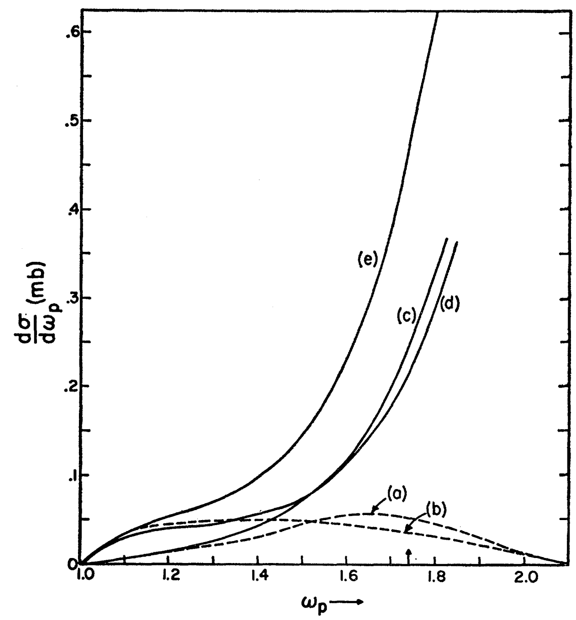

FIG. 2. The energy distribution of final pions is shown for pion lab kinetic energy of 200 MeV as a function of the total pion energy in pion mass units. (a) and (b) denote the contributions of the interaction current alone corresponding to Figs. 1(c) and (d), respectively. (c) and (d) represent the combined contribution of Figs. 1(a) and (c), and (b) and (d), respectively. The total is shown in the curve labeled (e). The arrow on the abscissa is the maximum pion energy allowed for a minimum photon energy of 50 MeV.

Naturally the low-energy behavior is extremely sensitive to the gamma-ray cutoff. The effect of using the correct flux is to lower the strictly static-nucleon theory cross section by 20 to 25%. Although the kinematical uncertainties in the theory might be as great as 20% the agreement with the experimental result seems quite satisfactory.

Deahl *et al.*[8] have found four events of the type $\pi^-+p\rightarrow\pi^-+p+\gamma$ ($E_\gamma>50$ MeV) for a lab pion energy of 224 MeV. The corresponding cross section is 0.04 mb.

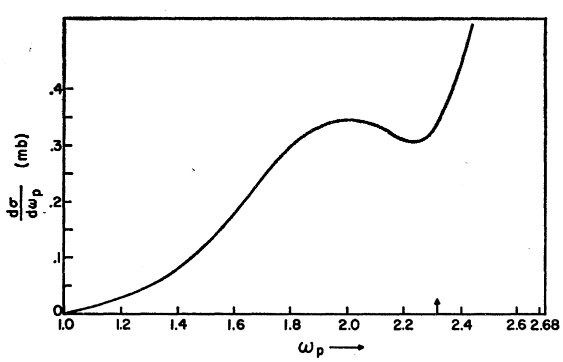

FIG. 3. The energy distribution of final pions is shown for pion lab kinetic energy of 300 MeV.

[8] J. Deahl, M. Derrick, J. Fetkovitch, T. Fields, and G. B. Yodh, Phys. Rev. **124**, 1987 (1961).

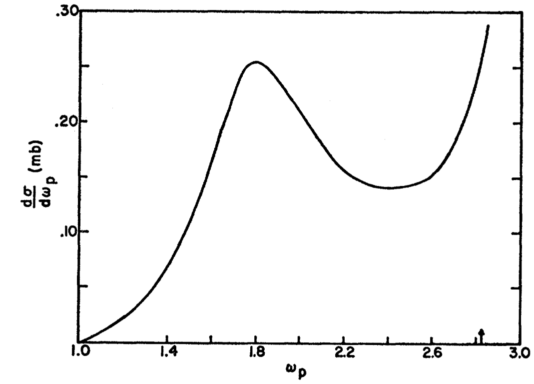

FIG. 4. The energy distribution of final pions is shown for pion lab kinetic energy of 400 MeV.

According to the model used here the $\pi^-+p\rightarrow\pi^-+p+\gamma$ cross section should be $\frac{1}{9}$ that of $\pi^++p\rightarrow\pi^++p+\gamma$. Thus we would predict 0.014 mb. As discussed by Cutkosky,[5] this value should be increased slightly by including properly the current of the proton. It is not clear whether any discrepancy can be claimed on the basis of only four events.

IV. DISCUSSION

Although the process under consideration has much intrinsic interest, we wish to emphasize a possibly more significant application as a probe of pion-nucleon dynamics. The traditional use of data on the photoproduction of pions from nucleons has been to elucidate the nature of the pion-nucleon interaction. Although it has proved possible to obtain qualitative information about the "higher resonances" in pion-nucleon scattering in this way the extra theoretical uncertainties in the photoproduction analysis have limited the unambiguous information obtainable by this technique.[9,10] It is somewhat surprising that more attention has not been given

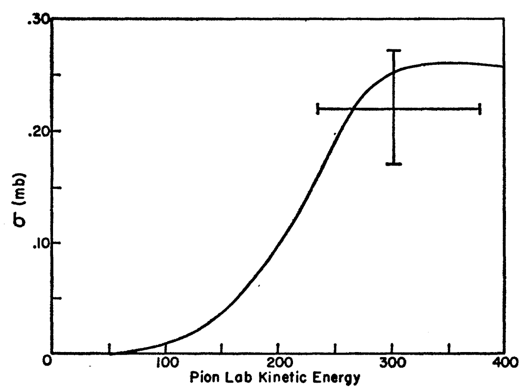

FIG. 5. The total theoretical cross section (for photons having energy greater than 50 MeV) is shown in the curve. The experimental point is taken from Ref. 1.

[9] R. F. Peierls, Phys. Rev. **118**, 325 (1960).
[10] G. Höhler and K. Dietz (to be published).

to radiative pion-nucleon scattering, in which more gentle photons are associated with the interesting phenomena in the resonance region. Although the cross section is typically less than one millibarn, present experimental techniques should suffice to give accurate information about this reaction. Certainly the static theory used in this paper is not adequate for the energies presently under discussion but we should remark that the development of a suitable theory does not appear to be difficult.

As an example, let us discuss the 900-MeV $F_{5/2}$ πN resonance. One might expect photon emission to be especially large at this resonance because of the large cross section for large-angle scattering. [For instance, we might note that the time corresponding to the width (100 MeV) of the resonance is just that required for an F-wave meson to go half a revolution about the nucleon at its classical impact parameter. On the basis of this simple picture, the large accelerations involved should emit substantial high-frequency radiation.] In the "mass spectrum" of the final pion-nucleon system, one expects to see the 600-MeV resonance ($D_{3/2}$), which could be reached by electric dipole emission and the 3-3 resonance, reached by magnetic dipole or electric quadrupole radiation (not to mention the decay $F_{5/2} \rightarrow$ nucleon+photon). Here the relative weight of the lower resonances is of interest. Even more informative (and difficult) would be accurate angular distribution data. This could be expected to tell, for example, something about the intricate mixture of amplitudes near the third resonance. It should be mentioned that Überall has investigated the bremsstrahlung in πN scattering for energies of several BeV and greater.[11] Another relevant proposal has been made by Yennie and Feshbach.[12]

[11] H. Überall, Phys. Rev. **126**, 861 (1962).

[12] D. R. Yennie and H. Feshbach, in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 219.

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Production of Hyperfragments by the Interactions of 1.5-GeV/c K^- Mesons in Lithium-Loaded Emulsions*

M. J. BENISTON, R. LEVI SETTI, W. PÜSCHEL,† AND M. RAYMUND

The Enrico Fermi Institute for Nuclear Studies and The Department of Physics, The University of Chicago, Chicago, Illinois

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Hyperfragment production by some 31 000 interactions of 1.5-GeV/c K^- mesons in Lithium-loaded $K5$ nuclear emulsions has been studied and compared with data from 0.8-GeV/c and existing data from 1.5-GeV/c K^- interactions in normal emulsions. The study of the prong number distributions of the hyperfragment parent stars provides a sensitive method for determining the production rates of hyperfragments by K^- interactions with light (C,N,0) and heavy (Ag,Br) emulsion nuclei; these production rates are found to be $(0.66\pm0.11)\%$ and $(5.20\pm0.20)\%$, respectively. An appreciable proportion of mesonic hyperfragments ($Z<6$) and Li^8 fragments have very short ranges ($R_{HF}\leq 10\,\mu$); this fact indicates the possibility of contaminations of "light" hypernuclides among the assumed mesic spallation hyperfragments. The predominant part of the hyperfragment production stars which shows the emission of "short" prongs involves the disintegration of heavy nuclei, thus indicating that Coulomb-barrier criteria cannot be used in discriminating among light or heavy hyperfragment parent stars at high K^- momenta. No double hyperfragment was observed. One K^- interaction emitted two hyperfragments decaying nonmesically. The π^+ decay of a ${}_{\Lambda}He^4$ hyperfragment has been found. An estimate of the branching ratio R of the π^+ decay and π^- decay modes for the ${}_{\Lambda}He^4$ hypernucleus gives $R\leq(2.7\pm1.1)\%$.

I. INTRODUCTION

THE production of hyperfragments by interactions of K^- mesons has been studied in nuclear emulsions at K^- momenta ranging from zero (absorptions at rest) up to 2.5 GeV/c.[1–6]

Characteristic of the experiments at high K^- momenta is the detection of large numbers of short-range hyperfragments attributed to heavy spallation products of silver and bromine nuclei recoiling after having

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† On leave from the Max-Planck-Institut für Physik, München, Germany.

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[5] J. Prem and P. H. Steinberg, presented at the International Conference on Hyperfragments, St. Cergue, Switzerland (unpublished).

[6] D. Abeledo, L. Choy, R. G. Ammar, N. Crayton, R. Levi Setti, M. Raymund, and O. Skjeggestad, Nuovo Cimento **22**, 1171 (1961).