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Radiative Muon Capture in Hydrogen

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It is shown that $\Gamma_{\rm rad}$, the rate (in sec^{-1}) of the radiative muon capture reaction $\mu^-+p\rightarrow\gamma+\nu+n$, is very different in the two eigenstates of total $[\mu^-p]$ spin (hyperfine effect in radiative muon capture). It is further shown that, in each of these two eigenstates, $\Gamma_{\rm rad}$ depends sensitively on the magnitudeo f the pseudoscalar form factor. In particular for $g_P^{(\mu)}=8g_A^{(\mu)}$ (pseudoscalar form factor dominated by a single pion pole term) the ratio, $R=\Gamma_{\rm rad}/\Gamma_{\rm nonrad}$, of radiative to nonradiative muon capture rates is given by: $R_{\rm sing}=4.96\times10^{-3}/634=0.079\times10^{-4}$ (spin-singlet $[\mu p]$ atom); $R_{\rm trip}=9.00\times10^{-2}/13.3=69\times10^{-4}$ (spin-triplet $[\mu p]$ atom); $R_{\rm mixt}=6.87\times10^{-2}/168=4.1\times10^{-4}$ (statistical mixture of spin-singlet and spin-triplet $[\mu p]$ atoms); $R_{\rm molec}=3.07\times10^{-2}/560=0.55\times10^{-4}$ (spin-doublet ortho $[p\mu p]$ molecule). If, on the other hand, $g_P^{(\mu)}=16g_A^{(\mu)}$ (with the additional contribution to the pseudoscalar form factor taken to be effectively independent of momentum transfer for $|q^2|\lesssim m_\pi^2$), the ratios R become: $R_{\rm sing}=14.6\times10^{-3}/503=0.29\times10^{-4}$; $R_{\rm trip}=11.8\times10^{-2}/37.0=32\times10^{-4}$; $R_{\rm mixt}=9.19\times10^{-2}/154=6.0\times10^{-4}$; $R_{\rm molec}=4.72\times10^{-2}/452=1.04\times10^{-4}$, so that the sensitivity mentioned is exhibited.

INTRODUCTION

RADIATIVE muon capture has been treated theoretically by many investigators[1–10] and has recently been studied experimentally in Fe[11], Cu[12], and Ca[13]. In the present paper we consider radiative muon capture in hydrogen, investigating in particular the *hyperfine effect*, i.e., the effect of the muon-proton relative spin orientation on the radiative muon capture rate. Such a hyperfine effect is now well known to exist in the rate of nonradiative muon capture by any nucleus with a nonvanishing spin[14–22]; the importance of an analogous hyperfine effect in radiative muon capture by a proton was first noted by Dye, Sen, Ho, and Tzu.[5]

The existence of a large hyperfine effect in radiative muon capture may be understood physically as follows. Imagine the $[\mu p]$ atom initially in a spin-singlet state and let $\mu^-\rightarrow\mu^-+\gamma$ be the first step in the radiative process. After this photon emission, the $[\mu p]$ atom must be in a spin-triplet S state—an S state because otherwise the μ^- and p will never be in spatial coincidence and a spin-triplet state as the photon carries off one unit of angular momentum. As a consequence the $\mu^-+p\rightarrow n+\nu$ second step of the radiative muon capture reaction proceeds from a $[\mu p]$ spin-triplet state and is therefore characterized by a relatively small rate.[14–22] On the other hand, if the $[\mu p]$ atom is in a spin-singlet S state the rate of $\mu^-+p\rightarrow n+\nu$ proceeding from such a state is relatively large.[14–22] We thus anticipate, and indeed find (see below), that the rate of $\mu^-+p\rightarrow\gamma+\nu+n$ is appreciably larger from an initial $[\mu p]$ spin-triplet S state, than from an initial spin-singlet S state; it is clear that an analogous though smaller difference also exists between the rate of $\mu^-+X_Z^A\ (I=1/2)\rightarrow\gamma+\nu+X_{Z-1}^A$ from an initial $[\mu X_Z^A\ (I=1/2)]$ spin-triplet S state and from an initial $[\mu X_Z^A\ (I=1/2)]$ spin-singlet S state $[X_Z^A\ (I=1/2)=F_9^{19}$, for example$]$.

The radiative muon capture rate in each of the two hyperfine states is expected to depend considerably more sensitively on the magnitude of the pseudoscalar form factor than the corresponding nonradiative muon capture rate. According to an argument of Manacher and Wolfenstein,[6,7] this more sensitive dependence arises largely from the fact that (1) single-pion exchange between the leptons (μ,ν) and the baryons (p,n) con-

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tributes importantly to the pseudoscalar form factor, and (2) that in the second and third steps of the sequence

$$p \rightarrow p+\gamma;\quad p \rightarrow \pi+n;\quad \pi+\mu \rightarrow \nu$$

[see Fig. (1h)], the pion propagator equals

$$\begin{aligned}\{(p-k-n)^2+m_\pi{}^2\}^{-1} &= \{(\nu-\mu)^2+m_\pi{}^2\}^{-1}=\{(-m_\mu{}^2+2m_\mu|\mathbf{v}|)+m_\pi{}^2\}^{-1} \\ &\cong\{(m_\mu{}^2-2m_\mu|\mathbf{k}|)+m_\pi{}^2\}^{-1}\end{aligned}$$

and is consequently enhanced, relative to its nonradiative value, by a factor which, at the high-energy end of the photon spectrum, is

$$(m_\pi{}^2+m_\mu{}^2)/(m_\pi{}^2-m_\mu{}^2)=3.7\,.$$

THE MATRIX ELEMENTS AND DISTRIBUTION FUNCTIONS

The transition matrix element for nonradiative muon capture by a proton is taken to be

$$M_{\text{nonrad}}=\frac{G}{\sqrt{2}}(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_\mu)\times\left(\bar{u}_n\left[g_V{}^{(\mu)}\gamma_\lambda+g_M{}^{(\mu)}\sigma_{\lambda\rho}\frac{(p-n)_\rho}{2m_N}+g_A{}^{(\mu)}\gamma_5\gamma_\lambda-ig_P{}^{(\mu)}\gamma_5\frac{(p-n)_\lambda}{m_\mu}\right]u_p\right),\quad(1)$$

where [23,24,4]

$$G=1.01\times10^{-5}m_N{}^2;\quad g_V{}^{(\mu)}\cong0.97;$$
$$g_A{}^{(\mu)}\cong1.0,\quad g_A{}^{(e)}\cong-1.22;$$
$$g_M{}^{(\mu)}=g_V{}^{(\mu)}\times(\mu_p-\mu_n);$$

$(\mu_p-\mu_n)\equiv$nucleon isovector anomalous magnetic moment$=1.79-(-1.91)=3.7$;

$$g_P{}^{(\mu)}=m_\mu\left(\frac{fg}{(p-n)^2+m_\pi{}^2}+C\right)=m_\mu\left(\frac{fg}{0.9m_\mu{}^2+m_\pi{}^2}+C\right)\approx8g_A{}^{(\mu)}+m_\mu C;\quad(2)$$

$g\equiv$pion-nucleon strong coupling constant $\cong(4\pi\times14)^{1/2}$; $f\equiv[\pi\rightarrow\mu+\nu]$ decay constant, with magnitude determined by

$$\Gamma(\pi\rightarrow\mu+\nu)=\frac{G^2f^2}{16\pi}(m_\pi{}^2-m_\mu{}^2)^2\frac{m_\mu{}^2}{m_\pi{}^3}$$

and with

$$fg/g_A{}^{(\mu)}>0;$$

[23] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 355 (1958); L. Wolfenstein, Nuovo Cimento **8**, 882 (1958).

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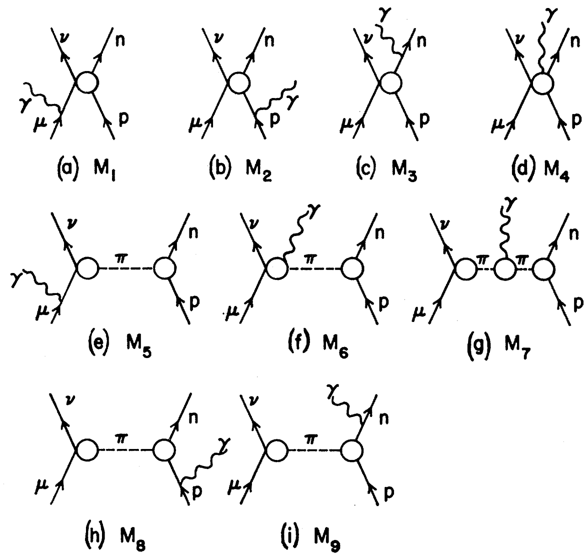

FIG. 1. The Feynman diagrams which correspond to the matrix elements employed in Eqs. (5a)–(5i).

p_λ, n_λ, μ_λ, $\nu_\lambda\equiv$proton, neutron, muon, neutrino momentum four-vectors.

It is to be noted that we suppose that only four form factors; polar vector ($g_V{}^{(\mu)}$), weak magnetism ($g_M{}^{(\mu)}$), axial vector ($g_A{}^{(\mu)}$) and pseudoscalar ($g_P{}^{(\mu)}$), are required to describe M_{nonrad}; this follows from the assumption that the hadronic polar-vector and axial-vector currents, $(j_V)_\lambda$ and $(j_A)_\lambda$, are, respectively, even and odd under isospace inversion[25]:

$$G(j_V)_\lambda G^{-1}=(j_V)_\lambda,\quad G(j_A)_\lambda G^{-1}=-(j_A)_\lambda\,.$$

The numerical value of $g_V{}^{(\mu)}$ and the relation between $g_M{}^{(\mu)}$ and $g_V{}^{(\mu)}$ follows from the assumption of the conservation of the hadronic polar-vector current[24,4]; this assumption also implies $G(j_V)_\lambda G^{-1}=(j_V)_\lambda$. The term $fg/[(p-n)^2+m_\pi{}^2]$ in $g_P{}^{(\mu)}$ exhibits explicitly the contribution of the single-pion exchange between the baryon and lepton to the pseudoscalar form factor[23]; the term C describes the corresponding contribution associated with the exchange of $0_1{}^{--}$ objects with mass$>3m_\pi$.

We proceed to set down the transition matrix elements for radiative muon capture by a proton. We have

$$M_{\text{rad}}=\frac{Ge}{\sqrt{2}}\sum_{\alpha=1}^{9}M_\alpha\,,\quad(3)$$

where, with

$$S^{(\alpha)}(q)\equiv\frac{i}{q\cdot\gamma-im_\alpha};\quad\Delta(q^2)\equiv\frac{1}{q^2+m_\pi{}^2};\quad(4)$$

$\epsilon_\lambda{}^*$ and $k_\lambda\equiv$photon polarization and photon-momentum

[25] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

four-vectors,

$$M_1=-i(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda S^{(\mu)}(\mu-k)\epsilon^*\cdot\gamma u_\mu)\left(\bar{u}_n\left[g_V{}^{(\mu)}\gamma_\lambda+g_M{}^{(\mu)}\sigma_{\lambda\rho}\frac{(p-n)_\rho}{2m_N}+g_A{}^{(\mu)}\gamma_5\gamma_\lambda\right]u_P\right), \tag{5a}$$

which corresponds to Fig. 1(a);

$$M_2=i(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_\mu)\left(\bar{u}_n\left[g_V{}^{(\mu)}\gamma_\lambda+g_M{}^{(\mu)}\sigma_{\lambda\rho}\frac{(p-k-n)_\rho}{2m_N}+g_A{}^{(\mu)}\gamma_5\gamma_\lambda\right]S^{(N)}(p-k)\left(\epsilon^*\cdot\gamma+i\frac{\mu_p k\cdot\gamma\epsilon^*\cdot\gamma}{2m_N}\right)u_p\right), \tag{5b}$$

which corresponds to Fig. 1(b);

$$M_3=i(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_\mu)\left(\bar{u}_n\frac{(i\mu_n k\cdot\gamma\epsilon^*\cdot\gamma)}{2m_N}S^{(N)}(n+k)\left[g_V{}^{(\mu)}\gamma_\lambda+g_M{}^{(\mu)}\sigma_{\lambda\rho}\frac{(p-k-n)_\rho}{2m_N}+g_A{}^{(\mu)}\gamma_5\gamma_\lambda\right]u_p\right), \tag{5c}$$

which corresponds to Fig. 1(c);

$$M_4=g_M{}^{(\mu)}(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_\mu)\left(\bar{u}_n\sigma_{\lambda\rho}\frac{\epsilon_\rho{}^*}{2m_N}u_p\right), \tag{5d}$$

which corresponds to Fig. 1(d).

The matrix elements M_1, M_2, M_3, M_4 describe the polar-vector, axial-vector, and weak-magnetism contributions to M_{rad}. The remaining matrix elements in M_{rad}: M_5, M_6, M_7, M_8, M_9 are pseudoscalar contributions and we write them in the approximation where we keep only the single-pion exchange terms ($\sim fg$). Then

$$M_5=fg(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda S^{(\mu)}(\mu-k)\epsilon^*\cdot\gamma u_\mu)(\mu-k-\nu)_\lambda\Delta((\mu-k-\nu)^2)(\bar{u}_n\gamma_5 u_p), \tag{5e}$$

which corresponds to Fig. 1(e);

$$M_6=-ifg(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_\mu)\epsilon_\lambda{}^*\Delta((\mu-k-\nu)^2)(\bar{u}_n\gamma_5 u_p), \tag{5f}$$

which corresponds to Fig. 1(f);

$$M_7=2ifg(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_\mu)(\mu-\nu)_\lambda\Delta((\mu-\nu)^2)(\mu-\nu)\cdot\epsilon^*\Delta((\mu-\nu-k)^2)(\bar{u}_n\gamma_5 u_p), \tag{5g}$$

which corresponds to Fig. 1(g);

$$M_8=-fg(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_\mu)(\mu-\nu)_\lambda\Delta((\mu-\nu)^2)\left(\bar{u}_n\gamma_5 S^{(N)}(p-k)\left(\epsilon^*\cdot\gamma+\frac{i\mu_p k\cdot\gamma\epsilon^*\cdot\gamma}{2m_N}\right)u_p\right), \tag{5h}$$

which corresponds to Fig. 1(h);

$$M_9=-fg(\bar{u}_\nu(1-\gamma_5)\gamma_\lambda u_\mu)(\mu-\nu)_\lambda\Delta((\mu-\nu)^2)\left(\bar{u}_n\mu_N\frac{ik\cdot\gamma\epsilon^*\cdot\gamma}{2m_N}S^{(N)}(n+k)\gamma_5 u_p\right), \tag{5i}$$

which corresponds to Fig. 1(i).

These matrix elements are similar to those used by Manacher.[7]

We note that M_{rad} is invariant under the gauge transformation $\epsilon_\rho{}^*\to\epsilon_\rho{}^*+\Lambda k_\rho$; in fact it is to ensure this invariance that the matrix elements M_4 and M_6 are adjoined to the other weak-magnetism and pseudoscalar matrix elements. We also note that using the Dirac equations for u_ν and u_μ we can express the whole pseudoscalar contribution to M_{rad}, i.e., $\sum_{\alpha=5}^{9}M_\alpha$, as a sum of matrix elements which correspond in form to the matrix elements which would be obtained with a γ_5 pseudoscalar coupling of the pion-lepton vertex; this result is in accordance with a theorem of Ruderman and Bludman.[26]

The expressions for M_{nonrad} and M_{rad} in Eqs. (1)–(5i) enable us to calculate the distribution functions and transition rates associated with nonradiative and radiative muon capture by a proton. The transition rate for nonradiative capture is given by

$$\Gamma_{\text{nonrad}}=\frac{G^2|\Phi(0)|^2}{2\pi}\frac{|\mathbf{v}|^2m_N}{m_N+m_\mu}$$
$$\times\left\{\left[(g_V{}^{(\mu)})^2+3(g_A{}^{(\mu)})^2+6\xi g_A{}^{(\mu)}(g_V{}^{(\mu)}-g_A{}^{(\mu)})+\frac{|\mathbf{v}|}{m_N}\big((g_V{}^{(\mu)}-g_A{}^{(\mu)})^2+2\xi(4g_A{}^{(\mu)}g_V{}^{(\mu)}-(g_A{}^{(\mu)})^2-(g_V{}^{(\mu)})^2)\big)\right]\right.$$
$$+(g_M{}^{(\mu)})^2 2\left(\frac{|\mathbf{v}|}{2m_N}\right)^2\left[1-\xi+\xi\frac{|\mathbf{v}|}{m_N}\right]+(g_P{}^{(\mu)})^2\left(\frac{|\mathbf{v}|}{2m_N}\right)^2$$

[26] M. Ruderman and S. Bludman, Phys. Rev. **101**, 910 (1956).

$$+\frac{|\mathbf{v}|}{2m_N}g_M{}^{(\mu)}\Big[-4g_A{}^{(\mu)}+\frac{|\mathbf{v}|}{m_N}g_V{}^{(\mu)}+\xi\Big(8g_A{}^{(\mu)}-4\frac{|\mathbf{v}|}{m_N}g_V{}^{(\mu)}-\frac{|\mathbf{v}|}{m_N}g_A{}^{(\mu)}-4g_V{}^{(\mu)}\Big)\Big]$$

$$-\frac{|\mathbf{v}|}{m_N}g_P{}^{(\mu)}\Big[(g_A{}^{(\mu)}-2\xi g_A{}^{(\mu)}+\xi g_V{}^{(\mu)})+\frac{|\mathbf{v}|}{2m_N}(g_A{}^{(\mu)}+3\xi g_V{}^{(\mu)})\Big]-g_P{}^{(\mu)}g_M{}^{(\mu)}\Big(\frac{|\mathbf{v}|}{m_N}\Big)^2\xi\Big[1-\frac{|\mathbf{v}|}{4m_N}\Big]\Big\}, \quad (6)$$

where $|\Phi(0)|^2$ is the probability of spatial coincidence of a muon and proton, and

$$\begin{aligned}\xi\equiv\tfrac{1}{3}\langle\boldsymbol{\sigma}_\mu\cdot\boldsymbol{\sigma}_p\rangle&=-1:\text{ spin-singlet }[\mu p]\text{ state}\\ &=\tfrac{1}{3}:\text{ spin-triplet }[\mu p]\text{ state}\\ &=0:\text{ statistical mixture of spin singlet and spin-triplet }[\mu p]\text{ states}\\ &=-\tfrac{2}{3}:\text{ spin-doublet ortho }[p\mu p]\text{ state.}^{4,27}\end{aligned} \quad (7)$$

Equations (6) and (7) for Γ_{nonrad} agree with previously obtained results.[4]

The distribution function in radiative muon capture for the emission of a photon with momentum in the range $d|\mathbf{k}|$ and solid angle $d\Omega_k$ and a neutrino in the solid angle $d\Omega_\nu$ works out to be

$$\omega(|\mathbf{k}|,\hat{p}\cdot\hat{k})d\Omega_\nu d\Omega_k d|\mathbf{k}|=\frac{G^2e^2}{4(2\pi)^5}\frac{|\Phi(0)|^2}{m_\mu^2}\frac{|\mathbf{v}|^2|\mathbf{k}|^2}{1-\mathbf{n}\cdot\hat{p}/m_N}\mathfrak{F}(|\mathbf{k}|,\hat{p}\cdot\hat{k})d\Omega_\nu d\Omega_k d|\mathbf{k}|, \quad (8)$$

where

$$\begin{aligned}&\mathfrak{F}(|\mathbf{k}|,\hat{p}\cdot\hat{k})\\ &=\Big\{[(g_V{}^{(\mu)})^2+3(g_A{}^{(\mu)})^2+((g_V{}^{(\mu)})^2-(g_A{}^{(\mu)})^2)\hat{p}\cdot\hat{k}+2\xi\{(g_A{}^{(\mu)})^2(1-\hat{p}\cdot\hat{k})-g_A{}^{(\mu)}g_V{}^{(\mu)}(1+\hat{p}\cdot\hat{k})\}]\\ &+\Big(\frac{fgm_\mu}{2m_N}\Big)^2[\Delta_2^2|\mathbf{n}|^2(1-\hat{p}\cdot\hat{k})+4m_\mu^2\Delta_1^2\Delta_2^2|\mathbf{v}|^2|\mathbf{n}|^2(1-(\hat{p}\cdot\hat{k})^2)+2m_\mu^2\Delta_1^2-2m_\mu\Delta_2^2\Delta_1|\mathbf{v}||\mathbf{n}|^2(1-(\hat{p}\cdot\hat{k})^2)\\ &+m_\mu\Delta_1\Delta_2(|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)+\xi\{|\mathbf{v}|(1-\hat{p}\cdot\hat{k})^2-2|\mathbf{k}|(1-\hat{p}\cdot\hat{k})\})-4m_\mu^2\Delta_1^2\Delta_2|\mathbf{v}|^2(1-(\hat{p}\cdot\hat{k})^2)]\\ &+2\Big(\frac{g_M{}^{(\mu)}}{2m_N}\Big)^2[(|\mathbf{v}|^2+|\mathbf{k}|^2)(1-\hat{p}\cdot\hat{k})-|\mathbf{v}||\mathbf{k}|(1-\hat{p}\cdot\hat{k})^2+\xi(|\mathbf{k}|+|\mathbf{v}|\hat{p}\cdot\hat{k})(|\mathbf{k}|-|\mathbf{v}|)(1-\hat{p}\cdot\hat{k})\\ &+m_\mu((|\mathbf{v}|-|\mathbf{k}|)(1-\hat{p}\cdot\hat{k})+\tfrac{1}{2}|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)+\xi\{(|\mathbf{v}|-|\mathbf{k}|)(1-\hat{p}\cdot\hat{k})-\tfrac{3}{2}|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)\})+m_\mu^2 2(1-\xi)]\\ &+2m_\mu\Big(\frac{m_\mu fg}{2m_N}\Big)[(-(g_A{}^{(\mu)}+\xi g_V{}^{(\mu)}+2\xi g_A{}^{(\mu)})|\mathbf{v}|^2-(g_A{}^{(\mu)}-\xi g_V{}^{(\mu)})|\mathbf{v}||\mathbf{k}|)(1-(\hat{p}\cdot\hat{k})^2)\Delta_1\Delta_2\\ &+(g_A{}^{(\mu)}(1-\hat{p}\cdot\hat{k})+\xi g_V{}^{(\mu)}(1+\hat{p}\cdot\hat{k})+2g_A{}^{(\mu)}\xi)\Delta_1]\\ &-2\Big(\frac{g_M{}^{(\mu)}}{2m_N}\Big)[2g_A{}^{(\mu)}((|\mathbf{v}|-|\mathbf{k}|)(1-\hat{p}\cdot\hat{k})+m_\mu+\xi\{(|\mathbf{v}|-|\mathbf{k}|)(1-\hat{p}\cdot\hat{k})-\tfrac{1}{2}|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)-m_\mu\hat{p}\cdot\hat{k}\})\\ &-2g_V{}^{(\mu)}\xi(\tfrac{1}{2}|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)+m_\mu(1+\hat{p}\cdot\hat{k}))]\\ &-2m_\mu\Big(\frac{g_M{}^{(\mu)}}{2m_N}\Big)\Big(\frac{fgm_\mu}{2m_N}\Big)[(\tfrac{1}{2}|\mathbf{v}|(1-\hat{p}\cdot\hat{k})^2-|\mathbf{k}|(1-\hat{p}\cdot\hat{k})+\xi\{(|\mathbf{k}|-|\mathbf{v}|)(1-\hat{p}\cdot\hat{k})+\tfrac{3}{2}|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)\})\Delta_2\\ &-|\mathbf{v}|^2\xi(1-(\hat{p}\cdot\hat{k})^2)(2m_\mu+|\mathbf{v}|+|\mathbf{k}|\hat{p}\cdot\hat{k})\Delta_1\Delta_2\\ &\quad+((|\mathbf{v}|-|\mathbf{k}|)(1-\hat{p}\cdot\hat{k})+\tfrac{1}{2}|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)+\xi\{(|\mathbf{v}|-|\mathbf{k}|)(1-\hat{p}\cdot\hat{k})+\tfrac{1}{2}|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)+4m_\mu\})\Delta_1]\Big\},\end{aligned} \quad (9)$$

where, by the conservation of momentum and energy

$$\begin{aligned}\mathbf{n}&=-(\mathbf{k}+\mathbf{v})\\ |\mathbf{v}|&\cong m_\mu-|\mathbf{k}|-|\mathbf{n}|^2/2m_N,\end{aligned} \quad (10)$$

and where

$$\Delta_1\equiv\Delta((\mu-\nu)^2),\quad \Delta_2\equiv\Delta((\mu-k-\nu)^2). \quad (11)$$

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[27] S. Weinberg, Phys. Rev. Letters 4, 575 (1960).

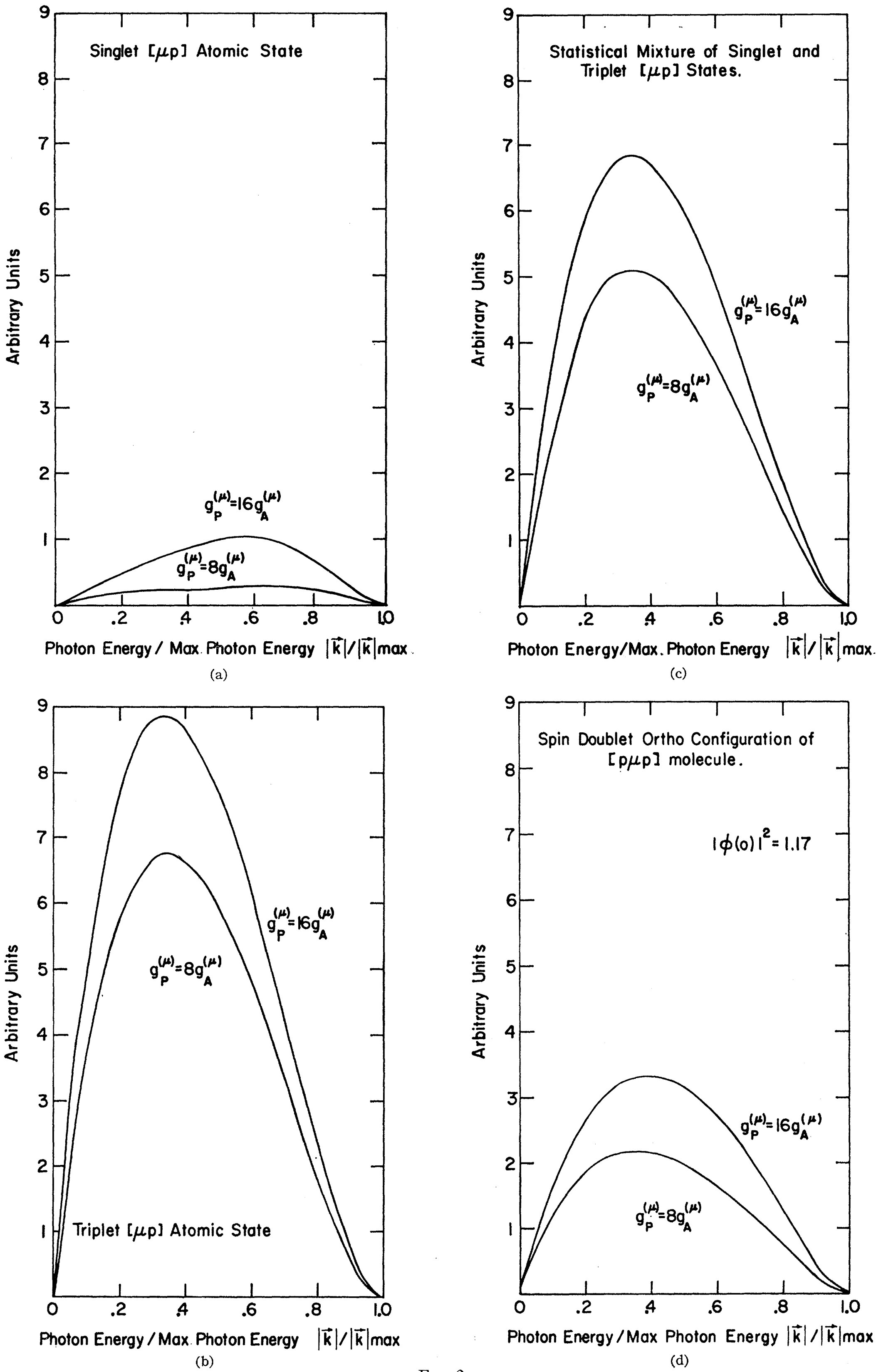
Singlet [μp] Atomic State
$g_P^{(\mu)}=16g_A^{(\mu)}$
$g_P^{(\mu)}=8g_A^{(\mu)}$
Arbitrary Units
Photon Energy / Max. Photon Energy |k⃗|/|k⃗|max
(a)
Statistical Mixture of Singlet and Triplet [μp] States.
$g_P^{(\mu)}=16g_A^{(\mu)}$
$g_P^{(\mu)}=8g_A^{(\mu)}$
Arbitrary Units
Photon Energy/Max. Photon Energy |k⃗|/|k⃗|max.
(c)
Triplet [μp] Atomic State
$g_P^{(\mu)}=16g_A^{(\mu)}$
$g_P^{(\mu)}=8g_A^{(\mu)}$
Arbitrary Units
Photon Energy / Max. Photon Energy |k⃗|/|k⃗|max
(b)
Spin Doublet Ortho Configuration of [pμp] molecule.
$|\phi(0)|^2=1.17$
$g_P^{(\mu)}=16g_A^{(\mu)}$
$g_P^{(\mu)}=8g_A^{(\mu)}$
Arbitrary Units
Photon Energy / Max Photon Energy |k⃗|/|k⃗|max
(d)

FIG. 2.

TABLE I. Transition rates for nonradiative and radiative muon capture by a proton in sec^{-1}.

	$g_P^{(\mu)}=8g_A^{(\mu)}$		$g_P^{(\mu)}=16g_A^{(\mu)}$	
	Γ_{nonrad}	Γ_{rad}	Γ_{nonrad}	Γ_{rad}
spin-singlet $[\mu p]$ atom	634	4.96×10^{-3}	503	14.6×10^{-3}
spin-triplet $[\mu p]$ atom	13.3	9.00×10^{-2}	37.0	11.8×10^{-2}
statistical mixture of spin-singlet spin-triplet $[\mu p]$ atoms	168	6.87×10^{-2}	154	9.19×10^{-2}
spin-doublet ortho $[p\mu p]$ molecule	560	3.07×10^{-2}	452	4.72×10^{-2}

In Eqs. (9) only the dominant terms have been exhibited, although in the numerical calculations terms of order (m_μ/m_N) higher than the dominant terms have been included (except the terms for weak-magnetism contributions). These terms are given in the Appendix.

It is to be noted that with neglect of all terms in (m_μ/m_N) and $(m_\mu/m_N)^2$.

$$\begin{aligned}\frac{1}{(4\pi)^2}\int\int\omega(|\mathbf{k}|,\hat{p}\cdot\hat{k})d\Omega_\nu d\Omega_k &\sim\frac{1}{2}\int\mathfrak{F}(|\mathbf{k}|,\hat{p}\cdot\hat{k})d(\hat{p}\cdot\hat{k})\\ &=((g_V^{(\mu)})^2+3(g_A^{(\mu)})^2+2\xi\{-g_V^{(\mu)}g_A^{(\mu)}+(g_A^{(\mu)})^2\})\\ &=(g_V^{(\mu)}+g_A^{(\mu)})^2; \qquad \xi=-1\text{: spin-singlet }[p\mu]\text{ state}\\ &=((g_V^{(\mu)}-(1/3)g_A^{(\mu)})^2+(32/9)(g_A^{(\mu)})^2; \quad \xi=\tfrac{1}{3}\text{: spin-triplet }[\mu p]\text{ state.}\end{aligned} \tag{12}$$

Thus, as anticipated from the qualitative discussion above, the radiative muon capture rate actually vanishes for the spin-singlet $[\mu p]$ state if the lepton-baryon weak coupling is characterized by $g_A^{(\mu)}=-g_V^{(\mu)}$ and by $g_M^{(\mu)}=0$, $g_P^{(\mu)}=0$ (so that $M_\alpha=0$ for $\alpha=4, 5, 6, 7, 8, 9$) and if M_2 and M_3 are dropped.

We have also studied the effect on Eqs. (5c)–(5i), (8), (9) of including the term in $g_P^{(\mu)}\sim C$. For the numerical evaluation of the photon spectrum and of Γ_{rad} we take $C=(fg)/(0.9m_\mu^2+m_\pi^2)$ so that $g_P^{(\mu)}=16g_A^{(\mu)}$. Such a value of $g_P^{(\mu)}>8g_A^{(\mu)}$ is suggested by the comparison of the theory with experiment in the radiative muon capture by Ca^{40}.[13] However, the effect of the term $\sim C$ on M_7 [Eq. (5g), Fig. 1(g)] is neglected since the (mass)^2 of the corresponding 0_1^{--} objects exchanged between the lepton and baryon is $>(3m_\pi)^2$ and appears twice as a factor in the denominator in M_7.

From Eqs. (8) and (9) we can calculate the photon momentum spectrum.

$$\begin{aligned}\omega(|\mathbf{k}|)d|\mathbf{k}| &= d|\mathbf{k}|\int\int\omega(|\mathbf{k}|,\hat{p}\cdot\hat{k})d\Omega_\nu d\Omega_k\\ &=8\pi^2 d|\mathbf{k}|\int_{-1}^{1}\omega(|\mathbf{k}|,\hat{p}\cdot\hat{k})d(\hat{p}\cdot\hat{k}).\end{aligned} \tag{13}$$

This integral is nontrivial since

$$\Delta_2\equiv[(\mu-K-\nu)^2+m_\pi^2]^{-1}\cong[m_\mu^2+2|\mathbf{k}|(m_\mu-|\mathbf{k}|)(\hat{p}\cdot\hat{k}-1)+m_\pi^2]^{-1} \tag{14}$$

and so depends substantially on $\hat{p}\cdot\hat{k}$ near the center of the photon spectrum. Finally the transition rate for radiative muon capture is given by

$$\begin{aligned}\Gamma_{rad}=\int_0^{|\mathbf{k}|_{max}}\omega(\mathbf{k})d|\mathbf{k}|; \quad |\mathbf{k}|_{max} &= m_\mu-\frac{|\mathbf{k}|_{max}^2}{2m_N}\\ &\cong m_\mu-\frac{m_\mu^2}{2m_N}.\end{aligned} \tag{15}$$

Numerical values obtained on the basis of Eqs. (6) and (7) for Γ_{nonrad}, and Eqs. (8)–(11), (13), (15) for $\omega(\mathbf{k})$ and Γ_{rad}, are set forth in Table I and Fig. 2.

FIG. 2. The photon-energy spectra correspond to (a) atomic $[\mu p]$ spin-singlet state, (b) atomic $[\mu p]$ spin-triplet state, (c) statistical mixture of atomic $[\mu p]$ spin-singlet and spin-triplet states, (d) spin-doublet ortho $[p\mu p]$ molecule with $|\Phi(0)|_{p\mu p}^2=2\times0.585=1.17$. The case $g_P^{(\mu)}=8g_A^{(\mu)}$ corresponds to Eq. (2) with $C=0$, i.e., only the single-pion exchange term present in the pseudoscalar interaction. The case $g_P^{(\mu)}=16g_A^{(\mu)}$ corresponds to Eq. (2) with $m_\mu C=8g_A^{(\mu)}$; this part of the form factor is essentially momentum-transfer independent and corresponds to the exchange of 0_1^{--} objects of higher mass ($\geq 3m_\pi$). The same arbitrary ordinate scale is used in all four graphs.

We have also investigated the γ-n correlation function; unfortunately this is dominated by phase space ($\mathbf{n}\approx-\mathbf{k}$ for high-energy photons) and as a result is not very sensitive to $g_P^{(\mu)}$.

DISCUSSION OF RESULTS

The most outstanding feature of the results is the anticipated large and sensitive-to-$g_P^{(\mu)}$, hyperfine effect in the transition rates for radiative muon capture by a proton. Thus, from Table I, we see that (all transition rates Γ in sec^{-1})

$$\frac{\Gamma_{\text{rad}}(\text{spin-triplet }[\mu p]\text{ atom})}{\Gamma_{\text{rad}}(\text{spin-singlet }[\mu p]\text{ atom})}=\begin{matrix} 9.00\times10^{-2}/4.96\times10^{-3}=18, & g_P^{(\mu)}=8g_A^{(\mu)} \\ 11.8\times10^{-2}/14.6\times10^{-3}=8.1, & g_P^{(\mu)}=16g_A^{(\mu)}\end{matrix} \tag{16}$$

$$R_{\text{mixt}}=\frac{\Gamma_{\text{rad}}(\text{mixt})}{\Gamma_{\text{nonrad}}(\text{mixt})}=\begin{matrix} 6.87\times10^{-2}/168 & =4.1\times10^{-4}, & g_P^{(\mu)}=8g_A^{(\mu)} \\ 9.19\times10^{-2}/154 & =6.0\times10^{-4}, & g_P^{(\mu)}=16g_A^{(\mu)},\end{matrix} \tag{17}$$

where mixt$\equiv$statistical mixture of spin-singlet, spin-triplet $[\mu p]$ atoms; we note that if all terms in (m_μ/m_N), $(m_\mu/m_N)^2$ are neglected, i.e., if $g_P^{(\mu)}=0$, $g_M^{(\mu)}=0$, (so that $M_\alpha=0$, $\alpha=4, 5, 6, 7, 8, 9$) and if M_2 and M_3 are dropped

$$R_{\text{mixt}}=\alpha/12\pi=1.9\times10^{-4}.$$

Further

$$\begin{aligned} R_{\text{molec}}=\Gamma_{\text{rad}}(\text{molec})/\Gamma_{\text{nonrad}}(\text{molec})&=3.07\times10^{-2}/560=0.55\times10^{-4}, \quad g_P^{(\mu)}=8g_A^{(\mu)} \\ &=4.72\times10^{-2}/452=1.04\times10^{-4}, \quad g_P^{(\mu)}=16g_A^{(\mu)},\end{aligned} \tag{18}$$

where molec$\equiv$spin-doublet ortho $[p\mu p]$ molecule.[4,27] These results agree with the calculations of Manacher.[7]

It is to be emphasized that both $\Gamma_{\text{rad}}(\text{molec})$ and $\Gamma_{\text{nonrad}}(\text{molec})$ are proportional to the *same* $|\Phi(0)|_{p\mu p}{}^2$ so that R_{molec} is independent of $|\Phi(0)|_{p\mu p}{}^2$. Since the present discrepancy between the predicted $\Gamma_{\text{nonrad}}(\text{molec})=560\ \text{sec}^{-1}$ ($g_P^{(\mu)}=8g_A^{(\mu)}$) and the observed[28] $\Gamma_{\text{nonrad}}(\text{molec})=(450\pm50)\ \text{sec}^{-1}$ may well be due to a miscalculation of $|\Phi(0)|_{p\mu p}{}^2$ [29] a measurement of $\Gamma_{\text{rad}}(\text{molec})$, and so an experimental determination of $R_{\text{rad}}(\text{molec})$, would be very helpful.

The predicted gamma-ray spectra for $g_P^{(\mu)}=8g_A^{(\mu)}$ and $g_P^{(\mu)}=16g_A^{(\mu)}$ are given in Fig. 2 for the various atomic $[\mu p]$ and molecular $[p\mu p]$ states.[30]

Of course, the predicted value of $\Gamma_{\text{rad}}(\text{molec})$, $3\times10^{-2}\ \text{sec}^{-1}$, is very small. However, with the hoped for advent in the not-too-distant future of very-high-intensity (pion factory produced) muon beams, one may contemplate a situation where, say, 10^6 negative muons per sec stop[31] in a liquid hydrogen target. Under these circumstances and with the above value of $\Gamma_{\text{rad}}(\text{molec})$ we can anticipate that a radiative muon capture event in liquid hydrogen will occur, on the average, every 15 sec.

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APPENDIX

In Eq. (9), the expression for $\mathfrak{F}(|\mathbf{k}|,\hat{p}\cdot\hat{k})$, terms of order m_μ/m_N higher than the dominant terms have been omitted. These terms are not unimportant numerically and have been included in the calculations, with the exception of those terms associated with the small weak magnetism contributions.

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[28] J. E. Rothberg, E. W. Anderson, E. J. Bleser, L. M. Lederman, S. L. Meyer, J. L. Rosen, and I. T. Wang [Phys. Rev. **132**, 2664 (1963)] give (464±42) sec^{-1}. R. H. Hildebrand [Bull. Am. Phys. Soc. **8**, 512 (1963)] gives (410±90) sec^{-1}. C. Rubbia [International Conference on Weak Interactions, Brookhaven, September 1963 (unpublished)] gives (450±50) sec^{-1}. The last two measurements are for 80% molec+20% spin-singlet atom.

[29] We used $|\Phi(0)|_{p\mu p}{}^2/|\Phi(0)|_{\mu p}{}^2=2\times0.585$ as given in Ref. 27 to calculate the numerical values of $\Gamma_{\text{nonrad}}(\text{molec})$ and $\Gamma_{\text{rad}}(\text{molec})$ in the last row of Table I or in Eq. (18).

[30] It is conceivable that nonradiative muon capture experiments will finally confirm $g_P^{(\mu)}=8$ while a better fit of the radiative muon capture data will ultimately be given by $g_P^{(\mu)}>8$. Such a situation will imply that the version of relativistic "perturbation" theory used in this paper to estimate the transitions associated in particular with diagrams $1d(M_4)$, $1g(M_7)$ is basically inadequate and that improved methods including a systematic treatment of multiple-pion exchange in diagrams $1d(M_4)$ and $1g(M_7)$ must be devised to calculate the structure dependent radiation. Some consideration of these questions have been given by Manacher (Ref. 7) on the basis of the static model description of the pion-nucleon interaction.

[31] E. R. Beringer, W. A. Blanpied, R. L. Gluckstern, V. W. Hughes, H. B. Knowles, S. Ohnuma, and G. W. Wheeler, International Conference on Sector Focussed Cyclotrons and Meson Factories CERN, 1963 (unpublished).

Let us denote these smaller terms by $\mathfrak{F}'(|\mathbf{k}|,\hat{p}\cdot\hat{k})$. We then have

$$
\begin{aligned}
&\mathfrak{F}'(|\mathbf{k}|,\hat{p}\cdot\hat{k})\\
&=\frac{m_\mu}{m_N}\Bigg[((g_V^{(\mu)})^2+(g_A^{(\mu)})^2)\bigg(\frac{(|\mathbf{v}|+|\mathbf{k}|)}{m_\mu}(1+\hat{p}\cdot\hat{k})+\xi\frac{|\mathbf{v}|}{m_\mu}(1-(\hat{p}\cdot\hat{k})^2)\bigg)-\xi g_V^{(\mu)}g_A^{(\mu)}(1+\hat{p}\cdot\hat{k})\frac{(|\mathbf{k}|+|\mathbf{v}|\hat{p}\cdot\hat{k})}{m_\mu}\\
&\quad-g_V^{(\mu)}g_A^{(\mu)}\bigg(2\frac{(|\mathbf{v}|-|\mathbf{k}|)}{m_\mu}(1-\hat{p}\cdot\hat{k})+\xi\frac{\{2|\mathbf{v}|-|\mathbf{k}|+3|\mathbf{k}|\hat{p}\cdot\hat{k}-|\mathbf{v}|\hat{p}\cdot\hat{k}(1-\hat{p}\cdot\hat{k})\}}{m_\mu}\bigg)\\
&\quad+(g_A^{(\mu)})^2\big((1-\hat{p}\cdot\hat{k}+2\xi)+(1+\mu_p)(2+\xi(1-\hat{p}\cdot\hat{k}))\big)-2g_V^{(\mu)}g_A^{(\mu)}\big((1-\xi\hat{p}\cdot\hat{k})+(1+\mu_p)(1+\xi+\xi(1-\hat{p}\cdot\hat{k}))\big)\\
&\quad+(g_V^{(\mu)})^2(1+2\xi-\xi(1+\mu_p))(1+\hat{p}\cdot\hat{k})+(g_V^{(\mu)}+g_A^{(\mu)})\mu_n(g_V^{(\mu)}\xi(1+\hat{p}\cdot\hat{k})+g_A^{(\mu)}(2-\xi-3\xi\hat{p}\cdot\hat{k}))\Bigg]\\
&\quad+\mu_n^2\left(\frac{m_\mu}{2m_N}\right)^2[(g_V^{(\mu)})^2+6g_V^{(\mu)}g_A^{(\mu)}\xi+3(g_A^{(\mu)})^2(1-2\xi)]\\
&\quad+\frac{m_\mu}{m_N}\left(\frac{fgm_\mu}{2m_N}\right)^2\bigg[-\frac{(\mu_p+1-\mu_n)}{2}\xi|\mathbf{v}|(|\mathbf{v}|+|\mathbf{k}|)(1-(\hat{p}\cdot\hat{k})^2)\Delta_1\Delta_2\\
&\quad-\frac{(\mu_p+1+\mu_n)}{2}\xi\{|\mathbf{v}|^2(1-\hat{p}\cdot\hat{k})^2+2|\mathbf{k}|^2(1-\hat{p}\cdot\hat{k})+|\mathbf{v}||\mathbf{k}|(3\hat{p}\cdot\hat{k}-1)(1-\hat{p}\cdot\hat{k})\}\Delta_1\Delta_2\\
&\quad+2(\mu_p+1+\mu_n)m_\mu(|\mathbf{k}|+|\mathbf{v}|\hat{p}\cdot\hat{k})\Delta_1^2\bigg]\\
&\quad+\frac{m_\mu}{m_N}\left(\frac{fgm_\mu}{2m_N}\right)\Delta_1[(\mu_p+1-\mu_n)\tfrac{1}{2}|\mathbf{v}|(g_V^{(\mu)}-\xi g_A^{(\mu)})(1-(\hat{p}\cdot\hat{k})^2)\\
&\quad+(\mu_p+1+\mu_n)\{(\xi g_V^{(\mu)}+g_A^{(\mu)}(1+2\xi))(|\mathbf{k}|+\hat{p}\cdot\hat{k}|\mathbf{v}|)+\tfrac{1}{2}(\xi g_V^{(\mu)}-g_A^{(\mu)})(|\mathbf{v}|+2|\mathbf{k}|\hat{p}\cdot\hat{k}+|\mathbf{v}|(\hat{p}\cdot\hat{k})^2)\}]\\
&\quad+\frac{m_\mu}{m_N}\left(\frac{fgm_\mu}{2m_N}\right)[\Delta_1\Delta_2|\mathbf{v}||\mathbf{n}|^2(1-(\hat{p}\cdot\hat{k})^2)(\xi g_V^{(\mu)}-g_A^{(\mu)})\\
&\quad+\tfrac{1}{2}\Delta_1\{g_A^{(\mu)}\big(|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)+\xi\{2(|\mathbf{k}|+|\mathbf{v}|)(1+\hat{p}\cdot\hat{k})-|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)\}\big)\\
&\quad+g_V^{(\mu)}\big(2(|\mathbf{k}|-|\mathbf{v}|)(1-\hat{p}\cdot\hat{k})+|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)+\xi\{4(|\mathbf{k}|+|\mathbf{v}|\hat{p}\cdot\hat{k})-|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)\}\big)\}\\
&\quad+\Delta_2\{\tfrac{1}{2}(g_V^{(\mu)}+g_A^{(\mu)})(2+\mu_p)(1-\xi)|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)\\
&\quad+\{(g_A^{(\mu)}+(1+\mu_p)g_V^{(\mu)})\xi-(g_V^{(\mu)}+(1+\mu_p)g_A^{(\mu)})\}(|\mathbf{v}|-|\hat{k}|)(1-\hat{p}\cdot\hat{k})\}\\
&\quad+2m_\mu\Delta_1\Delta_2|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)(|\mathbf{v}|+(1+\mu_p)|\mathbf{k}|)(\xi g_V^{(\mu)}-g_A^{(\mu)})+2m_\mu\Delta_1(1-(1+\mu_p)\hat{p}\cdot\hat{k})(g_A^{(\mu)}-\xi g_V^{(\mu)})\\
&\quad-\mu_n\Delta_2\{\tfrac{1}{2}g_V^{(\mu)}\big(|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)(1-\xi)+2\xi(|\mathbf{v}|-|\mathbf{k}|)(1-\hat{p}\cdot\hat{k})\big)\\
&\quad+g_A^{(\mu)}\big((|\mathbf{v}|-|\mathbf{k}|)(1-\hat{p}\cdot\hat{k})(1-2\xi)-\tfrac{1}{2}|\mathbf{v}|(1-(\hat{p}\cdot\hat{k})^2)(1-5\xi)\big)\}\\
&\qquad\qquad+2m_\mu\mu_n\Delta_1\Delta_2|\mathbf{v}||\mathbf{k}|(1-(\hat{p}\cdot\hat{k})^2)(g_A^{(\mu)}(2\xi-1)-\xi g_V^{(\mu)})+2m_\mu\mu_n\Delta_1\hat{p}\cdot\hat{k}((2\xi-1)g_A^{(\mu)}-\xi g_V^{(\mu)})].
\end{aligned}
$$

It will be observed that a term of order $(m_\mu/m_N)^2$ associated with radiation by the magnetic moment of the neutron has been retained. This term is significant numerically in the singlet $[\mu p]$ configuration by virtue of the fact that its coefficient is $(g_V^{(\mu)}-3g_A^{(\mu)})^2\approx 22$, a large coefficient.