Zero-Energy Intercepts of the ρ and R Trajectories, the pn Charge-Exchange Scattering, and the Difference in pp and np Total Cross Sections*

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(Received 30 December 1963)

It is shown that the existing high-energy data for the np charge-exchange differential cross section in the forward direction and the difference between np and pp total cross sections can be simultaneously explained in terms of two Regge trajectories. The two trajectories are the ρ and the R (using the notation of Pignotti in the preceding paper). The R has parity and G parity opposite to ρ but the same isotopic spin. These two sets of quantum numbers in the crossed channel are the only possible ones with nonvanishing contribution to the processes in question. The conjecture of real analyticity of the generalized coupling constants, together with the existing data, requires the existence of an R trajectory. In fitting the experimental data the intercepts are $\alpha_\rho(0)=0.57\pm0.1$ and $\alpha_R(0)=0.31\pm0.05$.

THE np charge-exchange experiment of Palevsky *et al.*[1] has received considerable attention from numerous authors. By assuming that only the ρ trajectory in the t channel is dominant, Muzinich[2] has obtained a rough fit to the narrow forward peak of the differential charge-exchange cross section at 2.85-BeV laboratory energy. However, Phillips subsequently showed that ρ exchange alone cannot explain simultaneously the energy dependence of $(\sigma_{pp}-\sigma_{np})$ and $d\sigma_{\text{c.e.}}(t=0)/d\Omega$.[3] He has also shown that experimental results contradict the real analyticity of the generalized coupling constants introduced by Gell-Mann[4] and by Gribov and Pomeranchuk.[5] Taking what amounts to a combination of a Regge-pole pion and a Regge-pole ρ, Islam and Preist have obtained a reasonable fit to the differential charge-exchange np cross section at 2.04 BeV.[6] However, their solution does not remove the difficulties pointed out by Phillips, because the pion contribution vanishes in the forward direction. The purpose of this paper is to show that by introducing the R trajectory of Pignotti,[7] we can explain the data and at the same time obtain the intercept values $\alpha(t=0)$ of the ρ and R trajectories. We shall also see that a second ρ trajectory, instead of R, does not suffice.

The possible quantum numbers of the particles which can be exchanged in the t channel for nucleon-nucleon scattering (s channel) have been tabulated by Muzinich.[8] It can be shown that of these twelve sets of quantum numbers only two give a nonvanishing contribution to the forward direction in the charge-exchange process. The two are

(i) ρ trajectory with $I=1$, $G=+1$, $P=-1$, and $\tau=-1$

and

(ii) R trajectory with $I=1$, $G=-1$, $P=+1$, and $\tau=+1$,

where I is the isospin, G is the G parity, P is the parity, and $\tau=(-1)^J$ is the signature.

We define a general two-pole, $I=1$ amplitude A at $t=0$ as

$$A=-\beta_1\frac{(2\alpha_1+1)}{\sqrt{s}}P_{\alpha_1}(-z)\frac{(1+\tau_1e^{-i\pi\alpha_1})}{\sin\pi\alpha_1}-\beta_2\frac{(2\alpha_2+1)}{\sqrt{s}}P_{\alpha_2}(-z)\frac{(1+\tau_2e^{-i\pi\alpha_2})}{\sin\pi\alpha_2}\,. \tag{1}$$

Here the Regge parameters β_1, β_2, α_1, and α_2 are evaluated at $t=0$, and we have $Z=(m+T)/m$ and $s=4m^2+2mT$, where T is the laboratory kinetic energy. Writing $\beta=Be^{i\pi\alpha}$, we have

$$A=-B_1\frac{(2\alpha_1+1)}{\sqrt{s}}P_{\alpha_1}(z)\frac{(1+\tau_1e^{-i\pi\alpha_1})}{\sin\pi\alpha_1}-B_2\frac{(2\alpha_2+1)}{\sqrt{s}}P_{\alpha_2}(z)\frac{(1+\tau_2e^{-i\pi\alpha_2})}{\sin\pi\alpha_2}\,, \tag{2}$$

where now B_1 and B_2 are real. The differential charge-exchange cross section in the forward direction is given by

$$d\sigma_{\text{c.e.}}(t=0)/d\Omega=|A|^2\,, \tag{3}$$

where Ω is the center-of-mass solid angle, and from the optical theorem we have

$$D\sigma\equiv\sigma_{pp}-\sigma_{np}=(4\pi/p)\,\text{Im}A\,, \tag{4}$$

where $p=(mT/2)^{1/2}$. From Eqs. (2), (3), and (4), the four real parameters α_1, B_1, α_2, and B_2 can be fitted to

* Work done under the auspices of the U. S. Atomic Energy Commission.

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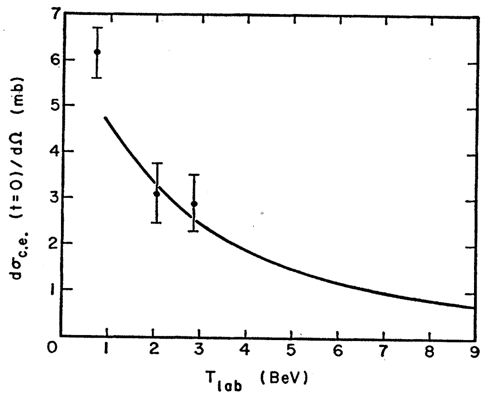

FIG. 1. The differential charge-exchange cross section in the forward direction versus laboratory energy.

the available experimental data of Palevsky *et al.*[1] and Diddens *et al.*[9]

Two solutions can be considered:

(1) One possibility is that both trajectories considered have the same quantum numbers as ρ. In this case the amplitude can be written as

$$A=-B_\rho\frac{(2\alpha_\rho+1)}{\sqrt{s}}P_{\alpha_\rho}(z)\frac{(1-e^{-i\pi\alpha_\rho})}{\sin\pi\alpha_\rho}-B_{\rho'}\frac{(2\alpha_{\rho'}+1)}{\sqrt{s}}P_{\alpha_{\rho'}}(z)\frac{(1-e^{-i\pi\alpha_{\rho'}})}{\sin\pi\alpha_{\rho'}}. \quad (5)$$

To explain the positive sign of $D\sigma$ between $T=1$ and $T=7$ BeV, we must take B_ρ and $B_{\rho'}$ to have opposite signs (B_ρ positive and $B_{\rho'}$ negative). This contradicts the real analyticity of the generalized coupling constants (see the arguments of Phillips[3]). However, the argument here does not deny existence of a ρ' trajectory. We shall return to this point later.

(2) The second possibility is to take a combination of the ρ trajectory [with the set of quantum numbers (i)] and the so called R trajectory [with the set of quantum numbers (ii)]. The R trajectory was proposed by Pignotti[7] in connection with SU_3 symmetry and the bootstrap mechanism. Here we point out that, aside from the SU_3 implications, it is the only other trajectory that contributes to np charge-exchange cross sections in the forward direction and, consequently, to $D\sigma$ through the optical theorem. In this case, the amplitude A is

$$A=-B_\rho\frac{(2\alpha_\rho+1)}{\sqrt{s}}P_{\alpha_\rho}(z)\frac{(1-e^{-i\pi\alpha_\rho})}{\sin\pi\alpha_\rho}-B_R\frac{(2\alpha_R+1)}{\sqrt{s}}P_{\alpha_R}(z)\frac{(1+e^{-i\pi\alpha_R})}{\sin\pi\alpha_R}, \quad (6)$$

a combination that is particularly suitable. As pointed out by Chew,[10] because ρ and R have opposite signatures we can fit the $D\sigma$ data taking B_ρ and B_R both positive provided that we have $\alpha_\rho>\alpha_R$. We should notice that experimentally $D\sigma$ is negative at low energies, becomes positive for laboratory momenta between 1.2 and 8 BeV/c, and then appears to become negative again.[9] Using Eqs. (3), (4), and (6), we vary the four parameters to fit simultaneously the $d\sigma_{\text{c.e.}}(t=0)/d\Omega$ data of Palevsky *et al.*[1] and the $D\sigma$ data of Diddens *et al.*[9]

The four parameters were fitted by numerical calculation with the help of the IBM-7094 computer of the Lawrence Radiation Laboratory. The results are:

$$\alpha_\rho=0.57\pm0.1, \quad \alpha_R=0.31\pm0.05, \quad B_\rho=0.8\pm0.2, \quad B_R=1.8\pm0.4. \quad (7)$$

The result for α_ρ is in fair agreement with the arguments of Ref. 11. Figure 1 shows the result of our fit $d\sigma_{\text{c.e.}}(t=0)/d\Omega$. The point at 710 MeV is from the charge-exchange experiment by Larsen.[12] We have not used this point in fitting the parameters. Figure 2 shows the fit to $D\sigma$ of Diddens *et al.*[9]

In this solution we have neglected the data below 2 BeV. To fit the $D\sigma$ data below 2 BeV, we would have to consider at least a third trajectory, a ρ'. This we have not done so far, one reason being that at these energies one may question the idea that the Regge poles of the t channel are dominant. The second reason for ignoring these low-energy points is that a third trajectory would allow a total of six parameters, and the present data are not sufficient for such an elaborate calculation. We could, of course, include $\sigma_{\bar{p}p}-\sigma_{pp}$ data as well; but then we should also have to include the ω trajectory. Such a possibility is subject to future investigation.

Finally, we remark that any future charge-exchange experiment should be valuable to our understanding of

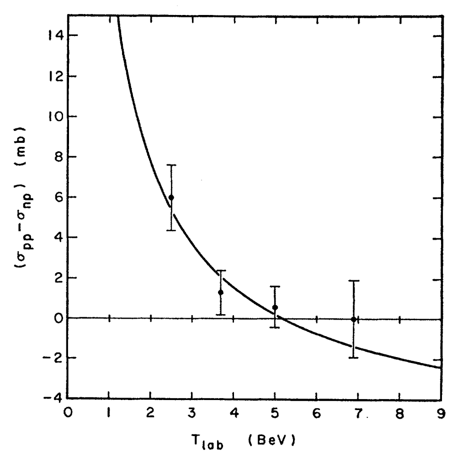

FIG. 2. The difference between pp and pn total cross sections versus laboratory energy.

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these trajectories. In particular, the measurement of high-energy pion-nucleon charge exchange should be encouraged. Because of G-parity conservation, the R trajectory would be absent here and, if the energy is high enough, the ρ alone should suffice.

ACKNOWLEDGMENTS

The author is indebted to Professor G. F. Chew for numerous helpful suggestions and encouragements. Thanks are also due to Dr. E. Leader and Dr. V. Teplitz and A. Pignotti for helpful discussions.

PHYSICAL REVIEW VOLUME 134, NUMBER 3B 11 MAY 1964

Determination of Pion-Nucleon S-Wave Scattering Lengths by the N/D Method

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(Received 5 December 1963)

The pion-nucleon S-wave scattering lengths are calculated using the method originally developed by Balázs, wherein an effective-range two-pole approximation is made for the numerator function. The residues of the effective-range poles are determined by matching the amplitude and its derivative with those calculated with a fixed energy dispersion relation. In calculating the latter the contribution of only the N^* and ρ, together with those of appropriate nucleon-pole terms, are retained. The calculated scattering length in the $T=\frac{3}{2}$ state is in excellent agreement with experimental result, while for the $T=\frac{1}{2}$ state, the calculated value, though of the right sign, is about twice the experimental scattering length.

SEVERAL approximation schemes[1–4] have recently been suggested to calculate the elements of the S matrix of strongly interacting particles. These attempts aim at constructing approximate solutions for the scattering amplitude consistent with the requirements of analyticity, elastic unitarity, and crossing symmetry. Among the methods, the one suggested by Balázs in which an effective-range approximation is made to represent the effect of the distant crossed-channel singularities, has the advantage of being free from the necessity of introducing arbitrary parameters into the theory. The Balázs method has been applied to the pion-nucleon problem by Singh and Udgaonkar[5] who have made a self-consistent calculation of the mass and width of the pion-nucleon (3,3) isobar, N^*. The present investigation which may be considered as a continuation of the work of these authors, is devoted to the study of the pion-nucleon S-wave amplitude using the aforementioned techniques. Our procedure is as follows: We use the N/D equations, and represent the N function by a two-pole effective-range formula. The residues of these poles, whose positions have been fixed *a priori*, are next evaluated by comparing the amplitude and its derivative at a suitably chosen point, with the values of the same quantities calculated with the help of a fixed energy dispersion relation. In calculating the latter only the contributions of N^* and ρ, together with those of the appropriate nucleon-pole terms are retained. In this way the partial-wave amplitude is completely determined and the S-wave scattering lengths may now be readily determined. The calculated scattering length in the $T=\frac{3}{2}$ state comes out to be in excellent agreement with the experimental result, while for the $T=\frac{1}{2}$ state the calculated scattering length, although of the right order and having the correct sign, is much too large. This may be due to our explicit neglect of the inelastic channels which are expected to be relatively more important in the $T=\frac{1}{2}$ state.

We follow the same notation as in Frautschi and Walecka.[6] Let us consider the S-wave amplitude f_{0+} normalized as

$$f_{0+}=\frac{W^2}{q}e^{i\delta}\sin\delta \tag{1}$$

and write it in the N/D form

$$f_{0+}=N(s)/D(s). \tag{2}$$

In (1), $W^2(=S)$ is the square of the total c.m. energy of the incoming particles and q the magnitude of c.m. 3-momentum. δ is the $S_{1/2}$ phase shift. In the two-pole effective-range approximation the $N(s)$ function may be written as[7]

$$N(s)=\frac{R_1}{s+m^2}+\frac{R_2}{s+16m^2}. \tag{3}$$

In (3), m is the nucleon mass. The pion mass has been

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