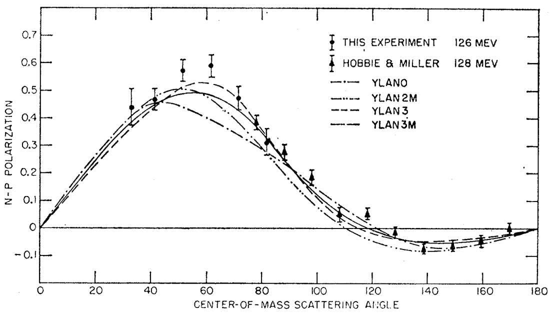

FIG. 7. The *n-p* polarization at 126 and 128 MeV (Ref. 1) in comparison to the Yale phase-shift predictions (Ref. 23). Not shown are solutions YLAN1 which follows the YLAN3M prediction and YLAN2 which is very similar to the prediction given by YLAN2M.

predictions, and seem to rule out the earlier solutions, YLAN0, YLAN2, and YLAN2M, at this energy. Of the remaining solutions, the polarization data at 126 MeV favor YLAN3 because of the higher peak in this prediction. In view of the discrepancies between the phase-parameter predictions and the earlier measurements,[21] however, these differences between the experimental points and the Yale phase-parameter curves should not be overemphasized. But it is interesting to note that the presently favored YLAN3M solution in this energy range was fitted to the *p-n* polarization derived from the inelastic *p-d* scattering experiment of Kuckes and Wilson at 143 MeV.[10] Theoretical corrections by Cromer and Thorndike[22] indicated that their peak polarization of 0.495 ± 0.017 should be raised by 0.03 ± 0.01 bringing it into closer agreement with the present experiment.

The measurement of the triple scattering parameter, D_t, at 128 MeV selects solution YLAN3M in preference to the other solutions, and since the YLAN3M solution, unlike the YLAN3 solution, can be joined smoothly to the quadrupole moment of the deuteron, we feel that the YLAN3M solution is an accurate representation of the data. However, modifications of the YLAN3M solution would result in better agreement with the double and triple scattering data at 126 MeV. With the differential cross section, polarization, and triple scattering data at this energy, the $T=0$ interaction should be well determined.

ACKNOWLEDGMENTS

We are grateful to the entire staff of the cyclotron laboratory for their assistance in making this experiment possible. The help of many research assistants in assembling apparatus and collecting data is gratefully acknowledged. One of us (A.S.C.) is grateful for the support of a National Science Foundation Graduate Fellowship for the years 1958 to 1962.

[22] A. Cromer and E. Thorndike (to be published).

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Tests of the One-Pion-Exchange Model

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The original Chew-Low proposal of a pion pole in the collision matrix at unphysical momentum transfer has evolved into a "generalized Born approximation" for one-pion exchange (OPE). This predicts that the collision amplitude will be independent of the total center-of-mass energy W for the reaction. The author describes tests of this prediction for the process $\pi+N\to N+2\pi$, where the final dipion mass is in the vicinity of the ρ resonance at 750 MeV, using data for incoming pion momenta of 1.4, 1.7, and 3.0 BeV/c. The results agree with the model except for a variation with W of the angular distribution of the dipion decay with respect to the incident π direction.

I. INTRODUCTION

IN recent years people have often tried to describe collisions of elementary particles by a "generalized Born approximation," that is, by writing a lowest order perturbation theory matrix element, in which the coupling constants at each vertex are replaced by "form factors," arbitrary functions of the invariants which may be formed from the four-momenta meeting at the vertex. As a rule, only one of the possible lowest order matrix elements is used because experimental precision is not yet great enough to justify a more elaborate analysis with superposition of several different terms in the collision amplitude. Thus, the method is only useful when one term seems to dominate the amplitude. I shall outline the development of one such model, the one-pion-exchange (OPE) model, for the process,

$$\pi^-+p\to\begin{cases}\pi^-+\pi^++n & \text{(1a)}\\ \pi^-+\pi^0+p\,, & \text{(1b)}\end{cases}$$

and describe some tests of the model. The tests are

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restricted to final dipion masses near the ρ resonance since events are too scarce away from the resonance. The methods used may be applied, with some modifications, to other models based on a generalized Born approximation.

II. THE ONE-PION-EXCHANGE MODEL

The following set of variables affords a convenient description of process 1, especially when the final pions come out in a resonant state. The initial and final nucleon states are specified by four-momenta p_i and p_f, and spinors u_i and u_f, respectively. The initial and final π^- are labeled by four-momenta p_1 and p_2, and the third pion, by p_3: $s=(p_1+p_i)^2$; $t=-(p_i-p_f)^2$; $M^2=(p_2+p_3)^2$; $x=\cos\theta=\hat{p}_1\cdot\hat{p}_2$, evaluated in the final dipion center-of-mass (c.m.) frame; $\varphi=\arccos\hat{n}_1\cdot\hat{n}_2$ where $\mathbf{n}_1=\mathbf{p}_1\times\mathbf{p}_2$ and $\mathbf{n}_2=\mathbf{p}_i\times\mathbf{p}_f$, all evaluated in the dipion c.m. frame.

The ordinary Born approximation for this process may be described by Feynman diagrams. The one assumed to dominate is shown in Fig. 1(a), which refers to pion production by exchange of a virtual pion, and corresponds to a production amplitude,

$$A_F=(\eta)^{1/2}Gu_f\gamma^5u_i[1/(t+\mu^2)]F\,, \qquad (2)$$

where G is the pion-nucleon coupling constant ($G^2/4\pi\approx15$), μ is the charged pion mass, and F is the coupling constant for π-π scattering. The factor $(\eta)^{1/2}$ comes from isotopic spin considerations. For charged π exchange one has $\eta=2$; for neutral π exchange, $\eta=1$.

Chew and Low[1] argued that, if all other variables were fixed and the production amplitude A continued in t to $t_0=-\mu^2$ (which would correspond to the physically impossible case of virtual exchange of a π having the mass of a free π), then in the neighborhood of t_0, A would be dominated by the pole term

$$A\approx(\eta)^{1/2}Gu_f\gamma^5u_i[1/(t+\mu^2)]a(M,x)\,, \qquad (3)$$

where $a(M,x)$ is the elastic π-π scattering amplitude. Averaging over initial and summing over final nucleon polarizations, gives the production cross section,

$$\Sigma(s,t,M^2,x)=\frac{d^3\sigma}{d\Omega dM^2dt}=\frac{\eta}{2\pi}\frac{G^2}{4\pi}\frac{1}{4m^2}\frac{1}{p_{1L}^2}\frac{t}{(t+\mu^2)^2}kM\sigma_{\pi\pi}(M,x)\,, \qquad (4)$$

where m is the mass of the nucleon, k is the momentum of the final π^- in the dipion c.m. ($k^2=M^2/4-\mu^2$), $\sigma_{\pi\pi}(M,x)$ is the elastic π-π scattering cross section for total c.m. energy M and scattering angle θ, and $d\Omega=dxd\varphi$. Finally, p_{1L} is the laboratory momentum of the incoming $\pi^-[4m^2p_{1L}^2=s^2-2s(m^2+\mu^2)+(m^2-\mu^2)^2]$. This is the only factor in the cross section which depends on s.

It is hard to make a rigorous comparison of the Chew-Low result with experiment, since one must extrapolate the data to an impossible value of t. One cannot do this reliably with the limited data available, especially since the function in question has a zero between t_0 and the physical region. Instead, people were tempted by these practical considerations to extrapolate the Chew-Low form to the physical region and to assume that it dominates the amplitude, at least for small t [see Fig. 1(b)]. For M in the neighborhood of the ρ resonance at 760 MeV, this assumption gave good agreement with the shape of the cross section plotted against t, for $t\lesssim10\mu^2$. However, the computed $\sigma_{\pi\pi}$ failed to reach the maximum associated with a P-wave resonance, falling short by about 30%.[2]

Nevertheless, the simplicity of the model made its rescue desirable. One could do this by assuming that the amplitude has an additional dependence on t, coming from the fact that the π-π scattering at physical values of t is "off the energy shell" because the exchanged π has an imaginary mass. A similar argument might apply to the form factor at the $NN\pi$ vertex (assumed constant above) and even to the pole factor, or propagator [see Fig. 1(c)].[3] This arbitrariness could be reduced by requiring consistency of the form factors for this process with those of other processes to which the OPE model may apply, but in this paper I only discuss the implications of the model with arbitrary form factors. In this form the model is an example of a generalized Born approximation. The resulting cross section is

$$\Sigma=p_{1L}^{-2}C(t,M^2,x)\,, \qquad (5)$$

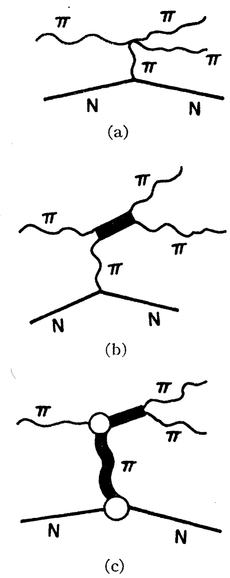


FIG. 1. (a) Feynman diagram for the one-pion-exchange contribution to the process $\pi+N\rightarrow2\pi+N$. (b) Symbolic expression for the Chew–Low form of OPE, when it is assumed to describe the amplitude in the physical region. The thick line represents the exact description of the elastic scattering in the 2π system. (c) Symbolic expression for the full "generalized Born approximation" for OPE. The open circles represent the additional arbitrary form factors at the vertices. The thick wavy line indicates possible modification of the pion propagator, or pole factor.

[1] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).

[2] A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters **6**, 628; **7**, 39 (E) (1961); E. Pickup, D. K. Robinson, and E. O. Salant, *ibid.* **7**, 192; 472 (E) (1961).

[3] F. Selleri, Phys. Letters **3**, 76 (1962). See this work for earlier references.

TABLE I. Ratio of neutral to charged dipion production.[a]

p_{1L} (BeV/c)	R	Range of M (MeV)	Approximate range of t [0.02 (BeV/c)^2]
1.38	2.4 ±0.3	680–830	3–10
1.59	1.70±0.26	700–810	2–8
3.0	1.93±0.27	720–805	$\frac{1}{2}$–10

[a] See Ref. 8.

where C is an arbitrary function. This yields the two unambiguous predictions: (a) Σ is independent of φ, (b) aside from a factor $1/p_{1L}^2$, Σ is independent of s. Treiman and Yang[4] were the first to state these facts.

As mentioned earlier, isotopic spin coupling coefficients at the nucleon vertex give charged pion exchange twice the weight of neutral pion exchange. In π-π scattering, the total cross section for $\pi^-+\pi^+\rightarrow\pi^-+\pi^+$ is

$$\sigma_{\pi^-\pi^+}=[1/(8\pi M)^2]\int d\Omega|(2/3)a_0+a_1+(1/3)a_2|^2, \quad (6a)$$

and for $\pi^-+\pi^0\rightarrow\pi^-+\pi^0$, it is

$$\sigma_{\pi^-\pi^0}=[1/(8\pi M)^2]\int d\Omega|a_1+a_2|^2, \quad (6b)$$

where a_I is the amplitude for total isotopic spin I. Bose statistics for pions imply a_I is even (odd) in x if I is even (odd).

Therefore we have the two results

$$\sigma_{\pi^-\pi^+}=\sigma_1+(1/9)(4\sigma_0+2\sigma_{02}+\sigma_2) \quad (7a)$$

$$\sigma_{\pi^-\pi^0}=\sigma_1+\sigma_2, \quad (7b)$$

where σ_{02} is an interference term, and the rest of the notation is obvious. The OPE model implies that the ratio of neutral to charged dipion production is

$$R=2\frac{\sigma_1+(1/9)(4\sigma_0+2\sigma_{02}+\sigma_2)}{\sigma_1+\sigma_2}. \quad (8)$$

Therefore, if even angular momentum contributions in the neighborhood of the P-wave ρ resonance are small, then we have $R\approx 2$. One might attempt a more complete analysis as follows. If the maximum (even) power of x in the dipion decay distribution is L, then there are $\bar{l}=L/2+[L/4]+2$ phase shifts $\delta(l,I)$ to be determined, with $0\leq l\leq L/2$ and $I=0$, 1, 2. (The quantity $[L/4]$ stands for the greatest integer in $L/4$.) On the other hand, there are L nontrivial coefficients in the $\pi^-\pi^+x$ distribution, and the same number in the $\pi^-\pi^0x$ distribution. For $L\geq 2$, the phase shifts are overdetermined even without using Eq. 8, and thus the OPE model might be tested by comparing the resulting value of R with experiment. This test has three limitations. First, the possibility of inelastic processes doubles the number of real constants to be determined. Secondly, there are large uncertainties in the coefficients of powers of x coming from statistical fluctuations of the limited data. These objections might be answered by more experiments. Thirdly, there is no reason why a phase-shift analysis should be justified when one of the incoming pions is virtual. This could account for deviations which are very hard to estimate. With these limitations in mind, one can only say that $R\approx 2$ is a comforting feature of the data presented below, though not vital for the OPE model, but R must be independent of s if OPE holds.

III. SUMMARY OF PREVIOUS TESTS OF OPE

A. The Treiman-Yang Angle

Pickup, Robinson, and Salant[5] examined the distribution of reaction 1(a) in φ, the Treiman-Yang angle. Within statistical uncertainty, they found isotropy in φ for all events together (within the ρ peak only, and for small t). However, when they divided the data for $\varphi<90°$ and $\varphi\geq 90°$, they observed a large change in the x distribution:

$$(F-B)/(F+B)=0.40\pm0.08 \quad (\varphi<90°)$$
$$(F-B)/(F+B)=0.08\pm0.09 \quad (\varphi\geq 90°),$$

where F is the number of events with $x\geq 0$, and B, with $x<0$, and $p_{1L}=1.38$ BeV/c.

At 1.59 BeV/c[6] and at 3.0 BeV/c[7] no statistically significant dependence on φ was found. Considering the earlier discussion, it is not surprising that the φ-dependence of reaction 1(a), a violation of the OPE model, should change with s, again violating the model.

B. The Ratio of Dipion Charge States

Table I shows the ratio R of the rate for neutral dipion production to that for negative dipion production as observed at several incoming π energies.[8] The result is compatible with OPE, although for a strict test the ranges in M and t should be the same for the three cases.

IV. TESTS OF THE ENERGY-DEPENDENCE IMPLICATIONS OF OPE

Equation 5 implies that $p_{1L}^2\Sigma(s,t,M^2,x)$ is independent of s. In principle one should test this statement for each s at every point in (t,M^2,x) space. However, the limited number of events available compels one to integrate Σ

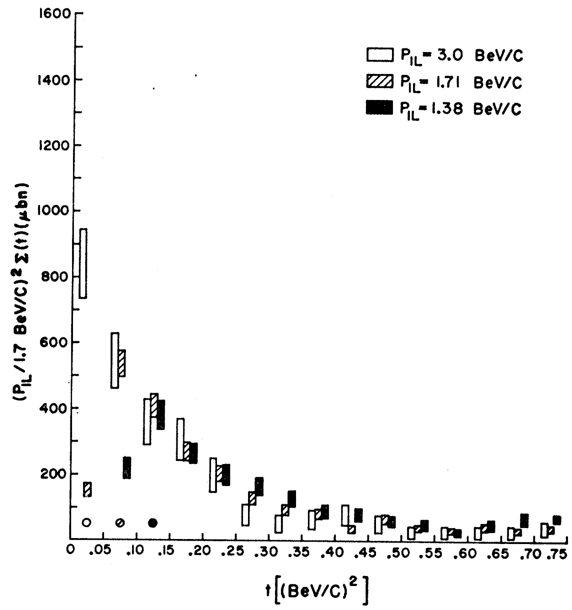
[4] S. B. Treiman and C. N. Yang, Phys. Rev. Letters **8**, 140 (1962).

[5] E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters **9**, 170 and 242 (E) (1962).

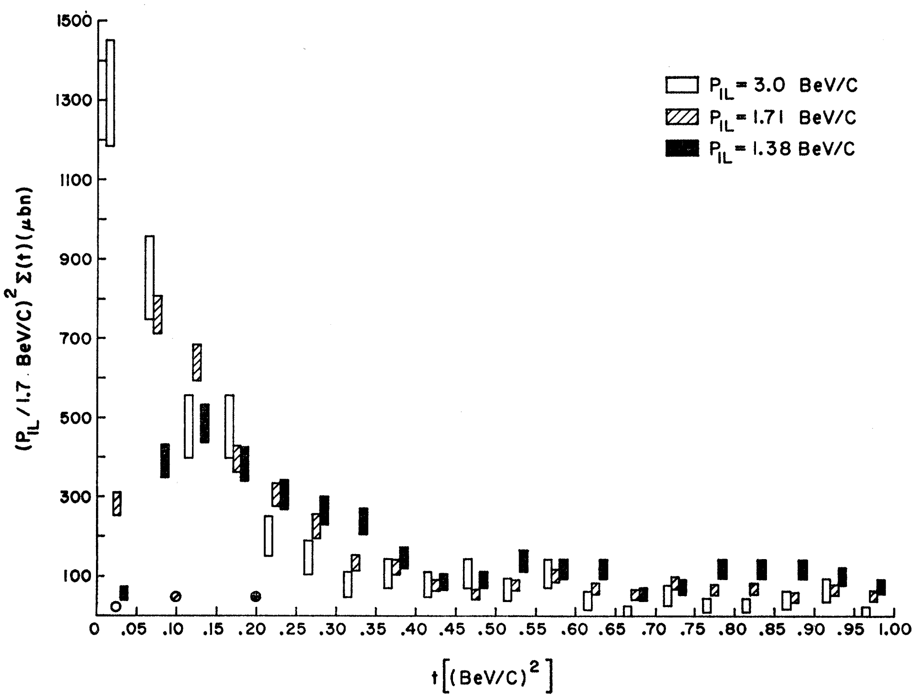
[6] Saclay–Orsay–Bari–Bologna collaboration, Nuovo Cimento **29**, 515 (1963).

[7] V. Hagopian and W. Selove, Phys. Rev. Letters **10**, 533 (1963) and private communication.

[8] The data of Table I are taken from the following sources: Pickup, *et al.*, Ref. 2 (1.38 BeV/c); Ref. 6 (1.59 BeV/c); V. Hagopian and W. Selove (private communication) (3.0 BeV/c).



(a)



(b)

FIG. 2. (a) Plots of $(p_{1L}/1.7\ \text{BeV}/c)^2\Sigma(t)$, where $\Sigma(t)$ is the cross section integrated over dipion decay angle, and over dipion masses between 700 and 840 MeV. The OPE model says this should be the same at all p_{1L}, as long as the M^2 integration range is the same. The bars are centered on the experimental values, and their vertical extents give their statistical uncertainty. The circles give the value of t at which some of the M^2 integration is cut off for the given p_{1L} by kinematic constraints. Each of the curves may be shifted up or down by five or ten percent because of experimental uncertainty in the normalization of the cross sections. (b) Same as case (a), except that here the M^2 integration extends from 600 to 900 MeV. See Ref. 9.

over large regions in two of the variables at a time in order to make meaningful comparisons of the distribution in the third variable. In carrying out integrations over t or M^2, one must remember an additional tacit dependence on s coming from momentum and energy conservation. For a given $M(t)$ the permissible range of $t(M)$ increases with s. For example, if one compares $\int_{t_1}^{t_2}\Sigma(M^2,t)dt$ at two values of s, one must observe the value of M for which the range (t_1,t_2) is no longer completely accessible at the lower s. For the dipion masses and momentum transfers of interest here, it is always high values of M and low values of t which begin to disappear at the lower s. In the figures these cutoff points are indicated for each s. Figures 2(a) and 2(b) give the t distributions for different ranges of M. Figure 3 gives the M^2 distributions for a single range

TABLE II. Angular distribution with respect to initial pion direction of final pion with same charge, in dipion c.m. frame.

p_{1L} (BeV/c)	Events per 100 in each x interval					Number of events	Range in M (MeV)	Approximate range in t [0.02 (BeV/c)^2]
(a) Neutral dipions[a]								
	−1 to −0.6	−0.6 to −0.2	−0.2 to 0.2	0.2 to 0.6	0.6 to 1			
1.38	25	12	3	20	40	120	700–800	3–8
1.59	10	14	13	18	45	174	710–810	3–8
1.71	17	9	12	25	37	292	700–800	2–8
3.0	18	7	12	22	41	154	720–805	$\frac{1}{2}$–10
	$\chi^2=27.1$ (12 degrees of freedom)							
(b) Singly charged dipions[b]								
1.25	29	20	6	15	29	412	735–810	4–8
1.59	25	19	7	14	35	223	710–810	2–5
3.0	28	12	12	15	33	80	720–805	$\frac{1}{2}$–10
	$\chi^2=8.5$ (8 degrees of freedom)							

[a] See Ref. 10.
[b] See Ref. 11.

in t.[9] Tables II(a)[10] and II(b)[11] give x distributions at different s for reactions (1a) and (1b), respectively.

Although there are suggestive trends in the figures, no violation of the OPE model appears there within experimental uncertainty. Furthermore, Table II(b) shows excellent agreement with the model. On the other hand, the $x=\cos\theta$ distributions of Table II(a) are incompatible at backward angles. Disagreement at the lowest energy is not surprising considering the result of the Treiman-Yang test there. Also the differences of range in M and t at the different s produce some uncertainty in the comparison. There is a strong dependence of the x distribution on M in the data for given energies.[12] To take a more positive view, the agreement of the data at 1.71 BeV/c with those at 3.0 BeV/c is quite striking, and these agree qualitatively with the data at 1.38 BeV/c. Perhaps the 1.59 BeV/c data indicate a departure from OPE for a small range of s.

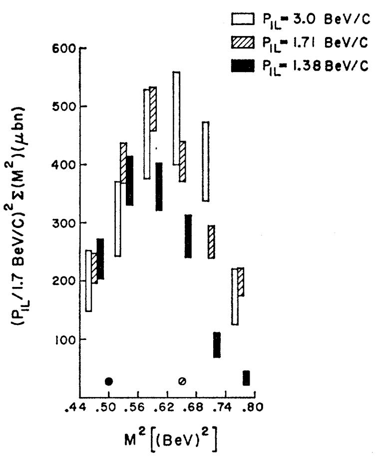


FIG. 3. Plots of $(p_{1L}/1.7\ \text{BeV}/c)^2\Sigma(M^2)$, where $\Sigma(M^2)$ is the cross section integrated over dipion decay angle, and over momentum-transfer-squared t between $2.5\,\mu^2$ and $12.5\,\mu^2$. The circles indicate the values of M above which some of the t integration is cut off at the given p_{1L}. (The circle for 3 BeV/c would be off the scale on the right.) See Ref. 9 and caption of Fig. 2.

V. DISCUSSION

The results quoted above show only two definite violations of the generalized Born approximation for OPE, the lack of isotropy in the Treiman-Yang angle at 1.38 BeV/c and the change with s in the back-angle π-π scattering distributions for the neutral dipion. Within the framework of the generalized Born approximation, one might be able to repair these deficiencies by superposing with the OPE amplitude other lowest-order diagrams, such as that for N^*(1238 MeV) production by ρ exchange (this particular diagram would be most important at the lowest s). Such a program may become worthwhile as further data accumulate and the restrictions on such additional amplitudes become definite. At present, the OPE model is consistent with the experiments quoted (for M near the ρ mass and $t\lesssim 10\mu^2$) as long as one integrates over solid angle for the dipion decay, and, at the higher two values of s consistent even without such integration.

ACKNOWLEDGMENTS

I wish to thank V. Hagopian, D. K. Robinson, and W. Selove for furnishing unpublished data for this analysis, and for helpful discussions, and S. B. Treiman for encouraging this work and for several vital suggestions.

[9] The results in Figs. 2 and 3 come from analysis of unpublished data of E. Pickup, W. J. Fickinger, D. K. Robinson, and E. O. Salant (1.38 and 1.71 BeV/c), and V. Hagopian and W. Selove (private communication) (3.0 BeV/c).

[10] The data in Table II(a) are taken from the following sources: Ref. 5 (1.38 BeV/c); Ref. 6 (1.59 BeV/c); D. K. Robinson (private communication) (1.71 BeV/c); V. Hagopian and W. Selove (private communication) (3.0 BeV/c).

[11] The data in Table II(b) are taken from the following sources: D. D. Carmony and R. T. Van de Walle, Phys. Rev. Letters **8**, 73 (1962), (1.25 BeV/c). [These data actually refer to $\pi^++p\rightarrow\pi^++\pi^0+p$, but if OPE is correct, this should have the same cross section as $\pi^-+p\rightarrow\pi^-+\pi^0+p$. Also, the data were weighted with the inverse of the momentum transfer factor in Eq. (4).] Saclay–Orsay–Bari–Bologna collaboration, Nuovo Cimento **25**, 365 (1962) (1.59 BeV/c). V. Hagopian and W. Selove (private communication) (3.0 BeV/c).

[12] This is shown by sources quoted in Refs. 7 and 11.